Correspondence

Comments on “Synthesis of Adaptive Monopulse Patterns”

Junhao Xie

In the recent paper,[1] Fante determines the weight vector w_Δ for the difference beam subject to the constant slope of the ratio Δ/Σ. The method is quite general in which the sum/difference beam weight vector should be simplified further as follows.

The weight vectors for the sum beam and difference beam in the paper[1] are given, respectively,

$$w = \frac{\Phi^{-1}s^*}{s^T\Phi^{-1}s^*} \tag{1}$$

$$w_\Delta = \Phi^{-1}H^*(H^T\Phi^{-1}H^*)^{-1}\rho \tag{2}$$

where

$$H^T = \begin{bmatrix} g^T(\theta_0 + \Delta\theta,\ f_0) \\ g^T(\theta_0,\ f_0) \\ g^T(\theta_0 - \Delta\theta,\ f_0) \end{bmatrix}$$

$$\rho = k_s \begin{bmatrix} w^T g(\theta_0 + \Delta\theta,\ f_0) \\ 0 \\ w^T g(\theta_0 - \Delta\theta,\ f_0) \end{bmatrix} \Delta\theta.$$

Since $1/s^T\Phi^{-1}s^*$ in (1) is a scalar, it can be denoted by μ; thus, the sum beam weight vector can be written as

$$w = \mu\overline{w} = \mu\Phi^{-1}s^*. \tag{3}$$

Equation (3) is actually the original expression in [1] in which μ is a nonzero complex number. Following the same procedure with the above formula, the corresponding difference beam weight vector $\overline{w}_\Delta$ is

$$w_\Delta = \mu\overline{w}_\Delta = \mu\Phi^{-1}H^*(H^T\Phi^{-1}H^*)^{-1}\overline{\rho} \tag{4}$$

where

$$\overline{\rho} = k_s \begin{bmatrix} \overline{w}^T g(\theta_0 + \Delta\theta,\ f_0) \\ 0 \\ \overline{w}^T g(\theta_0 - \Delta\theta,\ f_0) \end{bmatrix} \Delta\theta.$$

From (3) and (4), the simplified sum/difference weight vector should be

$$\overline{w} = \Phi^{-1}s^* \tag{5}$$

$$\overline{w}_\Delta = \Phi^{-1}H^*(H^T\Phi^{-1}H^*)^{-1}\overline{\rho}. \tag{6}$$

In general, although w, w_Δ in (1) and (2) and $\overline{w}$, $\overline{w}_\Delta$ in (5) and (6) all result in the same ratio Δ/Σ, only sum beam from the latter is suitable for target detection with the maximum of the output signal-to-noise ratio (SNR). In order to simplify analysis, we only consider the spatial processing for a linear array of *M* equispaced elements with half-wavelength distance between elements. Assuming a strong jammer with power σ_J^2 and azimuth θ_J added to the white noise with power σ_n^2 environment, thus

$$X = \sigma_J J + \sigma_n N$$

$$R_{XX} = E[X^*X^T] = \Phi = \sigma_J^2 J^* J^T + \sigma_n^2 E_M$$

where $J^T = [j_1 \quad j_2 \quad \ldots \quad j_M]$, $j_m = \exp(im\pi \sin\theta_J)$, and E_M is the $M \times M$ identity matrix.

The array output and power with weight vector $w_{\rm opt}$ are

$$Y(\theta) = X^T w_{\rm opt}$$

$$P(\theta) = w_{\rm opt}^H R_{XX} w_{\rm opt}. \tag{7}$$

With the sum beam weight vector in (1) and (5), the corresponding array spatial spectrum are given, respectively, by (depicted in Fig. 1 for $\theta_J = 0°$, $\sigma_J^2 = 100$, $\sigma_n^2 = 1$ and $M = 8$)

$$P(\theta) = 1/s^T\Phi^{-1}s^*$$

$$\overline{P}(\theta) = s^T\Phi^{-1}s^*. \tag{8}$$

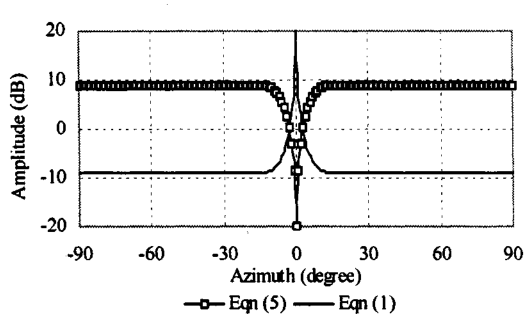

Fig. 1. Spatial spectrum of (1) and (5) for a jammer added to the white noise environment.

