Spaced Directive Antennas for Mobile Communications by the Fourier Transform Method

Rodney Vaughan, *Senior Member, IEEE*

***Abstract*—The Fourier relations between channel transfer function and scattering distribution can apply to personal and mobile communications where multipath is a prevalent phenomena. In this paper, the transform relations are reviewed and interpreted for the mobile radio channel. The effective scattering distribution is the vector product of the antenna pattern and the incident waves and is a scalar function of angle and delay time. The space base-band frequency correlation function transforms with the averaged power of the effective scattering distribution. If the angular power density marginal of the effective scattering distribution is known, then the transform relations can be used for configuring antennas for spatial diversity. Similarly, if the delay time marginal is known, then conditions for frequency diversity are available. The two-dimensional (2-D) transform gives a convenient route for assessing tradeoffs between combined frequency and space diversity. Using modeled distributions, solutions are given for spaced directive antennas and an example is discussed for the space-frequency tradeoff.**

***Index Terms*—Land mobile radio, land mobile radio diversity systems, multipath interference.**

I. INTRODUCTION: MULTIPATH CHANNELS

A. Background

THE use of mobile radio channels is expected to continue to grow quickly, driven by a combination of consumer demand for mobile services and advances in the enabling electronic and infrastructural technology. A limiting factor is that the many users must share the finite wireless spectral resource. In mobile communications, efficient use of the spectrum comes down to controlling the radiowave launching and reception in complicated multipath environments. The antenna, together with the associated adaptive signal processing, become the key elements of the link. Basic antenna configurations must be as compact as possible, but also be able to retrieve from the multipath a variety of signals, which have different or uncorrelated multipath degradation. This paper presents the basic Fourier relations for the multipath of mobile communications and applies them to finding the spacing requirement for directive antennas. The spacing requirement is well known for the situation at a mobile where omnidirectional (in two spatial dimensions) antennas are often used and where less than a half-wavelength spacing provides diversity action [1]–[3]. However, at the base station and also with some mobile terminals, the antennas are directive and the conditions for space diversity are different, with the larger directionality requiring larger spacing. This paper also connects the spacing requirements and the directionality through the Bello channel relations [4] as applied to the mobile channel. Previous and recent work pertaining to the area of Doppler spectra or spatial/diversity relations in mobile communications include [4]–[21].

The remainder of this section reviews the mobile channel and its statistical relationships. Section II follows up with finding the spacing requirements for directional antennas or directional angular power distributions and Section III covers the joint use of frequency diversity and space diversity for general directive antennas.

B. Multipath Channels

The open circuit voltage at the receiving antenna terminal is a function of the incident fields $\boldsymbol{E}(\theta,\phi)$ and the receiving pattern $\boldsymbol{h}(\theta,\phi)$. It is defined here, through introducing the base-band frequency ω and mobile terminal position z-dependence as

$$V_{oc}(\omega,z)=\int_0^{2\pi}\int_0^{\pi}\boldsymbol{E}(\omega,z;\theta,\phi)\cdot\boldsymbol{h}(\omega;\theta,\phi)\sin\theta\, d\theta\, d\phi. \tag{1}$$

The pattern applies a complex weight to the waves coming from different directions and can be considered as a spatial filter. The open circuit voltage is a scalar and can be described as the *electromagnetic propagation channel signal transfer function*, which is referred to from here on just as the transfer function.

C. Channel Model

Over a space of typically many wavelengths, the transfer function for a mobile terminal is modeled as a summation over many discrete, constant, scalar, effective sources, i.e., $H(\omega,z)=V_{oc}(\omega,z)=\Sigma_i\, a_i e^{j\psi_i}\cdot e^{-jk_R r_i}$, where $a_i e^{j\psi_i}$ is the complex amplitude of the ith source, k_R is the wavenumber of the radio frequency (i.e., the carrier plus the base-band frequency) and r_i is an equivalent distance to the ith source. The assumptions for this model are that: the direction to, and the magnitude and phase of, each scatterer are constant; that there is no distance dependence for the magnitude of the waves from the sources; and that the incident waves from the scatterers are plane. So only the phase mixing process of the wave contributions is modeled. As long as there are several effective sources and none of these are dominant, then the process results in Rayleigh distributed fading of the received

Manuscript received May 12, 1999; revised March 10, 2000.

The author is with the Industrial Research Limited, Lower Hutt, 6009 New Zealand.

Publisher Item Identifier S 0018-926X(00)07272-0.

signal envelope. If there is a dominant source, then the fading is Rician, which may alter the correlation results below; however, there is a decreasing need for diversity with an increasingly dominant source bearing the wanted signal.

The electrical distance to the ith scatterer can be approximated [8] by $k_R r_i \approx \omega\tau_i - u_i z$ where τ is the delay and $u_i = k_C \cos\theta_i$ is the direction expressed as the spatial Doppler frequency in rads/m. (The subscript C indicates the carrier frequency.) The equivalent base-band transfer function of the channel is, therefore, approximated by the discrete Fourier transform $H(\omega, z) = \Sigma_i\ a_i e^{j\psi_i} e^{j(-\omega\tau_i + u_i z)}$.

