A Novel Implementation of Multilevel Fast Multipole Algorithm for Higher Order Galerkin's Method

Kalyan C. Donepudi, Jiming Song, *Senior Member, IEEE*, Jian-Ming Jin, *Senior Member, IEEE*, Gang Kang, and Weng Cho Chew, *Fellow, IEEE*

***Abstract*—A new approach is proposed to reduce the memory requirements of the multilevel fast multipole algorithm (MLFMA) when applied to the higher order Galerkin's method. This approach represents higher order basis functions by a set of point sources such that a matrix–vector multiply is equivalent to calculating the fields at a number of points from given current sources at these points. The MLFMA is then applied to calculate the point-to-point interactions. This permits the use of more levels in MLFMA than applying MLFMA to basis-to-basis interactions directly and, thus, reduces the memory requirements significantly.**

***Index Terms*—Fast solvers, Galerkin's method.**

I. INTRODUCTION

THE electromagnetic (EM) interactions with arbitrarily shaped objects can be characterized by seeking the solution of an integral equation whose unknown function is the induced current. In this process, the integral equation is first discretized into a matrix equation using the Galerkin-based method of moments (MoM) with subdomain basis functions such as rooftop functions for curvilinear quad patches and Rao–Wilton–Glisson (RWG) functions [1] for triangular patches. It is convenient to model objects with arbitrary shape using triangular patches; hence, RWG functions are widely used for representing unknown current distributions.

When iterative solvers are used to solve the MoM matrix equation, the calculations of matrix–vector multiplys can be accelerated using fast multipole method (FMM) or multilevel fast multipole algorithm (MLFMA) [2]–[4]. The FMM reduces the complexity of a matrix–vector multiply from $O(N^2)$ to $O(N^{1.5})$, where N is the number of unknowns. The MLFMA further reduces the complexity to $O(N \log N)$. The memory required by MLFMA is only $O(N \log N)$; hence, it can solve very large problems on existing computers. Integral-equation solvers that use RWG basis functions and employ the standard implementations of MLFMA such as FISC [5], [6] and ScaleME [7]–[9] have been very successful in solving large-scale problems. However, these basis functions have a poor convergence and need a large number of unknowns for a desired accuracy. For example, a sampling rate of ten unknowns per wavelength (about 200 unknowns per square wavelength for quad patches or 340 unknowns per square wavelength for triangular patches) typically results in an accuracy of 0.1 dB in the radar cross section (RCS) calculation. The average edge size of the patches is about 0.1λ. As a result, it is very expensive to achieve a high accuracy by using RWG bases or any other low-order functions. Hence, it is imperative to use higher order basis functions [10], [11] with vastly superior convergence properties to obtain accurate results using only a fraction of the unknowns required by low-order basis functions. It has been demonstrated [11], [12] that using higher order basis functions, the sampling rate for an accuracy of 0.1 dB in RCS can be reduced to $40 \sim 70$ unknowns per square wavelength with an average edge length of $0.5 \sim 1\lambda$.

However, when MLFMA is used to accelerate the solution of the higher order Galerkin-based MoM [12], its performance is limited for the following reason. In MLFMA, only the near interactions of the MoM matrix are computed explicitly whereas the far interactions are computed implicitly by using the multipole expansion of current distributions. In this procedure, the scatterer is first enclosed in a large cube, which is then divided into eight equally sized small cubes. Each of the small cubes is further divided into eight smaller cubes recursively until the smallest cube size is about several times bigger than the longest patch edge. Thus, the edge length limits the number of levels used in MLFMA.

In Galerkin-based MoM using RWG basis functions since the edge length is about 0.1λ, the finest cube is about a quarter of a wavelength. However, because of the large patch size used with higher order basis functions, the number of levels in MLFMA is at least one or two less than that for RWG basis functions. In MLFMA, the radiation pattern of each basis is calculated and stored. The number of samplings (K) in the radiation pattern is given by

$$K = 2L^2, \qquad L = kd + \alpha(kd)^{1/3} \tag{1}$$

where:

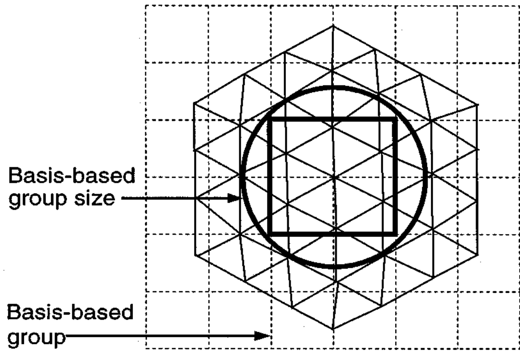
- k wavenumber;
- α depends on the accuracy;
- d diameter of the group size, which is bigger than the cube as shown in Fig. 1(a).

