Ultimate Thickness to Bandwidth Ratio of Radar Absorbers

Konstantin N. Rozanov

***Abstract*—Analytic properties of the reflection coefficient of a multilayer metal-backed slab are considered. The result is a new form of the dispersion relationship, which characterizes the integral of the reflectance over wavelength in terms of the total thickness and averaged static permeability of the slab. The relation may be transformed to an inequality, which produces the least thickness to bandwidth ratio achievable for a physically realizable radar absorber. The particular cases of broad-band and narrow-band absorbers are discussed. The least thickness of a 10-dB broad-band dielectric radar absorber is shown to be 1/17 of the largest operating wavelength. The discussion also involves the results of numerical study.**

***Index Terms*—Microwave absorbers.**

I. Introduction

METHODS of radar absorbers design have provoked great interest since the invention of the radar (see [1]–[3] and references therein). This paper deals with the absorbers intended to reduce the reflection when a plane monochromatic electromagnetic wave is normally incident to an infinite plane metal surface coated with the absorber. These materials are conventionally characterized by the thickness d and by the largest value of the module of the voltage reflection coefficient ρ_0 within the operating waveband $\lambda_{\min}, \ldots, \lambda_{\max}$.

The standard objective for the radar absorber design is to obtain the absorber of the least thickness having the lowest possible reflectance ρ_0 within the widest operating waveband. These requirements are contradictory to each other: actual radar absorbers are known to have good performance only within a limited waveband [4], [5]. A single-layer dielectric absorber made of a material with frequency dependent permittivity ε is a simple example illustrating this fact. If ε varies with the wavelength λ as $\mathrm{Re}(\varepsilon) \propto (\lambda/d)^2$, $\mathrm{Im}(\varepsilon) \propto \lambda/d$, the reflectance is independent of frequency [4]. With proper selection of the coefficients of proportionality, any reflectance at any thickness of the absorber within any prescribed band can be provided. However, these coefficients cannot be selected arbitrarily because the frequency dependencies of the real and imaginary parts of the permittivity of any actual material are correlated to each other by means of the Kramers–Kronig relations. The agreement of these dependencies with the Kramers–Kronig relations at all wavelengths yields $|\rho| \equiv 1$. Otherwise, the operating waveband of the radar absorber is limited.

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For some particular occasions, analytical equations are known that produce the relation between d, ρ_0, and the operating bandwidth $\Delta\lambda = \lambda_{\max} - \lambda_{\min}$. For example, for a single-layer radar absorber (Dallenbach screen) with $\rho_0 \ll 1$, the largest possible bandwidth is [1]

$$\frac{\Delta\lambda}{\lambda_0} = \frac{2\rho_0}{(\pi\, d/\lambda_0) \cdot |\varepsilon - \mu|} \tag{1}$$

where λ_0 is the middle of the operating waveband and ε, μ are the relative permittivity and permeability of the layer, which are assumed to be independent of frequency. Equation (1) can be simplified if the layer is thin and the waveband is narrow ($d \ll \lambda_0, \Delta\lambda \ll \lambda_0$). Assuming $|\varepsilon| \gg |\mu|$ and taking into account that $2\pi\, d\sqrt{\varepsilon\mu}/\lambda_0 \approx \pi/2$, (1) readily transforms to

$$\Delta\lambda \approx \frac{32}{\pi}\rho_0 \mathrm{Re}(\mu)\, d. \tag{2}$$

Equation (2) validates the well-known fact that the performance of a single-layer absorber can be greatly improved by increasing the permeability of the layer. However, microwave permeability of known magnetic materials do not exceed several units and other approaches to broaden the bandwidth of radar absorbers attract great attention.

Conventional methods of creating broad-band nonmagnetic absorbers employ multilayer absorbing structures or dielectrics with frequency dispersion of the permittivity, though no analytical expressions relating bandwidth, thickness, and reflectance of the absorbers are known for these cases. The usual approach to the design of such absorbers is the numerical optimization of performance with the use of some parameterized frequency dependence of the constitutive parameters that does not contradict the Kramers–Kronig relations [5]–[7]. For example, numerical optimization of the multilayer broad-band absorber ($\lambda_{\min} \ll \lambda_{\max}$) with the permittivity given by a sum of resonance dependencies produced $d = \lambda_{\max}/14.5$ for the 10-dB reflectance level [6].