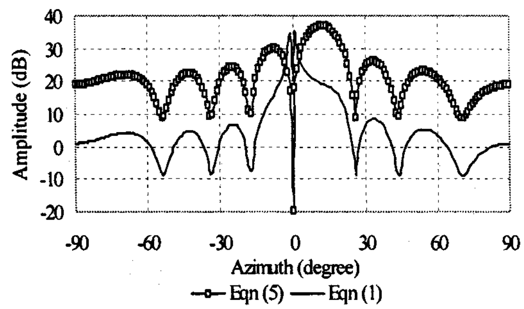

Fig. 2. Spatial spectrum of (1) and (5) for a jammer and a target added to the white noise environment

Fig. 1 shows that weight vector $\overline{w}$ greatly suppresses the jammer, however, weight vector w wrongly lets the jammer pass just like target and acts as super resolution method minimum variance distortionless response (MVDR) [2] because the interference covariance matrix Φ is the same as array covariance matrix R_{XX} from the aforementioned assumption.

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[1] R. L. Fante, *IEEE Trans. Antennas Propagat.*, vol. 47, pp. 773–774, May 1999.

If there is an actual target signal with power σ_s^2 and azimuth θ_s added to the above noise environment, thus, (8) becomes

$$P(\theta) = \frac{s^T \Phi^{-1} R_{XX} \Phi^{-1} s^*}{(s^T \Phi^{-1} s^*)^2}$$
$$\overline{P}(\theta) = s^T \Phi^{-1} R_{XX} \Phi^{-1} s^*. \tag{9}$$

Fig. 2 depicts spatial spectrum in (9) when $\sigma_s^2 = \sigma_J^2$ and $\theta_s = 10°$ that means the interference is located in the main beam of the target. It is obvious from Fig. 2 that weight vector $\overline{w}$ is correct and there exists distortion in the main beam of the target result from suppression of the interference within the main beam of the target.

References

[1] L. E. Brennan and I. S. Reed, "Theory of adaptive radar," *IEEE Trans. Aerosp. Electron. Syst.*, vol. AES-9, pp. 237–252, Mar. 1973.

[2] S. Haykin, *Adaptive Filter Theory*, 3rd ed. Englewood Cliffs, NJ: Prentice-Hall, 1996.

Author's Reply

Ronald L. Fante

The comment by Xie points out that one must be careful with normalization when computing an adapted sum pattern. That is, even though normalization is unimportant when computing Δ/Σ, it can be important when computing Σ alone. This is correct. However, in practice, we never compute Σ alone. Rather, we always compute Σ/I, the signal-to-interference ratio (SIR) in the sum beam. When this quantity is computed, normalization is not an issue, because both the numerator and denominator of Σ/I are quadratic functions of the weights, so that any weight normalization factor cancels. The reader is referred to [1] for additional details.

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References

[1] R. L. Fante, "Ground and airborne target detection with bistatic adaptive space-based radar," *IEEE AES Syst. Mag.*, vol. 14, Oct. 1999.

[2] ——, "Ground and airborne target detection with bistatic adaptive space-based radar," in *Proc. 1999 Radar Conf.*, Waltham, MA, also contained in.

Errata for "Wide-Band Radar Operating Considerations for Low-Altitude Surveillance in the Presence of Specular Multipath"

J. G. Teti, Jr.

The above paper[1] contained typographical errors in (4). The corrected equation is as shown at the bottom of the page.

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[1] J. G. Teti, Jr., *IEEE Trans. Antennas Propagat.*, vol. 48, pp. 176–191, Feb. 2000.

$$p_1(t - T_1) = f(\varphi_d)\left[f(\varphi_d)s(t - T_1)e^{-j2\pi f_{D_s} t} * \xi_d(t)e^{-j2\pi f_{D_t} t}\right] \tag{4a}$$

$$p_2(t - T_2) = f(\varphi_b)\left[\left(f(\varphi_d)s(t - T_2)e^{-j2\pi f_{D_s} t} * \xi_b(t)e^{-j2\pi f_{D_t} t}\right) * \rho_{rms}(-t)e^{-j2\pi f_{D_{tc}} t}\right] \tag{4b}$$

$$p_3(t - T_3) = f(\varphi_d)\left[\left(f(\varphi_b)s(t - T_3)e^{-j2\pi f_{D_s} t} * \rho_{rms}(t)e^{-j2\pi f_{D_{sc}} t}\right) * \xi_b(t)e^{-j2\pi f_{D_t} t}\right] \tag{4c}$$

$$p_4(t - T_4) = f(\varphi_b)\left[\left(\left(f(\varphi_b)s(t - T_4)e^{-j2\pi f_{D_s} t} * \rho_{rms}(t)e^{-j2\pi f_{D_{sc}} t}\right) * \xi_{2b}(t)e^{-j2\pi f_{D_t} t}\right) * \rho_{rms}(-t)e^{-j2\pi f_{D_{tc}} t}\right] \tag{4d}$$