Each higher order basis needs much more samplings than the RWG basis. The average number of unknowns required is about 1.5 per patch for RWG basis, 5 and 10.5 for the first- and second-order basis functions, respectively. Thus, the number of unknowns in each group for higher order basis functions is larger than that for RWG basis. Therefore, more near interactions have to be calculated and stored for each higher order

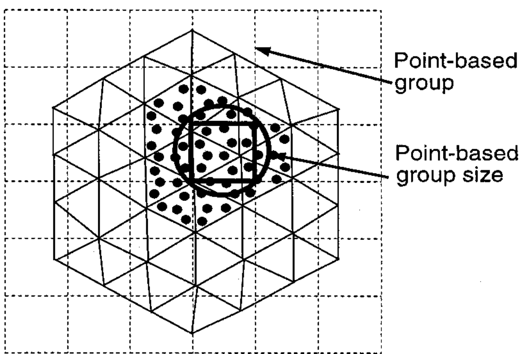
Manuscript received February 15, 2000; revised June 14, 2000. This work was supported by a grant from AFOSR via the MURI Program under Contract F49620-96-1-0025.

The authors are with the Center for Computational Electromagnetics, Department of Electrical and Computer Engineering, University of Illinois at Urbana-Champaign, Urbana, IL 61801-2991 USA (e-mail: j-jin1@uiuc.edu).

Publisher Item Identifier S 0018-926X(00)07704-8.



(a)



(b)

Fig. 1. Applying MLFMA to (a) basis-based sources and (b) point-based sources.

basis. Consequently, MLFMA cannot work very efficiently with higher order Galerkin's method.

To overcome this problem, we propose to implement MLFMA based on point-to-point interactions, instead of the traditional basis-to-basis interactions. When we calculate the matrix elements for which the testing and source bases are not close to each other, we can apply Gaussian quadrature to evaluate the integrals. This process can be interpreted as replacing a continuous source distribution with discrete sources as shown in Fig. 1(b). Thus, one matrix–vector multiply is similar to the calculation of the electromagnetic fields for a given distribution of N source bases and then testing them with these bases. In this implementation, we first find Q equivalent point sources from these N source bases, then calculate electromagnetic fields at these Q points, and finally test them with each testing basis. The value of Q depends on the number of patches and the quadrature rule used for each patch. The MLFMA is used to calculate electromagnetic fields at Q points generated by Q point sources. By doing so, the number of levels used is not limited by the size of basis functions, making MLFMA more efficiently. Furthermore, the near interaction part of the MoM matrix is redefined as the difference between the original matrix and the interactions calculated by MLFMA. Consequently, the memory requirement can be reduced as well. The proposed scheme can be interpreted as a procedure for reducing Galerkin-based formulation for the solution of integral equations to Nyström-type discretizations [13]. In the following section, this approach is first described in detail. Analysis of computational complexity is conducted to show the memory and complex operation counts required by this approach. Numerical results validating the point-based method are then presented, which are followed by the conclusion.

II. Applying MLFMA to Point-Based Sources

By testing the electric field integral equation (EFIE) with the basis function $\boldsymbol{t}_j$, the matrix element Z_{ji} is written as

$$Z_{ji} = \int dS \boldsymbol{t}_j(\boldsymbol{r}) \cdot \int dS' \overline{\boldsymbol{G}}(\boldsymbol{r}, \boldsymbol{r}') \cdot \boldsymbol{t}_i(\boldsymbol{r}') \tag{2}$$

where the dyadic Green's function is given by

$$\overline{\boldsymbol{G}}(\boldsymbol{r}, \boldsymbol{r}') = \frac{ik\eta}{4\pi} \left[\overline{\boldsymbol{I}} - \frac{\nabla\nabla'}{k^2} \right] \frac{e^{ik|\boldsymbol{r}-\boldsymbol{r}'|}}{|\boldsymbol{r}-\boldsymbol{r}'|}$$

and η denotes the impedance. When the testing basis $(\boldsymbol{t}_j)$ is sufficiently far away from the source basis $(\boldsymbol{t}_i)$, the integrals can be evaluated by using Gaussian quadrature to give

$$\begin{aligned} Z_{ji}^{\text{far}} &= \sum_{p=1}^{Q} W_{jp} \boldsymbol{t}_j(\boldsymbol{r}_p) \cdot \sum_{q=1}^{Q} \overline{\boldsymbol{G}}(\boldsymbol{r}_p, \boldsymbol{r}_q) \cdot W_{iq} \boldsymbol{t}_i(\boldsymbol{r}_q) \\ &= \sum_{p=1}^{Q} \boldsymbol{J}_j(\boldsymbol{r}_p) \cdot \sum_{q=1}^{Q} \overline{\boldsymbol{G}}(\boldsymbol{r}_p, \boldsymbol{r}_q) \cdot \boldsymbol{J}_i(\boldsymbol{r}_q) \end{aligned}$$

where:

Q	total number of quadrature points on all patches;
W_{jp}	quadrature weight at the pth point, which is nonzero only on the patch supporting the basis $\boldsymbol{t}_j$;
$\boldsymbol{J}_j(\boldsymbol{r}_p)$	is defined as $W_{jp}\boldsymbol{t}_j(\boldsymbol{r}_p)$.