The principal drawback of the numerical approach is that it does not produce any data on whether the found solution is the best. The optimization of complicated absorbing schemes is a multi-extremum problem, whereas known optimization methods produce only local minimums but not a global one. Besides, there always is an opportunity that some other scheme of the absorber (or some other kind of parameterization) would be able to yield better results. Therefore, the numerical methods provide the upper bound for the thickness to bandwidth ratio of absorbers.

Since the idea of the perfect radar absorber contradicts the Kramers–Kronig relations, which represent analytic properties of the frequency dependence of constitutive parameters, it seems to be helpful to exploit the theory of analytic functions to establish the fundamental limitation for the $d/\Delta\lambda$ ratio. This paper presents the results of this approach discussing, therefore, the lower bound for the thickness to bandwidth ratio. In other words, the range of the bandwidths is determined that can be never attained at a specified thickness and reflectance of the absorber.

II. Integral Relation for the Reflection Coefficient of a Metal-Backed Slab

We consider a slab of thickness d, permittivity $\varepsilon = \varepsilon' - i\varepsilon''$, and permeability $\mu = \mu' - i\mu''$, overlying a perfectly reflecting plane and illuminated at normal incidence by a monochromatic plane wave. Let the complex reflection coefficient $\rho(\omega)$ be analytically extended into the plane of complex frequencies, where ω is the circular frequency. If electromagnetic fields are assumed to have the $\exp(i\omega t)$ time dependence, $\rho(\omega)$ is analytic in the lower half-plane of complex ω [8].

If ρ is treated as a function of the free-space wavelength $\lambda = c/\omega$, where c is the speed of light, it has no poles in the upper half-plane of complex wavelengths, but may have nulls there. If the nulls of $\rho(\lambda)$ located in the upper half-plane are $\lambda_1, \ldots, \lambda_n, \ldots$, then the ancillary function

$$\rho'(\lambda) = \rho(\lambda)\frac{(\lambda - \lambda_1^*)\ldots(\lambda - \lambda_n^*)\ldots}{(\lambda - \lambda_1)\ldots(\lambda - \lambda_n)\ldots} \tag{3}$$

where sign $*$ stands for the complex conjugation, has neither nulls nor poles at $\mathrm{Im}(\lambda) > 0$. Hence, the logarithm of $\rho'(\lambda)$ is an analytic function in the upper half-plane of complex wavelengths and the Cauchy theorem is valid for it: the integration over any closed contour C yields zero if the contour is located within the upper half-plane of λ. Let the contour consist of the whole axis of real wavelengths and the closing semicircle C_∞, which belongs to the upper half-plane and has infinite radius as shown in Fig. 1. Note that $|\rho'(\lambda)| = |\rho(\lambda)|$ at real wavelengths and the real part of $\ln\rho$ is an even function of λ. Therefore, the real part of the Cauchy integral transforms to

$$\begin{aligned}\mathrm{Re}\int_C \ln\rho'\, d\lambda &= 2\int_0^\infty \ln|\rho|\, d\lambda + \mathrm{Re}\int_{C_\infty} \ln\rho\, d\lambda \\ &\quad + \mathrm{Re}\int_{C_\infty} \frac{(\lambda - \lambda_1^*)\ldots(\lambda - \lambda_n^*)\ldots}{(\lambda - \lambda_1)\ldots(\lambda - \lambda_n)\ldots}\, d\lambda = 0.\end{aligned} \tag{4}$$

As $|\lambda| \to \infty$ at the contour C_∞, the reflection coefficient of a metal-backed slab given by the Fresnel law [9] is approximately equal to $\rho = -1 + 4\pi i\mu d/\lambda$. Only the first-order term in λ^{-1} is retained here, because other terms do not contribute to the second integral in (4). Inserting this to (4) yields

$$\int_0^\infty \ln|\rho(\lambda)|\, d\lambda = -2\pi^2\mu_s d + \pi\sum_i \mathrm{Im}\lambda_i \tag{5}$$

where $\mu_s = \mu'|_{\lambda\to\infty}$ is the static permeability of the slab. Since all λ_i are in the upper half-plane, Im $\lambda_i > 0$ for any i, and two terms in the right part of (5) have opposite signs. The module of the reflection coefficient is not larger than unity, hence, the left part of (5) is not larger than zero. Therefore,

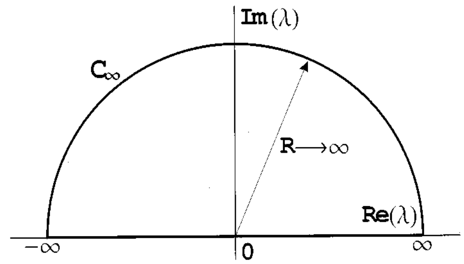

Fig. 1. The contour for the integration in (4)

$$\left|\int_0^\infty \ln|\rho(\lambda)|\, d\lambda\right| \le 2\pi^2\mu_s d \tag{6}$$

which is held for the reflection coefficient of any metal-backed magnetodielectric layer.