D. Fourier Relations

In general, the summation for all the delays and ray directions can be expressed using its integral form [8], i.e.,

$$H(\omega, z) = \frac{1}{2\pi} \int_0^\infty \int_{-k_c}^{k_c} a(\tau, u) e^{-j\omega\tau} e^{juz}\, du\, d\tau \tag{2}$$

where $a(\tau, u)$ is the distribution density of signals (open circuit voltages) received by the antenna from the scattering medium at delay τ and direction u. This distribution is scalar, comprising complex amplitudes, and is referred to here as the *effective scatterer distribution*. The 2π factor is included in the integral form here to formalize the two-dimensional (2-D) Fourier transform with mixed exponents. The exponents can be made the same sign by defining the positive spatial direction as along the negative z-axis. The inverse transform is

$$a(\tau, u) = \frac{1}{2\pi} \int_{-\infty}^\infty \int_{-\infty}^\infty H(\omega, z) e^{j\omega\tau} e^{-juz}\, d\omega\, dz. \tag{3}$$

The importance of the Fourier transform pair is the insight it offers: models of the effective scatterer distribution can be used conveniently to synthesize mobile channel transfer functions or measured transfer functions can produce an effective scatterer distribution. If the receiving pattern is known, then some insight into the distribution of wave arrival $\boldsymbol{E}(\theta, \phi)$ is available, but this is restricted by the fact that this is a vector (both polarizations, in general) and the waves arrive from within three spatial dimensions (two polar angles), whereas the transfer function is scalar and the effective scatterers are in two spatial dimensions (one polar angle).

There can be four complex functions carrying the same information for characterization of the channel:

- $a(\tau, u)$ scattering function in the time-delay Doppler domain (referred to as the effective scattering distribution);
- $h(\tau, z)$ impulse response in the delay space domain (spatial spectrum);
- $A(\omega, u)$ transfer function in the base-band frequency Doppler domain (frequency spectrum);
- $H(\omega, z)$ transfer function in the base-band frequency space domain (space-frequency spectrum)

and the single variable transformations follow conventional one-dimensional Fourier formulas.

E. Correlation Functions

Major assumptions are now made to allow simplification. Ergodicity is assumed so that the statistics remain second order. If the process is assumed to be *wide sense stationary in the frequency domain* (i.e., the mean and correlation of the frequency transfer functions do not depend on the choice of frequency) then the process is *uncorrelated in the delay domain*. These conditions imply each other. Similarly, *wide sense stationarity in the space domain* corresponds to *uncorrelated scattering in the Doppler domain*. The correlation function of the effective scattering distribution R_a now relates to the space-frequency correlation function R_H by

$$\begin{aligned} R_a(\tau_1, \tau_2; u_1, u_2) &= \frac{1}{2\pi} \iint R_H(\Delta\omega, \Delta z) \\ &\quad \cdot e^{j(-\Delta\omega\tau_2 + u_2\Delta z)}\, d\Delta\omega\, d\Delta z \\ &\quad \cdot \delta(\tau_2 - \tau_1) 2\pi\delta(u_2 - u_1) \quad \text{(4a)} \\ &= P(\tau_2, u_2)\delta(\tau_2 - \tau_1) 2\pi\delta(u_2 - u_1). \quad \text{(4b)} \end{aligned}$$

The averaged power of the effective scattering distribution is

$$\begin{aligned} P(\tau, u) &= E\{|a(\tau, u)|^2\} \quad \text{(5a)} \\ &= \frac{1}{2\pi} \iint R_H(\Delta\omega, \Delta z) e^{-j(\Delta\omega\tau - u\Delta z)}\, d\Delta\omega\, d\Delta z. \quad \text{(5b)} \end{aligned}$$

With $a(\tau, u)$ being the vector product of the antenna pattern and the incident waves, it follows that $P(\tau, u)$ is the averaged product of the antenna power pattern and the copolarized incident power pattern. This means that the antenna power pattern and the copolar incident power distribution have the same effect on the correlation function. The correlation function is the inverse transform of (5)

$$R_H(\Delta\omega, \Delta z) = \frac{1}{2\pi} \iint P(\tau, u) e^{j(\Delta\omega\tau - u\Delta z)}\, d\tau\, du. \tag{6}$$

So the above wide sense stationarity conditions lead to the simplified result that the space-frequency correlation function and the average power density distribution in time delay and spatial Doppler frequency are Fourier pairs.

Bello [4] coined the term wide sense stationary uncorrelated scattering (WSSUS) to describe such channels. Many other details extending to situations outside of mobile communications may be found in Bello's classical paper. However, the above relations are the relevant ones for mobile radio. Here, the WSS refers to the delta function description in Doppler frequency (WSS in position or time for constant velocity of the mobile terminal) and the US refers to the delta function in delay time (sources at different delays are mutually uncorrelated). Bello's work related to tropospheric channels, which contained scintillating scatterers being illuminated by static terminals. For mobile channels, at least one of the terminals is moving and the scatterers are essentially static and the validity of the WSSUS model should always be questioned for a given situation. The channel can be arranged to be "sufficiently WSSUS" for gaining useful insight and inferring channel behavior, by arranging the ensemble averaging. This averaging is often taken as several sampled records over short distances while staying within a given physical environment, followed by the averaging. Statistically, the ensemble averaging implies many "realizations." This can be interpreted as several sampled records which should have different (i.e., independent) spatial paths within the same physical environment, or else as several records in different (independent) physical environments. The two cases are different. One averages within a single physical environment such as a given room. In fact, this could be argued to imply nonindependent records because the same physical scatterers are supplying the power or, in simulation terms, the same seed is used for the environment realization. The second case averages over many different environments, which offers unambiguous independence. The issue is that for a single data record calculated over a static scenario, which is typically a few wavelengths, then the (assumed constant) instantaneous effective scattering distribution defines the transfer function of the record. For averaged quantities, the correlation functions reveal averaged parameters such as the decorrelation distance and frequency and these may not apply accurately to any one of the instantaneous environments.