Correspondingly, the matrix–vector multiply for the far interactions is written as

$$\sum_{i=1}^{N} Z_{ji}^{\text{far}} x_i = \sum_{p=1}^{Q} \boldsymbol{J}_j(\boldsymbol{r}_p) \cdot \sum_{q=1}^{Q} \overline{\boldsymbol{G}}(\boldsymbol{r}_p, \boldsymbol{r}_q) \cdot \sum_{i=1}^{N} \boldsymbol{J}_i(\boldsymbol{r}_q) x_i \tag{3}$$

where x_i is the coefficient of the ith basis function. By defining

$$\boldsymbol{E}_p = \sum_{q=1}^{Q} \overline{\boldsymbol{G}}(\boldsymbol{r}_p, \boldsymbol{r}_q) \cdot \boldsymbol{J}_q, \qquad \boldsymbol{J}_q = \sum_{i=1}^{N} \boldsymbol{J}_i(\boldsymbol{r}_q) x_i \tag{4}$$

Equation (3) is simplified as

$$\sum_{i=1}^{N} Z_{ji}^{\text{far}} x_i = \sum_{p=1}^{Q} \boldsymbol{J}_j(\boldsymbol{r}_p) \cdot \boldsymbol{E}_p.$$

The above formulation means that for a given distribution of N source bases, calculate the electric fields and test them with these bases. The MLFMA can be used to calculate the fields generated by current sources very efficiently. Defining a variable I_{pq} such that

$$I_{pq} = \begin{cases} 1, & p\text{–}q \text{ interaction is calculated by MLFMA} \\ 0, & \text{otherwise} \end{cases}$$

the matrix-vector multiply calculated by MLFMA is modified as

$$\sum_{i=1}^{N} Z_{ji}^{\text{far}} x_i = \sum_{p=1}^{Q} \boldsymbol{J}_j(\boldsymbol{r}_p) \cdot \sum_{q=1}^{Q} I_{pq} \overline{\boldsymbol{G}}(\boldsymbol{r}_p, \boldsymbol{r}_q) \cdot \sum_{i=1}^{N} \boldsymbol{J}_i(\boldsymbol{r}_q) x_i. \tag{5}$$

Now, the near interaction part of the MoM matrix is redefined as the difference between the original matrix and the interactions calculated by MLFMA, so we have

$$Z_{ji}^{\text{near}} = Z_{ji} - Z_{ji}^{\text{far}} \tag{6}$$

where

$$Z_{ji}^{\text{far}} = \sum_{p} \boldsymbol{J}_j(\boldsymbol{r}_p) \cdot \sum_{q} I_{pq} \overline{\boldsymbol{G}}(\boldsymbol{r}_p, \boldsymbol{r}_q) \cdot \boldsymbol{J}_i(\boldsymbol{r}_q).$$

Hence

$$\begin{aligned} Z_{ji}^{\text{near}} &= Z_{ji} - Z_{ji}^{\text{far}} \\ &= \int dS \boldsymbol{t}_j(\boldsymbol{r}) \cdot \int dS' \overline{\boldsymbol{G}}(\boldsymbol{r}, \boldsymbol{r}') \boldsymbol{t}_i(\boldsymbol{r}') - \sum_{p} \boldsymbol{J}_j(\boldsymbol{r}_p) \\ &\quad \cdot \sum_{q} I_{pq} \overline{\boldsymbol{G}}(\boldsymbol{r}_p, \boldsymbol{r}_q) \cdot \boldsymbol{J}_i(\boldsymbol{r}_q). \end{aligned}$$

If the testing basis $\boldsymbol{t}_j$ and the source basis $\boldsymbol{t}_i$ belong to the nearby groups but are not very close to each other, the same quadrature rule used for evaluating the far interactions is used to calculate the first term of the above equation. If all point–point interactions are calculated by MLFMA, Z_{ji}^{near} equals zero. Therefore, Z^{near} is a sparse matrix and has less nonzero elements than the matrix for near interactions between bases.

Since the current vector in (4) has only two tangential components, we write it in terms of two independent tangential vectors as

$$\boldsymbol{t}_i(\boldsymbol{r}_q) = \sum_{l'=1}^{2} \boldsymbol{\Lambda}_{l'}(\boldsymbol{r}_q) y_{il'}(\boldsymbol{r}_q). \tag{7}$$

Therefore, current $\boldsymbol{J}_q$ at the qth source point $\boldsymbol{r}_q$ is written as

$$\begin{aligned} \boldsymbol{J}_q &= \sum_{i=1}^{N} W_{iq} \boldsymbol{t}_i(\boldsymbol{r}_q) x_i \\ &= \sum_{i=1}^{N} W_{iq} \sum_{l'=1}^{2} \boldsymbol{\Lambda}_{l'}(\boldsymbol{r}_q) y_{il'}(\boldsymbol{r}_q) x_i \\ &= \sum_{l'=1}^{2} \boldsymbol{\Lambda}_{l'}(\boldsymbol{r}_q) Y_{l'}(\boldsymbol{r}_q) \end{aligned} \tag{8}$$

where

$$Y_{l'}(\boldsymbol{r}_q) = \sum_{i=1}^{N} W_{iq} y_{il'}(\boldsymbol{r}_q) x_i.$$