The treatment can be readily extended to the case of a multilayer slab. With the wavelengths tending to infinity, any thickness of the material is small compared to λ. Therefore, a multilayer slab can be treated as a stratified medium and be characterized by the effective permeability [9]. This immediately yields

$$\left|\int_0^\infty \ln|\rho(\lambda)|\, d\lambda\right| \le 2\pi^2\sum_i \mu_{s,\,i}\, d_i \tag{7}$$

instead of (6), where d_i, $\mu_{s,\,i}$ are the thickness and the static permeability of ith layer of the multilayer slab.

The above derivation is similar to that yielding the Bode–Fano theorem [10] with the only difference that (6) involves the reflection coefficient treated as a function of λ rather than of ω.

III. Application to Radar Absorbers

If the module of the reflection coefficient of a radar absorber is less than ρ_0 within the operating waveband $\lambda_{\min}, \ldots, \lambda_{\max}$, then

$$|\ln\rho_0|(\lambda_{\max} - \lambda_{\min}) < \left|\int_0^\infty \ln|\rho|\, d\lambda\right|. \tag{8}$$

Taking this into account, we finally arrive at

$$|\ln\rho_0|(\lambda_{\max} - \lambda_{\min}) < 2\pi^2\sum_i \mu_{s,\,i}\, d_i. \tag{9}$$

Inequality (9) can be useful for estimating the ultimate performance of radar absorbers. It gives the largest bandwidth that is possible with a prescribed thickness and reflectance of the absorber.

As an example, we treat broad-band absorbers, where $\lambda_{\max} \gg \lambda_{\min}$ and the latter value may be neglected. Then (9) can be rewritten as

$$\lambda_{\max}\Gamma_0 \le 40\pi^2\sum_i \mu_{s,\,i}\, d_i \Big/ \ln 10 \approx 172\sum_i \mu_{s,\,i}\, d_i \tag{10}$$

where the decibel scale of the reflectance is introduced: $\Gamma_0 = 20\log\rho_0$. For nonmagnetic broad-band absorbers ($\mu_{s,i} \equiv 1$), it follows from (10) that the application of any multilayer slab made of dielectrics with any physically realizable frequency dependence of the permittivity cannot provide 10-dB reflectance level, if the thickness of the absorber is less than $\lambda_{\max}/17.2$.

Inequality (9) specifies the lower bound for the $d/\Delta\lambda$ ratio. An important problem is whether a real radar absorber can exist in which this bound is achieved. This depends on whether the limiting value of the integral of ln$|\rho|$ over the wavelength can be obtained in an actual material producing the equality in (6) and whether all absorption of the electromagnetic energy can be localized within prescribed finite waveband $\lambda_{\min}, \ldots, \lambda_{\max}$ to produce the equality in (9).

It follows from (5) that the equality in (6) is attained when $\rho(\lambda)$ has no nulls in the upper half-plane of complex λ. Such functions correspond to the minimum phase-shift frequency dependence [10] for which the variation of the phase of the reflection coefficient with λ changing from zero to ∞ does not exceed 2π. A single-layer absorber can possess the minimum phase shift dependence if, for example, it is made of a dielectric exhibiting the Debye law of the frequency dispersion of the permittivity. As to the equality in (9), it is impossible to obtain because this would require designing the wavelength dependence of the reflection coefficient as a piecewise-linear function. In some cases, this function can be approximated rather well by physically realizable dependencies $\rho(\lambda)$ and the thickness to bandwidth ratio is close to the limiting value predicted by (9). An illustration for this is given by the data [6] on broad-band dielectric radar absorbers, which produces the $\Delta\lambda/d$ ratio very close to that yielded by (10). However, there are some occasions when the lower bound given by (9) may be improved to give more precise results.