II. Correlation Distance for Directional Antennas or Scenarios

The spatial correlation distance for a given angular distribution of effective sources can be found directly from the single-dimensional Fourier relation between the correlation function $R_H\ (\Delta z)$ of the channel $H(z)$ and the Doppler power profile written as

$$R_H(\Delta z) \Leftrightarrow P(u). \tag{7}$$

It is emphasised that the Doppler profile is for the effective sources, i.e., the antenna pattern is included, with the physical incident power distribution multiplied by the power pattern of the antenna. The relationship can, therefore, be used for solving omnidirectional antennas in the presence of directional scenarios (e.g., vehicular mobile cases) or for directional antennas in the presence of a constant angular power density or a combination of these.

For either directional scenarios or directional antennas, it is convenient to use a circular function; the $\cos^n\theta$ form, proposed in the context of mobile communications by Lee [6], is the simplest. For very narrow beams, the asymptotic form for large n can be found from using the small θ approximation $\ln(\cos\theta) \approx \ln(1-\theta^2/2) \approx -\theta^2/2$, which results in $\cos^n\theta \approx \exp(-n\theta^2/2)$, i.e., the cosine form becomes similar to Gaussian.

An amplitude pattern defined as

$$g_1(\theta) = \cos^n\left(\frac{\theta-\theta_0}{2}\right), \qquad |\theta-\theta_0| \le \pi \tag{8}$$

is single lobed over the complete angular region with a maximum at θ_0. Any sidelobe structure of a real-world pattern is assumed to have a secondary effect. If we take a model with the 2-D scenario of uniform incident power [1], with pdf $p_\theta(\theta) = 1/2\pi$, then the power of the effective scattering distribution is

$$P_\theta(\theta) \propto p_\theta(\theta) g^2(\theta) \propto g^2(\theta) \qquad |\theta-\theta_0| \le \pi. \tag{9}$$

Converting to a function of u,

$$P(u) = \frac{K}{\sqrt{k_c^2-u^2}} \cos^{2n}\left(\frac{\cos^{-1}\frac{u}{k_c} - \cos^{-1}\frac{u_0}{k_c}}{2}\right) \tag{10}$$

where K is a scaling constant and $u_0 = k_c\cos\theta_0$ and so the spatial correlation function is

$$R_h(\Delta z) = K \int_{-k_c}^{k_c} \frac{1}{\sqrt{k_c^2-u^2}} \cdot \cos^{2n}\left(\frac{\cos^{-1}\frac{u}{k_c} - \cos^{-1}\frac{u_0}{k_c}}{2}\right) e^{-ju\Delta z}\, du. \tag{11}$$

In this example, there are singularities at $u = \pm k_c$ occurring for $n = 0$, but these can be handled by their analytic integration and numeric integration is not required for $n = 0$ or for any n with $\theta_0 = \pi/2$ (see below). Also, the correlation function is purely real when the beam maximum is $\theta_0 = \pi/2$ or for $n = 0$. In summary, (11) is the correlation between the voltages from antennas whose patterns are given by $g(\theta)$ and are separated by a distance Δz. An interpretation of the situation is given in Fig. 1.

Another way to find the correlation function is to state directly the correlation between the spaced antenna patterns for a given angular power distribution [7]. The pattern from an identical antenna spaced at a distance Δz (see Fig. 1) can be written

$$g_2(\theta) = g_1(\theta) e^{jk\Delta z\cos\theta}. \tag{12}$$

Then the correlation between the voltages for the two-dimensional (2-D) case can be written

$$R_H(\Delta z) = \langle g_1(\theta) g_2(\theta) \rangle = \int_0^{2\pi} p_\theta(\theta) g_1(\theta) g_2^*(\theta)\, d\theta \tag{13}$$

which gives the same result as (11). This is a more general approach, allowing diversity antenna derivation, which is not restricted to space diversity.

For measured patterns and most modeled patterns, the integral is solved numerically. However, the choice of pattern modeling function can be arranged to give simplified results. For example, in the special case where the patterns are broadside ($\theta_0 = \pi/2$) and using a pattern function of $g(\alpha) = \cos^n(\alpha)$, where $\alpha = \theta - \pi/2$ is the angle from broadside, over the half-space only, gives the standard integral

$$R_H(\Delta z) = \int_{-\pi/2}^{\pi/2} \cos^{2n}(\alpha) e^{jk_c\Delta z\sin\alpha}\, d\alpha = \sqrt{\pi}2^n\Gamma(n+1/2)\frac{J_n(k_C\Delta z)}{(k_C\Delta z)^n}. \tag{14}$$

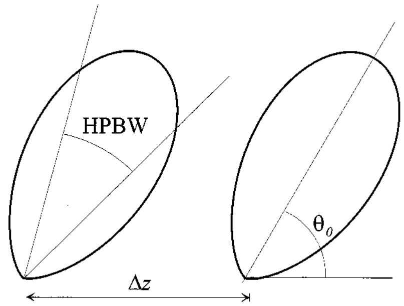


Fig. 1. An example of the $\cos^n((\theta-\theta_0)/2)$ amplitude pattern with the z-axis the spatial direction, and θ_0 the mean angle measured from the z-axis.