Substituting the above expression into (3) yields

$$\begin{aligned} \sum_{i=1}^{N} Z_{ji}^{\text{far}} x_i &= \sum_{p=1}^{Q} \sum_{l=1}^{2} W_{jp} y_{jl} \boldsymbol{\Lambda}_l(\boldsymbol{r}_p) \cdot \sum_{q=1}^{Q} \overline{\boldsymbol{G}}(\boldsymbol{r}_p, \boldsymbol{r}_q) \\ &\quad \cdot \sum_{l'=1}^{2} \boldsymbol{\Lambda}_{l'}(\boldsymbol{r}_q) Y_{l'}(\boldsymbol{r}_q). \end{aligned} \tag{9}$$

Applying the addition theorem to the dyadic Green's function [2], [3] yields

$$\begin{aligned} \overline{\boldsymbol{G}}(\boldsymbol{r}_p, \boldsymbol{r}_q) &= \frac{ik\eta}{4\pi} \left[\overline{\boldsymbol{I}} - \frac{\nabla\nabla}{k^2} \right] \frac{e^{ik|\boldsymbol{r}_p - \boldsymbol{r}_q|}}{|\boldsymbol{r}_p - \boldsymbol{r}_q|} \\ &= \int d^2\hat{k} \left(\overline{\boldsymbol{I}} - \hat{k}\hat{k} \right) e^{-j\boldsymbol{k}\cdot(\boldsymbol{r}_{pm} - \boldsymbol{r}_{qm'})} \alpha(\boldsymbol{k}, \boldsymbol{r}_{mm'}) \end{aligned} \tag{10}$$

where

$$\begin{aligned} \alpha(\boldsymbol{k}, \boldsymbol{r}_{mm'}) &= \frac{ik\eta}{4\pi} \sum_{l=0}^{L} (-j)^l (2l+1) \\ &\quad \cdot h_l^{(2)}(kr_{mm'}) P_l \left(\hat{r}_{mm'} \cdot \hat{k} \right) \\ \boldsymbol{r}_{pq} &= \boldsymbol{r}_p - \boldsymbol{r}_q = \boldsymbol{r}_p - \boldsymbol{r}_m + \boldsymbol{r}_m - \boldsymbol{r}_{m'} - \boldsymbol{r}_q \\ &= \boldsymbol{r}_{pm} + \boldsymbol{r}_{mm'} - \boldsymbol{r}_{qm'} \end{aligned}$$

and $\boldsymbol{r}_m$ and $\boldsymbol{r}_{m'}$ are the group centers for the points p and q, respectively. Consequently, the far interactions of the matrix–vector multiply can be written as

$$\begin{aligned} &\sum_{i=1}^{N} Z_{ji}^{\text{far}} x_i \\ &\quad = \sum_{p=1}^{Q} \sum_{l=1}^{2} U_{jpl} \int d^2\hat{k} \boldsymbol{V}_{mlp}(\boldsymbol{k}) \cdot \sum_{m'} \alpha(\boldsymbol{k}, \boldsymbol{r}_{mm'}) \\ &\quad\quad \cdot \sum_{q \in G_{m'}} \sum_{l'=1}^{2} \boldsymbol{V}_{m'ql'}^{*}(\boldsymbol{k}) Y_{l'}(\boldsymbol{r}_q) \end{aligned} \tag{11}$$

where $U_{jpl} = W_{jp} y_{jl}$, the summation over m' is carried out for all nonneighbor groups, denoted by $G_{m'}$ and the radiation and receiving patterns are given by

$$\boldsymbol{V}_{mlp}(\boldsymbol{k}) = e^{i\boldsymbol{k}\cdot\boldsymbol{r}_{pm}} \left(\overline{I} - \hat{k}\hat{k} \right) \cdot \boldsymbol{\Lambda}_l(\boldsymbol{r}_p). \tag{12}$$

Since the point sources do not have any spatial extent, we can refine the finest cube in MLFMA as small as a quarter of a wavelength. There are two advantages in applying MLFMA to point-to-point interactions. First of all, the group size is the same as the cube and there is no overlapping in sources as shown in Fig. 1. Second, smaller group size means that much less samplings of radiation patterns are needed. As a consequence, the memory requirements for the radiation pattern can be reduced.