In particular, if the absorber possesses magnetic properties, inequality (9) can produce extremely wide operating waveband. Indeed, there is a number of materials, the static permeability of which ranges up to hundreds and thousands. To obtain more accurate estimate notice that in this instance the low-frequency dispersion of the permeability is not negligible and contributes essentially to the right part of (8). Since this contribution corresponds to the low-frequency absorption, it is useless for increasing the absorption within the operating waveband, which lies in the region of higher frequencies. For simplicity, we consider a single-layer absorber and assume that the magnetic losses in the low-frequency region are associated with an isolated absorption band so that the imaginary part of the permeability is negligibly small at the low-frequency boundary of the operating band $\lambda_{\max}$. Then the integral involved in (6) can be separated into two parts, one associated with the low-frequency magnetic absorption and another related to the absorption within the operating waveband. The former part may be treated in assumption $d \ll \lambda$ that yields ln$|\rho| \approx -4pi\mu''\, d/\lambda$. After that, making use of the well-known summation law [11]

$$\mu'(\infty) = \mu'(\lambda_{\max}) + \frac{2}{\pi}\int_{\lambda_{\max}}^{\infty} \frac{\mu''(x)}{x}\, dx \tag{11}$$

which relates integral loss within an absorption band to the difference of the real parts of the permeability at its boundaries and performing a simple manipulation, we arrive at

$$|\ln \rho_0|(\lambda_{\max} - \lambda_{\min}) < 2\pi^2 \mu'(\lambda_{\max})\, d \tag{12}$$

instead of (9). Inequality (12), which can be readily extended to multilayer absorbers, means that the operating bandwidth depends on the real part of the permeability at the low-frequency boundary of the operating waveband rather than on the static permeability. Note that (12) is valid only if the imaginary part of the permeability is negligibly small at $\lambda_{\max}$. In this case, $\mu'(\lambda_{\max}) < \mu_s$, and the right part of (12) is less than that of (9).

Another case when estimate (9) can be improved relates to narrow-band absorbers. Here, it is unavoidable high-frequency energy absorption that cannot be utilized. Let the operating band be located near the first interference minimum of reflection coefficient, i.e., the absorber is a quarter-wavelength one. If the absorber is thin enough, higher interference minima are well separated from the operating band and have a negligible effect on the performance. Note that it is of no importance whether the higher interference minima introduce into the left or right part of (5), because this does not affect the integral of the reflection coefficient over the wavelength. Therefore, we can neglect the losses and the frequency dispersion of the constitutive parameters beyond the operating waveband and replace the high-frequency permittivity and permeability by some real constants ε_∞, μ_∞ without affecting the absorption within the operating waveband.

Then the problem can be solved with the help of the result given in the Appendix, which concerns the location of reflection nulls at the plane of complex λ in the case when the slab has no dielectric and magnetic losses. As it follows from the foregoing treatment, the integral of ln$|\rho|$ over the first interference minimum is equal to the difference of $2\pi^2\mu_s\, d$ and sum (A.4), in which the first term ($n = 0$) is excluded and $\mu = \mu_\infty$ is assumed. For simplicity, let the treatment be limited to the case of a thin absorber with $|\varepsilon| \gg |\mu|$. Then, Artanh $Z_0 \approx Z_0$, (A3) produces 16 $\mu d/\pi$ for the first term of (A.4), which corresponds to the first interference minimum, and we arrive at

$$|\ln \rho_0|(\lambda_{\max} - \lambda_{\min}) < (2\pi^2(\mu_s - \mu_\infty) + 16\, \mu_\infty)\, d. \tag{13}$$

If the frequency dispersion of permeability is negligible ($\mu_s = \mu_\infty$), (16) can be simplified as

$$\frac{\Delta\lambda}{d} < \frac{16\, \mu_s}{|\ln \rho_0|}. \tag{14}$$

IV. Numerical Results

To find whether the estimates obtained above can be attained in practicable radar absorbers we employ the numerical methods. We treat the case of narrow-band ($\Delta\lambda/\lambda_0 = 1/10$, where λ_0 is the middle of the operating waveband) single-layer nonmagnetic absorbers and search numerically for the dielectric dispersion curve that allows the layer to be of the least thickness providing that its reflectance is less than the prescribed value ρ_0 within a specified waveband. The frequency dependence of the permittivity is assumed to be a sum of n resonance terms

$$\varepsilon(f) = \varepsilon_\infty + \sum_{j=1}^{n} \frac{A^{(j)}}{1 - \mathrm{i}f/f_{\mathrm{rel}}^{(j)} - \left(f/f_{\mathrm{res}}^{(j)}\right)^2} \tag{15}$$

where f is the frequency and other parameters determine resonance features and are varied without restrictions provided that their values are positive, with the exception of ε_∞, which must not be less than unity. These limitations follow from fundamental properties of constitutive parameters and cannot be overcome in realizable materials. Law (15) is valid, for example, for composites filled with conducting fibers [12]. The details of the numerical method are given elsewhere [13].