However, for finding the correlation spacing for antennas, as given below, the inverse of the normalized form of R_H is required and this has to be undertaken numerically. So closed-form solutions for correlation distances are not normally possible.

In general, the correlation coefficient function of the Gaussian channel (the real and imaginary parts of the transfer function have zero mean, normally distributed amplitudes) is complex, denoted by

$$\rho_H(\Delta z) = \frac{R_H(\Delta z) - \langle |H| \rangle^2}{R_H(0) - \langle |H| \rangle^2} = \frac{R_H(\Delta z)}{R_H(0)} = \rho_{II}(\Delta z) - j\rho_{IQ}(\Delta z). \quad (15)$$

As noted above, an imaginary part arises when the mean angle is nonzero, i.e., when the Doppler spectrum is asymmetric, and this phenomenon is clear from basic Fourier transform theory. The correlation coefficient of the power signal is

$$\rho_{|H|^2} = |\rho_H|^2 = \rho_{II}^2 + \rho_{IQ}^2. \quad (16)$$

An example of the Doppler and correlation functions are given in Fig. 2 where ρ_{II}, ρ_{IQ} and $\rho_{|H|^2} = |\rho_H|^2$ are given for a directional scenario with the $\cos^n$ amplitude pattern, which has half-power beamwidth (HPBW)

$$\text{HPBW (rads)} = 4\cos^{-1}(2^{-(1/2n)}). \quad (17)$$

A snapshot result from simulation of the scenario is given in Fig. 3, which includes the estimates of the correlation components, the power signal correlation coefficient, and the magnitude squared of the signal correlation coefficient, $|\rho_H|^2$. The figure shows a typical difference between these two correlation coefficients for finite sample sizes, which is small for large correlations but increases for small correlations at the larger displacements. The power signal correlation estimate can be negative whereas $|\rho_H|^2$ must always be positive of course. An ensemble average of correlations from many simulations sees both correlation coefficients converge to be the same.

As the scenario parameters (the mean direction and HPBW) are varied, the real and imaginary components hold their form, with their relative amplitudes and periodicity varying. For single-beam scenarios, the correlation is well behaved in the sense that the correlation distance d_d given by $\rho_{|H|^2}(\Delta z = d_d) = 0.5$ is well defined. This distance is plotted in Fig. 4 and this graph can be used as a guideline for spatial correlation distances.

The special case of the omnidirectional scenario is given by the HPBW $= 360°$ giving $d_d \approx 0.18\lambda$ from $J_0^2(k_c d_d) = 0.5$. As the directionality increases, the decorrelation distance increases, with the rate of increase greater for mean angles approaching zero, i.e., when the mean direction is along the spacing axis.

For the region where the HPBW is less than twice the mean angle, the straight line graphs demonstrate that the HPBW is inversely proportional to the decorrelation distance (cf. the uncertainty principle, where the product of the spreads of a Fourier pair are bounded $\sigma_u \sigma_{\Delta z} \geq 1/2$). From a least mean square solution, the relationship can be expressed, with the HPBW and mean angle in degrees, approximately as

$$\text{HPBW} \cdot \frac{\Delta z}{\lambda} = F(\theta_0), \quad \text{HPBW} < \sim\theta_0 \quad (18)$$

where

$$F(\theta_0) = \frac{2200}{(\theta_0+1)^{1.3}} + 11, \qquad 10° \leq \theta_0 \leq 90°. \quad (19)$$

For scenarios or antennas with more complicated directionality, problems can arise with the definition of the decorrelation distance. This is because the function may not be monotonic around the correlation value of interest, here 0.5. From familiar Fourier transform behavior, this occurs, for example, when the majority of energy of the Doppler spectrum is near the support edges, $u = \pm k_c$. An example is for a double-lobed directional scenario, obtained, for example, by a horizontally oriented dipole-like pattern in a horizontal omnidirectional scenario. The correlation coefficient is given in Fig. 5, which shows the potential confusion for the value of the 0.5 deccorrelation distance. (Mathematically, the correlation functions can always have a properly defined decorrelation distance, but in an engineering sense, the interpretation of results must include a recall that there are considerable assumptions in the model and that this combined with the oscillatory nature of the function create practical uncertainty in the decorrelation distance.) In a limiting case, we have a single ray, which is a delta function in the Doppler spectrum, and this has a correlation function which is always of magnitude one and the decorrelation distance does not exist.

III. Frequency Correlation and Combined Frequency and Space Correlation

In an analogous way to the distance decorrelation, the power-delay profile gives the decorrelation frequency (known as the coherence bandwidth), through the single Fourier relationship $P(\tau) \Leftrightarrow R_H(\Delta\omega)$. For example, an exponential power delay profile model and its corresponding correlation function are

$$P(\tau) = \frac{1}{s}e^{-\tau/s}, \quad (\tau \geq 0) \Leftrightarrow \frac{1}{1+j\Delta\omega s} = R_H(\Delta\omega) \quad (20)$$