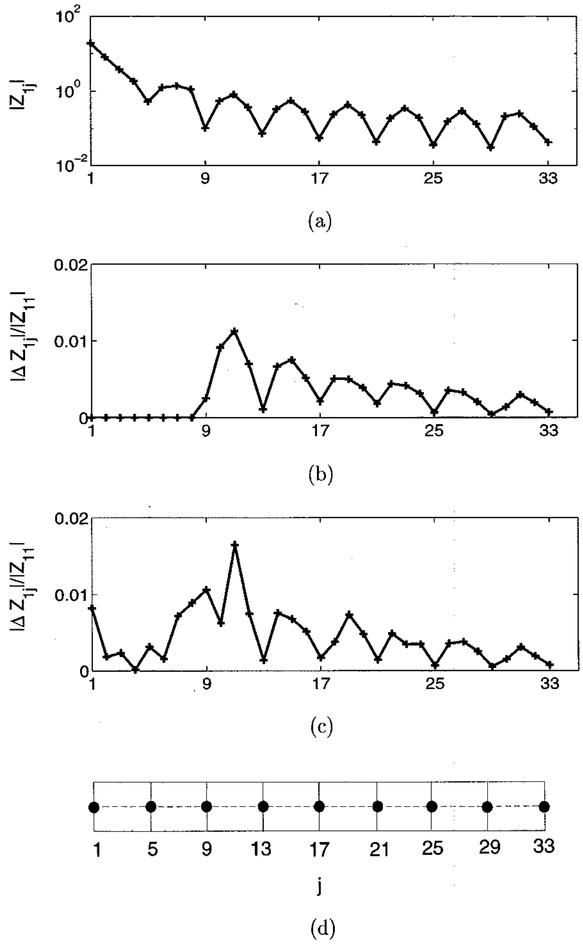


Fig. 2. (a) Absolute value of matrix elements Z_{1j}. (b) Relative error in the basis-based approach. (c) Relative error in the point-based approach. (d) Location of Z_{1j}.

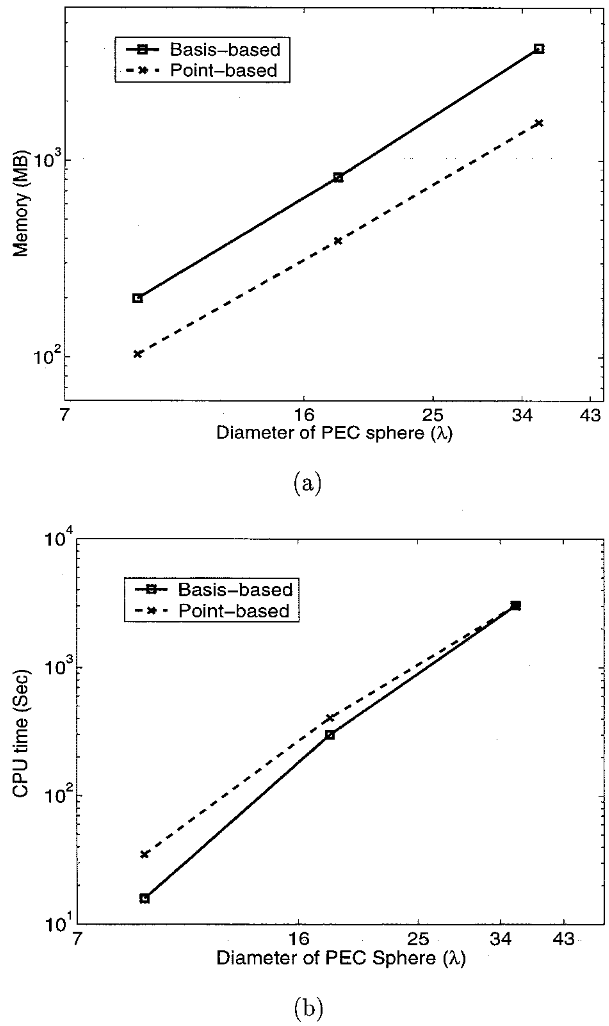


Fig. 3. (a) Memory requirements and (b) CPU time for a matrix–vector multiply of MLFMA as applied to the basis-based and point-based sources.

Further reductions can be obtained if the radiation patterns are calculated on the fly since no numerical integration is needed to calculate the radiation pattern.

III. Complexity Analysis

The memory requirements in the MLFMA implementation have three parts: one for near interaction elements Z^{near}, another for the radiation and receiving patterns for each basis or point as given by (12), and the third for the radiation patterns for all nonempty groups at each level. The first part is unchanged from the traditional basis-based approach to the point-based approach. This, however, is not the case for the second part. When the second-order basis is used, the number of levels in the point-based MLFMA is increased by two compared to that for the basis-based MLFMA. The cubic size in the finest level is then reduced to one fourth, and by (1), the number of samplings of the radiation and receiving patterns for each point is reduced to one eighth. Since the number of points is about twice of the number of bases, the second part of the memory requirement is reduced to one fourth. From one level to a coarser level, the samplings in the radiation patterns are increased by a factor of four and since the number of nonempty cubes is reduced to one fourth, the memory requirement is a constant for all nonempty cubes at each level. Consequently, the point-based approach needs more memory than the traditional basis-based approach in the third part; but, this increase is insignificant compared to the reduction achieved in the second part. Hence, if we write the total memory required as $C_1 N \log N + C_2 N$, the proposed method maintains C_1 while it reduces C_2 by a factor of four.