The results obtained are shown in Figs. 2 and 3. Points in Fig. 2 are the largest $\Delta\lambda/d$ values given as a function of ρ_0 and obtained at $n = 1$ and two in (15). Dotted curve a) corresponds to the limit (14) and curve b) —to the dependence

$$\Delta\lambda = \frac{32}{\pi}\frac{\rho_0}{1-\rho_0^2}d \tag{16}$$

which determines the ultimate bandwidth for a single-layer absorber made of dielectrics with frequency independent permittivity [14] and is a generalization of (2) to the case of arbitrary ρ_0.

The solid lines in Fig. 2 show the dependence

$$\frac{\Delta\lambda}{d} = \frac{32}{\pi}C_n\frac{\rho_0^{1/(n+1)}}{1-\rho_0^{2/(n+1)}} \tag{17}$$

where $C_n = 3/(n+2)$ at $n = 1$ and 2. The figure reveals excellent agreement of the numerical data with (17). Fig. 3 shows the values of C_n obtained from the calculation results for $n = 1\ldots 5$ and $\rho_0 = 0.316$ (-10 dB) in assumption that the dependence $\Delta\lambda/d(\rho_0)$ is given by (17). The agreement is very good again. Thus, (17) provides a good estimate for the least values of the $d/\Delta\lambda$ ratio, which are achievable in narrow-band dielectric absorbers exhibiting resonance dispersion law (15).

Note that any physically realizable dispersion law can be fitted by (15) with any prescribed accuracy at $n \to \infty$. Therefore, corresponding limit of the right part of (17) yields the least thickness of a single-layer absorber that may be made of a dielectric possessing physically realizable types of the frequency dispersion of the permittivity. Tending the right part of (17) to infinity with the account for $C_n = 3/(n+2)$ it is easy to obtain $\Delta\lambda/d \to (3/\pi)(16/|\ln\rho_0|)$, which differs from (14) less than by 5%. This indicates that (14) is accurate enough in producing the ultimate value of the thickness to bandwidth ratio for narrow-band dielectric absorbers, i.e., it can be improved only on the base of some additional assumptions.

For example, in actual dielectrics it is difficult to attain high values of the permittivity, or high quality factor of the resonance [12]. For this reason, the thickness of real absorbers employing the dielectric resonance may be less than that predicted by (17) in some occasions depending on the values ρ_0, $\lambda_{\min}$, and $\lambda_{\max}$ specified for the absorber.

It is interesting to express in figures the characteristics of the absorbers discussed above. For the 10-dB reflectance level, the ultimate $d/\Delta\lambda$ value of a nonmagnetic Dallenbach screen is 1/3.2 as it can be seen from (2). Inequality (14) produces 1/13.9 for this value, and the largest possible bandwidth of a narrow-band dielectric absorber is about 4.4 times larger than that of the Dallenbach screen of the same thickness. For a single-layer dielectric absorber with the resonance dielectric dispersion, the ultimate $d/\Delta\lambda$ value given by (17) is 1/8.4 that is 2.6 times larger than in the Dallenbach screen. For two dielectric resonances involved, $d/\Delta\lambda$ may be as low as 1/9.7. For the 20-dB narrow-band dielectric absorbers, the ultimate value of $d/\Delta\lambda$ is 1 for the Dallenbach screen, 1/3.6 for the case of single dielectric resonance, 1/4.5 for two resonances, and 1/6.95 for a multilayer dielectric absorber with any physically realizable frequency dependence of the permittivity.

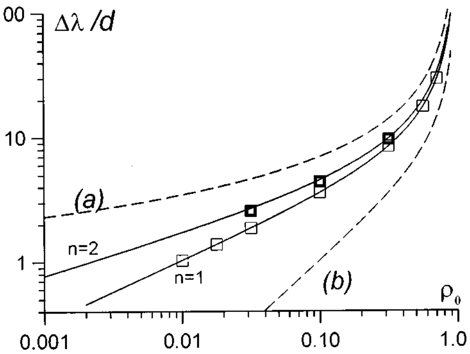

Fig. 2. The value of $\Delta\lambda/d$ as a function of ρ_0 for single-layer dielectric radar absorbers with frequency dielectric dispersion of the resonance type. Boxes are the numerical results. Solid lines correspond to (17) at $n = 1$ and $n = 2$; dotted line (a) corresponds to (14); line (b) to (16).