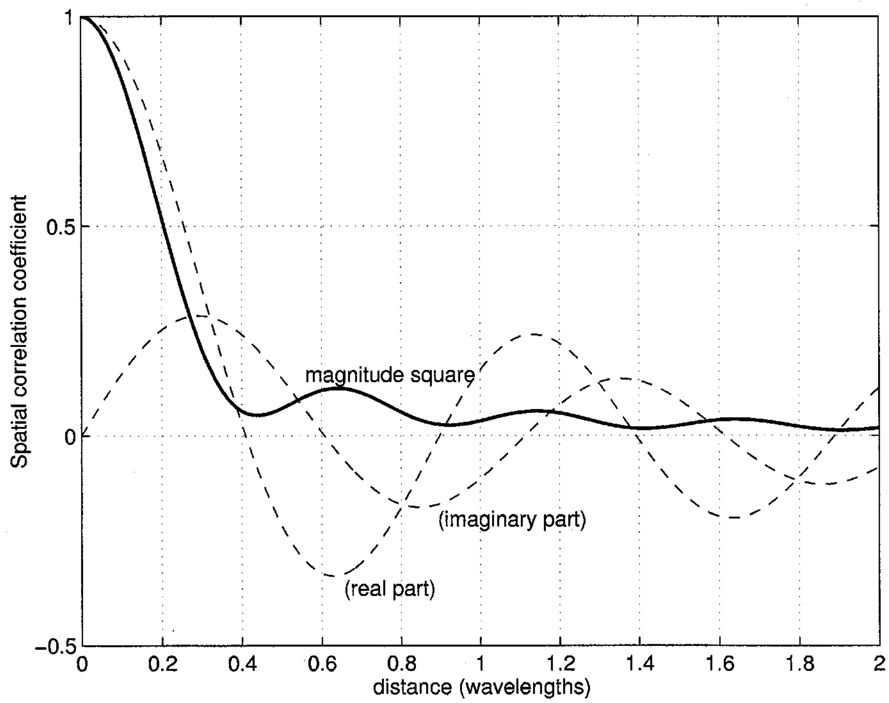


Fig. 2. The spatial power correlation coefficient including the real and imaginary components for a directional scenario. The example is similar to that of a pattern of Fig. 1 above: the mean angle is $\theta_0 = 45°$, but here $n = 1$ which is a scenario with HPBW $= 180°$. The function is well behaved (monotonic) for correlations above about 0.1 and the 0.5 decorrelation distance is well defined.

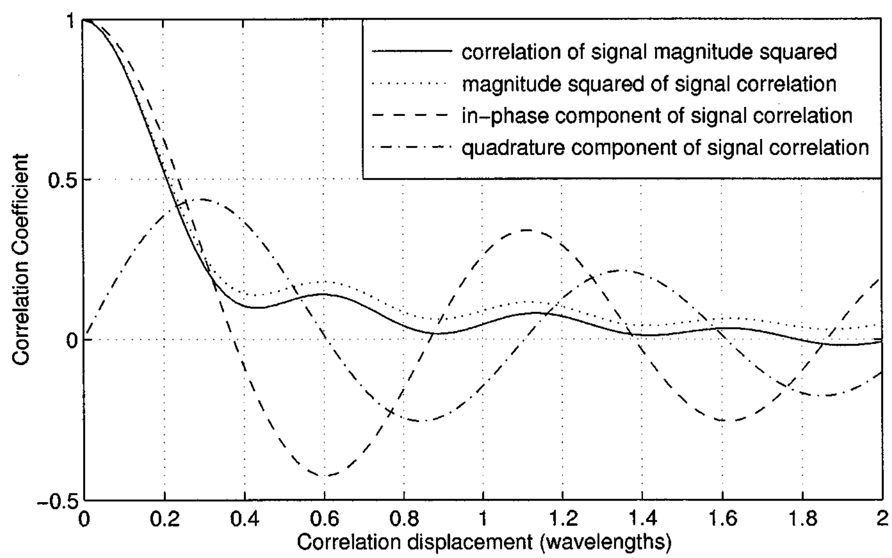


Fig. 3. The correlation coefficients as calculated from a computer simulation. $\rho_{|H|^2}(\Delta z)$ is different to $|\rho_H(\Delta z)|^2$ because of the finite sample size.

respectively, where s is the delay spread. This gives the 0.5 coherence bandwidth, which is where the power correlation coefficient falls to 0.5, i.e., when

$$|\rho_H(\Delta\omega)|^2 = \left|\frac{C_H(\Delta\omega)}{C_H(0)}\right|^2 = \left|\frac{R_H(\Delta\omega)}{R_H(0)}\right|^2 = 0.5 \qquad (21)$$

which is when $\Delta\omega^{(0.5)} = 1/s$.

The relation $R_H(\Delta z, \Delta\omega) \Leftrightarrow P(u, \tau)$ can be used to study the combination of both the frequency diversity and the space diversity. The spatial and frequency correlation coefficients can be separated [13]

$$|\rho_H(\Delta\omega, \Delta z)|^2 = |\rho(\Delta\omega)|^2 \cdot |\rho(\Delta z)|^2 \qquad (22)$$