The number of operations in each matrix–vector multiply consists of three parts similar to the memory requirements: one for near interactions, another for calculating the radiation patterns for each group from its points or bases at the finest level corresponding to the last two summations in (11) and evaluating the fields at each basis corresponding to the first two summations and the integral in (11), and the third for the translation at each level and interpolation/anterpolation from one level to another [4]. Since a large number of groups is needed in the translation, which is the summation over m' in (11) and the interpolation/anterpolation are required for both θ and ϕ, the number of operations in the third part is dominant. Hence, the point-based approach needs more operations than the basis-based approach

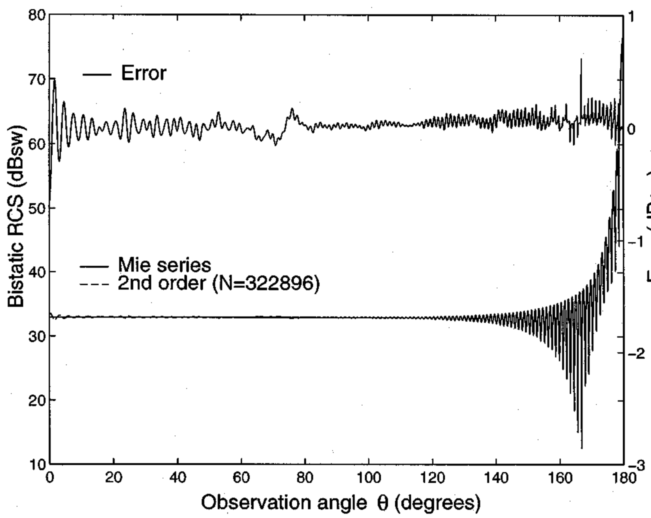


Fig. 4. Bistatic RCS of a PEC sphere having a diameter of 50λ using the point-based approach (seven-level MLFMA, 5.43 Gb of memory).

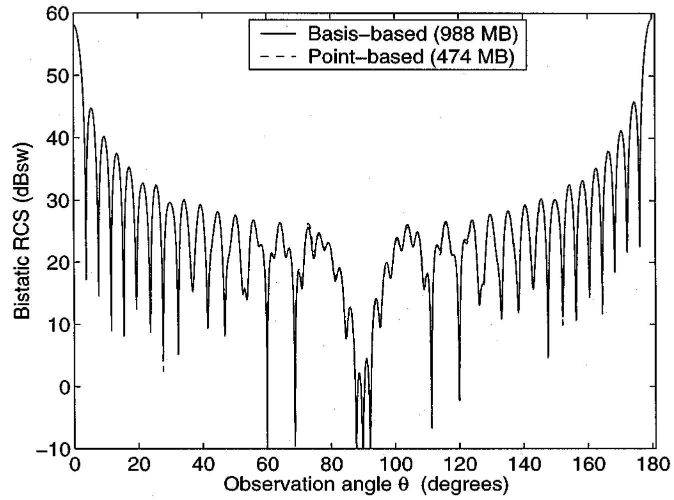


Fig. 5. Bistatic RCS of a PEC cube having a side length of 15λ calculated by the MLFMA as applied to the basis-based and point-based sources, respectively.

because of the added levels. If we write the total number of operations as $C_3 N \log N + C_4 N$, the proposed method maintains C_3 while it increases C_4.

IV. Numerical Results

The point-based approach is verified by first computing the matrix elements Z_{ij} of a thin conducting strip using a five-level MLFMA and then obtaining the error with respect to the direct computation. The actual size of the second-order basis functions would permit only three levels of MLFMA, but the use of the point-based approach allows two additional levels. The errors in the real and imaginary parts of the matrix elements are plotted in Fig. 2 for the first row of the matrix Z_{1j} as a function of j. It is observed that the error in the point-based MLFMA is similar to that in the basis-based MLFMA.

The inherent advantage in applying MLFMA to the point-based sources is shown by computing the RCS of a perfectly electric conducting (PEC) sphere with different diameters using the second-order basis functions [12] and then comparing the memory needed for applying MLFMA to the basis-based sources. The computation is carried out on a 16-node linux cluster called as *Orion* [14]. Each node of the cluster consists of a 100-MHz motherboard and a 350-MHz AMD K6-2 processor. Each board is equipped with a 256 SDRAM card leading to a total of 4 Gb of dynamic random access memory. The MLFMA kernel used is ScaleME [7]–[9]. The discretization size is kept constant at 33 unknowns per square wavelength and the diameter of the sphere is increased from 9λ to 36λ. The memory required is shown in Fig. 3(a), which clearly indicates that the memory requirements are reduced by a factor of two. The corresponding CPU times recorded for a matrix–vector multiply are plotted in Fig. 3(b). The CPU time of the point-based approach is initially increased for small problems, but eventually converges to that of the basis-based approach for larger problems. This is due to the increased memory access time in the basis-based approach because of its larger memory requirements.

Next, the RCS of a 50λ-diameter sphere is calculated using 322 896 second-order basis functions. The root mean square (rms) error is about 0.0877 dB and the corresponding bistatic RCS pattern in the E-plane is plotted in Fig. 4. Seven levels of the point-based MLFMA with radiation patterns computed on fly are used, which requires 5.43 Gb of memory. The use of the basis-based approach requires 12.69 Gb of memory.

Finally, the RCS of a conducting cube having a side length of 15λ is calculated using 45 320 second-order basis functions. The corresponding bistatic RCS patterns in the E-plane are plotted in Fig. 5. In this case, we use six levels of MLFMA with radiation patterns computed on fly. The point-based approach requires 474 MB of memory, whereas the basis-based approach requires 988 MB of memory.