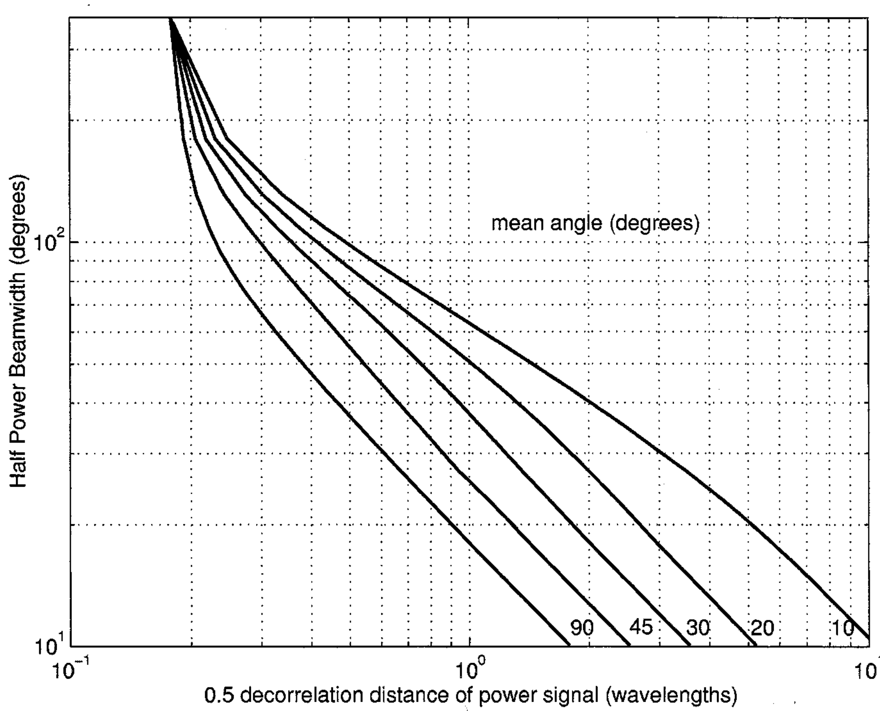


Fig. 4. The decorrelation distance for directional scenario. The directionality is due to the product of the incident angular power distribution at the antenna and the pattern of the antenna. The lines are approximately straight when the HPBW is less than the mean angle.

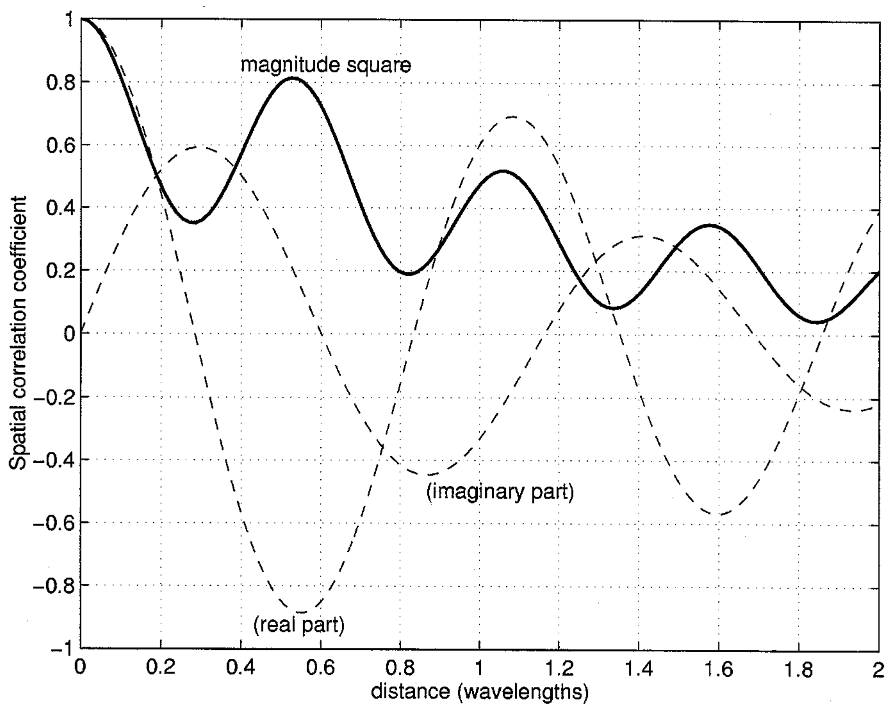


Fig. 5. The correlation coefficient function $|\rho_H(\Delta z)|^2$ for double-lobed antenna power pattern (or scenario), given by $\cos^{2n}(\theta - \theta_0)$ for the example HPBW $= 54°$, $\theta_0 = 20°$.

only if the angle of arrival and delay time of the waves are independent. References [11] and [17] discuss models which may not support this assumption. For a time-averaged scenario at a mobile, however, the assumption seems reasonable. If this assumption can hold, then for a given scenario, the distance and frequency spacing for a given decorrelation can be found. As

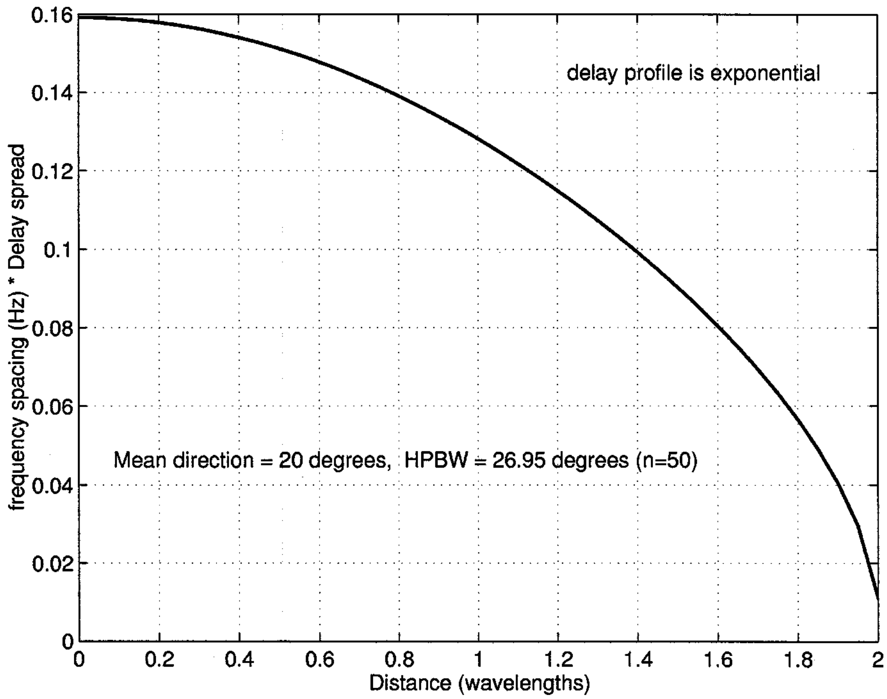


Fig. 6. The frequency and distance spacings for a directional antenna with a HPBW of 27° ($n = 50$) and mean angle of $\theta_0 = 20°$. It is assumed that the angle of arrival and delay times of the scenario are independent.

an example, the exponential delay profile model and $\cos^n\langle(\theta - \theta_0)/2\rangle$ directional scenario, the $|\rho_H|^2 = 0.5$ spacings are from

$$0.5 = \frac{1}{1+(\Delta\omega s)^2} \cdot \left| K \int \frac{1}{\sqrt{k^2-u^2}} \cdot \cos^{2n}\left(\frac{\cos^{-1}\left(\frac{u}{k}\right) - \cos^{-1}\left(\frac{u_0}{k}\right)}{2}\right) e^{-ju\Delta z}\, du \right|^2. \quad (23)$$

A sample solution, with $\theta_0 = 20°$ and $\text{HPBW} = 27°$, is plotted in Fig. 6.

When the frequency spacing is zero, the distance spacing (about two wavelengths) corresponds to purely space diversity. Similarly, for spatially coincident beams, the required purely frequency diversity is an angular frequency spacing equal to the inverse of the delay spread s. A tradeoff example for the specific situation here is to have a distance spacing of one wavelength and a frequency spacing of $0.13/s$ Hz $= 0.8/s$ rads/s.

Such curves need to be derived for any particular scenario to get the particular results. The algorithm is: for a range of spacings Δz_i, calculate a set of spatial correlation coefficients $\rho(\Delta z_i)$ and then, from the Fourier transform of the delay profile, the corresponding frequency spacings can be found. For the exponential delay profile model used here, the frequency spacings are given by

$$\Delta\omega_i = \sqrt{\frac{|\rho_H(\Delta z)|^2}{|\rho_0|^2} - 1} \quad (24)$$

where ρ_0 is the correlation coefficient value ($|\rho_0|^2 = 0.5$ in the above examples).

The trend of the design curves for different patterns behaves intuitively. As the mean angle approaches broadside, i.e., $\theta_0 = 90°$, the curve of Fig. 6 moves to intercept the space axis closer to the origin. For example, for $\theta_0 = 70°$, the intercept is at 0.7 wavelengths, with the angular frequency axis intercept unchanged. A larger beamwidth results in a similar change of the curve.

IV. Conclusion

The Fourier transform method for finding the conditions for diversity is convenient and insightful. The derivation of the transform relations are summarized and the quantities are discussed in order to clarify assumptions and the physical interpretation for the mobile communications case. The transfer function of the mobile channel as a function of frequency and position, is the transform of an effective scattering distribution, which is a function of delay time and spatial Doppler frequency (proportional to the directional cosine). The effective scattering distribution is the incident wave distribution at the antenna weighted by the pattern of the antenna. It is not possible, in general, to relate the complete physical scattering of the environment to the effective scattering distribution. This is because the polarization information of the physical environment is reduced to a scalar by the action of the antenna. The transform relations can be used with models of the scenario for diversity design and examples are given for spaced directive antennas and space-frequency diversity. The assumptions of the scenario models for the results presented are: wide sense stationarity; Rayleigh fading; a smooth-lobed directive antenna pattern; a uniform angular distribution of incident power; an exponential distribution of power decay with delay time; and the multipath angle-of-arrival and delay time uncorrelated.

REFERENCES

[1] R. H. Clarke, "Multiple diversity theory of mobile radio reception," *Bell Syst. Tech. J.*, pp. 967–1000, July/Aug. 1968.

[2] W. C. Jakes, *Microwave Mobile Communications*, NJ: AT&T Press, 1974.

[3] R. G. Vaughan and N. L. Scott, "Closely spaced monopoles for mobile communication," *Radio Sci.*, vol. 28, no. 6, pp. 1259–1266, Nov. 1993.

[4] P. A. Bello, "Characterization of randomly time-variant linear channels," *IEEE Trans. Circuits Syst.*, vol. CS-11, pp. 360–393, Dec. 1963.

[5] M. J. Gans, "A power-spectral theory of propagation in the mobile radio environment," *IEEE Trans. Veh. Technol.*, vol. VT-21, pp. 27–38, Feb. 1972.

[6] W. C.-Y. Lee, "Effects on correlation between two mobile radio base-station antennas," *IEEE Trans. Commun.*, vol. 21, pp. 1214–1224, Nov. 1973.