V. Conclusion

A new approach is presented to make the higher order Galerkin's method work more efficiently with the MLFMA. The far interactions in the matrix–vector multiply are calculated by applying MLFMA to the point interactions. This approach allows one to use more levels in MLFMA than applying MLFMA to basis interactions directly and, hence, reduces the memory requirements significantly.

Acknowledgment

The authors would like to thank Dr. S. Velamparambil for the specialization of ScaleME and his personal communications.

References

[1] S. M. Rao, D. R. Wilton, and A. W. Glisson, "Electromagnetic scattering by surfaces of arbitrary shape," *IEEE Trans. Antennas Propagat.*, vol. AP-30, pp. 409–418, May 1982.
[2] R. Coifman, V. Rokhlin, and S. Wandzura, "The fast multipole method for the wave equation: A pedestrian prescription," *IEEE Antennas Propagat. Mag.*, vol. 35, no. 3, pp. 7–12, June 1993.

[3] J. M. Song and W.C. Chew, "Multilevel fast multipole algorithm for solving combined field integral equations of electromagnetic scattering," *Microwave Opt. Tech. Lett.*, vol. 10, no. 1, pp. 14–19, Sept. 1995.

[4] J. M. Song, C.C. Lu, and W. C. Chew, "MLFMA for electromagnetic scattering from large complex objects," *IEEE Trans. Antennas Propagat.*, vol. 45, pp. 1488–1493, Oct. 1997.

[5] J. M. Song, C. C. Lu, W. C. Chew, and S. W. Lee, "Fast Illinois solver code (FISC)," *IEEE Antennas Propagat. Mag.*, vol. 40, no. 3, pp. 27–34, June 1998.

[6] J. M. Song and W. C. Chew, "The fast Illinois solver code: Requirements and scaling properties," *IEEE Comput. Sci. Eng.*, vol. 5, no. 3, pp. 19–23, July–Sept. 1998.

[7] S. Velamparambil, J. M. Song, and W. C. Chew, "A portable parallel multilevel fast multipole solver for scattering from perfectly conducting bodies," in *IEEE APS Int. Symp. Dig.*, vol. 1, Orlando, FL, July 1999, pp. 648–651.

[8] S. Velamparambil and W. C. Chew, "ScaleME: Application interface. A programmer's guide and reference," Univ. Illinois, Urbana-Champaign, Tech. Rep. CCEM-27-99, Oct. 1999.

[9] S. Velamparambil, J. M. Song, and W. C. Chew, "ScaleME: A portable, distributed memory multilevel fast multipole kernel for electromagnetic and acoustic integral equation solvers," Univ. Illinois, Urbana-Champaign, Tech. Rep. CCEM-23-99, Sept. 1999.

[10] R. D. Graglia, D. R. Wilton, and A. F. Peterson, "Higher order interpolatory vector bases for computational electromagnetics," *IEEE Trans. Antennas Propagat.*, vol. 45, pp. 329–342, Mar. 1997.

[11] K. Donepudi, G. Kang, J. M. Song, J. M. Jin, and W. C. Chew, "Higher order MoM implementation to solve integral equations," in *IEEE APS Int. Symp. Dig.*, vol. 3, Orlando, FL, July 1999, pp. 1716–1719.

[12] K. Donepudi, J. M. Jin, S. Velamparambil, J. M. Song, and W. C. Chew, "A higher order parallelized multilevel fast multipole algorithm for 3-D scattering," IEEE Trans. Antennas Propagat., to be published.

[13] L. S. Canino, J. J. Ottusch, M. A. Stalzer, J. L. Visher, and S. Wandzura, "Numerical solution of the Helmholtz equation in 2-D and 3-D using a high-order Nyström discretization," *J. Comput. Phys.*, vol. 146, pp. 627–663, 1998.

[14] S. Velamparambil, J. Schutt-Aine, J. Nickel, J. M. Song, and W. C. Chew, "Solving large scale electromagnetic problems using a linux cluster and parallel MLFMA," in *IEEE APS Int. Symp. Dig.*, vol. 1, Orlando, FL, July 1999, pp. 636–639.

Kalyan C. Donepudi received the B.Tech degree in electrical engineering from the Nagarjuna University, Guntur, India, in 1995, and the M.S. degree from the University of Illinois, Urbana-Champaign, in 1998. He is currently working toward the Ph.D. degree at the University of Illinois in electrical engineering.

His research interests are in general areas of computational electromagnetics.

Jiming Song (S'92–M'95–SM'99) received the B.S. and M.S. degrees, both in physics, from Nanjing University, China, in 1983 and 1988, respectively, and the Ph.D. degree in electrical engineering from Michigan State University, East Lansing, in 1993.

From 1983 to 1985, he was with the Department of Microwave Engineering, Beijing Broadcasting College, China. From 1993 to 2000, he worked as a Postdoctoral Research Associate, a Research Scientist and Visiting Assistant Professor at the University of Illinois at Urbana-Champaign. From 1996 to 2000, he worked as a Research Scientist at SAIC-DEMACO. His research has dealt with modeling and simulations of RF components, the wave scattering using fast fast algorithms, the wave interaction with inhomogeneous media, transient electromagnetic field, and high-Tc superconductive electronics. He was the principal author of the Fast Illinois Solver Code (FISC). He is currently a Staff Engineer/Scientist at Digital DNA Research Lab. of Motorola, Phoenix, AZ.