[7] R. G. Vaughan and J. B. Andersen, "Antenna diversity in mobile communications," *IEEE Trans. Veh. Technol.*, vol. VT-36, pp. 149–172, Nov. 1987.

[8] J. B. Andersen, S. L. Lauritzen, and C. Thommesen, "Distributions of phase derivatives in mobile communications," in *Proc. Inst. Elect. Eng.*, vol. 137, 1990, pp. 197–201.

[9] Y. Ebine, T. Takahashi, and Y. Yamada, "A study of vertical space diversity for a land mobile radio," *Electron. Commun. Jpn.*, vol. 76, no. 10, pp. 68–76, 1991.

[10] P. C. F. Eggers, "Angular dispersive mobile radio environments sensed by highly directive base station antennas," in *Proc. 6th Int. Symp. Personal, Indoor, Mobile Radio Commun. (PIMRC 95)*, Toronto, Canada, Sept. 1995, pp. 522–526.

[11] J. C. Liberti and T. S. Rappaport, "A geometrically based mode for line-of-sight multipath radio channels," in *Proc. 46th IEEE Veh. Technol. Conf.*, Atlanta, GA, May 1996, pp. 844–848.

[12] K. I. Pedersen, P. E. Mogensen, and B. H. Fleury, "Power azimuth spectrum in outdoor environments," *Electron. Lett.*, vol. 33, no. 18, pp. 1583–1584, 1997.

[13] M. Kalkan and R. H. Clarke, "Prediction of the space-frequency correlation function for base station diversity reception," *IEEE Trans. Veh. Technol.*, vol. 46, pp. 176–184, Feb. 1997.

[14] J. Fuhl, A. F. Molisch, and E. Bonek, "Unified channel model for mobile radio systems with smart antennas," *Inst. Elect. Eng. Proc. Radar, Sonar, Navigat.*, vol. 145, no. 1, pp. 32–41, Feb. 1998.

[15] T. Fulghum and K. Molnar, "The jakes fading model incorporating angular spread for a disk of scatterers," in *Proc. 48th Veh. Technol. Conf.*, Ottawa, Canada, May 1998, pp. 489–493.

[16] P. L. Perini and C. L. Holloway, "Angle and space diversity comparisons in different mobile radio environments," *IEEE Trans. Antennas Propagat.*, vol. 46, pp. 764–775, June 1998.

[17] O. Nørklit and J. B. Andersen, "Diffuse channel model and experimental results for array antennas in mobile communications," *IEEE Trans. Antennas Propagat.*, vol. 46, pp. 834–840, June 1998.

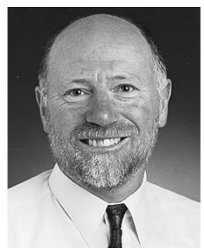
[18] R. G. Vaughan, "Pattern translation and rotation in uncorrelated source distributions for multiple beam antennas design," *IEEE Trans. Antennas Propagat.*, vol. 46, pp. 982–990, July 1998.

[19] G. Durgin and T. S. Rappaport, "Effects of multipath angular spread on the spatial cross-correlation of received voltage envelopes," in *Proc. 49th IEEE Veh. Technol. Conf.*, Houston, TX, May 1999, pp. 996–1000.

[20] O. Nørklit and R. G. Vaughan, "Angular partitioning to yield equal doppler contributions," *IEEE Trans. Veh. Technol.*, vol. 48, pp. 1437–1442, Sept. 1999.

[21] R. Raich and R. G. Vaughan, "Source power distribution for multipath environment using near field circular array beamformer," *Electron. Lett.*, vol. 35, no. 22, pp. 1893–1894, Oct. 1999.

[22] W. C. Jakes, *Microwave Mobile Communications*. Piscataway, NJ: IEEE Press, 1994.



Rodney Vaughan (M'84–SM'89) received the B.E. and M.E. degrees from the University of Canterbury, New Zealand, in 1975 and 1976, respectively, and the Ph.D. degree from Aalborg University, Denmark, in 1985, all in electrical engineering.

In 1972, he joined the New Zealand Post Office (now Telecom NZ Ltd.), where he worked on communications network analysis and telephone traffic forecasting. In 1978, he joined the Physics and Engineering Laboratory, New Zealand Department of Scientific and Industrial Research (DSIR). Here, he helped to pioneer New Zealand industrial applications of microprocessor technology through the design and development of industrial equipment ranging from hardware solutions to software protocol simulators for networks. In 1992, the DSIR was restructured and he transferred to the New Zealand Institute of Industrial Research Limited (IRL, formerly part of the DSIR). He initiated and developed a research program on communications technology, which receives funding support from the New Zealand Foundation for Research, Science, and Technology. He founded the Communications Team at IRL, running several scientific and industrial research projects. His research projects revolve around multipath communications theory (electromagnetic, line, and acoustic media), diversity design, signal and sampling theory, and signal processing. Recent industrial projects have included the design and development of specialist antennas for personal and satellite communications. His current research involves multipath propagation mechanisms, multipath analysis and prediction, space-time processing, and DSP applications.

Dr. Vaughan received a DSIR Public Service Award for Ph.D. study in 1982 (at Aalborg University, Denmark). He is a registered engineer in New Zealand, an URSI correspondent and is the New Zealand URSI Commission B (Fields and Waves) representative.