Dr. Song is a member of Phi Kappa Phi and was the recipient of the Outstanding Academic Award from the College of Engineering, Michigan State University, in 1992.

Jian-Ming Jin (S'87–M'89–SM'94) received the B.S. and M.S. degrees in applied physics from Nanjing University, Nanjing, China, in 1982 and 1984, respectively, and the Ph.D. degree in electrical engineering from the University of Michigan, Ann Arbor, in 1989.

He is an Associate Professor of electrical and computer engineering and Associate Director of the Center for Computational Electromagnetics, University of Illinois at Urbana-Champaign. He has authored or coauthored over 90 papers in refereed journals and several book chapters. He has also authored *The Finite Element Method in Electromagnetics* (New York: Wiley, 1993) and *Electromagnetic Analysis and Design in Magnetic Resonance Imaging* (Boca Raton, FL: CRC, 1998), and coauthored *Computation of Special Functions* (New York: Wiley, 1996). He currently serves as an associate editor of *Radio Science* and is also on the editorial board for *Electromagnetics Journal* and *Microwave and Optical Technology Letters*. His current research interests include computational electromagnetics, scattering and antenna analysis, electromagnetic compatibility, and magnetic resonance imaging.

Dr. Jin is a member of Commission B of USNC/URSI, Tau Beta Pi, and the International Society for Magnetic Resonance in Medicine. He was a recipient of the 1994 National Science Foundation Young Investigator Award and the 1995 Office of Naval Research Young Investigator Award. He also received a 1997 Xerox Research Award presented by the College of Engineering, University of Illinois at Urbana-Champaign, and was appointed the first Henry Magnuski Outstanding Young Scholar in the Department of Electrical and Computer Engineering in 1998. He served as an Associate Editor of the IEEE TRANSACTIONS ON ANTENNAS AND PROPAGATION (1996–1998). He was the symposium cochairman and technical program chairman of the Annual Review of Progress in Applied Computational Electromagnetics in 1997 and 1998, respectively. His name is often listed in the University of Illinois at Urbana-Champaign's *List of Excellent Instructors*.

Gang Kang was born in 1967 in Shaanxi Province, China. He received the B.S. and M.S. degrees from Xidian University, Xi'an, China, in 1988 and 1991, respectively, and the Ph.D. degree from Peking University, China, in 1998.

He was a Faculty Member of Xidian Unviersity from 1991 to 1994 and Peking University from 1997 to 1998. From June 1998 to December 1999, he was a Research Associate in the Center for Computational Electromagnetics, University of Illinois at Urbana-Champaign. He is currently a Research Associate in the Department of Electrical Engineering, University of Utah, Salt Lake City. His interest includes numerical techniques for electromagnetics and their applications in scattering, bioelectromagnetics, and electromagnetic compatibility.

Weng Cho Chew (S'79–M'80–SM'86–F'93) was born on June 9, 1953, in Malaysia. He received the B.S. degree in 1976, the M.S. and Engineer's degrees in 1978, and the Ph.D. degree in 1980, all in electrical engineering from the Massachusetts Institute of Technology, Cambridge.

He is currently a Professor in the Department of Electrical and Computer Engineering, and Director of the Center for Computational Electromagnetics and Electromagnetics Laboratoryat the University of Illinois at Urbana-Champaign. His recent research interest has been in the area of wave propagation, scattering, inverse scattering, and fast algorithms related to scattering, inhomogeneous media for geophysical subsurface sensing, and nondestructive testing applications. Previously, he had also analyzed electrochemical effects and dielectric properties of composite materials, microwave and optical waveguides, and microstrip antennas. From 1981 to 1985, he was with Schlumberger-Doll Research, Ridgefield, CT. While there, he was a Program Leader and later a Department Manager. From 1985 to 1990, he was an Associate Professor with the University of Illinois at Urbana-Champaign. He was an Associate Editor of the *International Journal of Imaging Systems and Technology* (1989–1994), and has been a guest editor of *Radio Science* (1986), *International Journal of Imaging Systems and Technology* (1989), and *Electromagnetics* (1995). He was also an Associate Editor of the *Journal of Electromagnetic Waves and Applications* (1196–present) and *Microwave Optical Technology Letters* (1996–present). From 1989 to 1993, he was the Associate Director the Advanced Construction Technology Center at the University of Illinois. He has authored *Waves and Fields in Inhomogeneous Media*, published over 200 scientific journal articles, and presented over 270 conference papers.

Dr. Chew is a member of Eta Kappa Nu, Tau Beta Pi, URSI Commisions B and F, and an active member with the Society of Exploration Geophysics. He was the NSF Presidential Young Investigator for 1986. He was also an AdCom member IEEE Geoscience and Remote Sensing Society and is presently an Associate Editor of the IEEE TRANSACTIONS ON GEOSCIENCE AND REMOTE SENSING (1984–present). His name is listed many times in the *List of Excellent Instructors* on campus. He is the winner of the 2000 IEEE Graduate Teaching Award and is a Founder Professor, College of Engineering, University of Illinois.