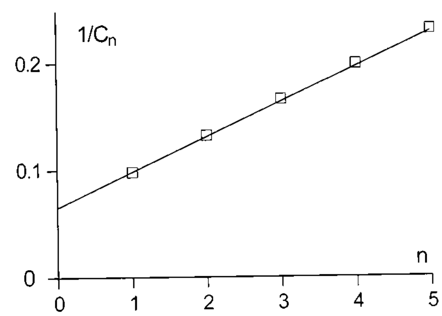

Fig. 3. The values of $1/C_n$ obtained numerically at $n = 1\ldots 5$ (points). The line corresponds to the dependence $1/C_n = (n+2)/3$.

V. Conclusion

We discuss the analytic properties of the reflection coefficient of a multilayer slab backed by a perfectly conducting plane. The result is a new form of the dispersion relationship indicating that the logarithm of the reflection coefficient of the slab integrated over the wavelength from zero to infinity is limited by the quantity, which is proportional to the thickness of the structure and to its averaged static permeability. The analysis of this relation produces a useful information concerning the ultimate characteristics of radar absorbers.

The results related to the performance of the radar absorbers can be summarized as following. Simple analytical estimate (9) is obtained of the ultimate bandwidth of radar absorbers. It follows from (9) that in broad-band nonmagnetic radar absorbers (including multilayer ones) the thickness to bandwidth ratio cannot be less than 1/17.2 at the 10-dB reflectance level. For narrow-band and magnetic absorbers, the estimate can be improved as given by inequalities (14) and (12), respectively. The numerical study yields (17) for the ultimate bandwidth of an absorber made of a dielectric with generalized resonance dispersion law of the permittivity. In narrow-band dielectric absorbers, the optimal selection of the frequency dispersion of permittivity cannot provide more than 4.4-fold broadening of the 10-dB bandwidth in comparison with Dallenbach screen of the same thickness, while the resonance dielectric dispersion can produce not more than 2.6-fold broadening of the operating bandwidth.

APPENDIX

We consider the location of nulls of the reflection coefficient at the plane of complex wavelengths for the case when the magnetodielectric layer under treatment has no dielectric and magnetic losses. Hence, its permittivity and permeability have real positive values and do not depend on frequency. The input impedance of the layer backed by a metal substrate is given by [9]

$$Z = \sqrt{\frac{\mu}{\varepsilon}}\,\tanh\left(\frac{2\pi \mathrm{i} d}{\lambda}\sqrt{\varepsilon\mu}\right) = Z_0\tanh(A+\mathrm{i}B) \quad \text{(A.1)}$$

where $Z_0 = \sqrt{\mu/\varepsilon}$ is the characteristic impedance of the layer and A and B stand for the real and imaginary parts of the argument of the hyperbolic tangent.

The complex wavelengths λ_n at which the reflection coefficient is zero are the solutions of the equation $Z(\lambda_n) = 1$. In general, there are two solutions: $tgB = 0$, $\tanh A = 1/Z_0$ and $tgB = \infty$, $\tanh A = Z_0$. As $|\tanh A| \leq 1$; only one of these is feasible in practice, depending on whether the module of Z_0 is larger than unity or not. In the majority of the practical cases $|\varepsilon| > |\mu|$, and we need in the second solution. By denoting $\lambda_n = \lambda_n' + \mathrm{i}\,\lambda_n''$, this solution transforms to a pair of equations governing the real and imaginary parts of λ_n

$$B = \frac{2\pi\, d\sqrt{\varepsilon\mu}(\lambda_n')}{(\lambda_n')^2 + (\lambda_n'')^2} = \pi(n+1/2)$$

$$A = \frac{2\pi\, d\sqrt{\varepsilon\mu}(\lambda_n'')}{(\lambda_n')^2 + (\lambda_n'')^2} = \text{Artanh } Z_0. \quad \text{(A.2)}$$

The solution of (A.2) for λ_n'' is

$$\lambda"_n = \frac{1}{\text{Artanh } Z_0}\,\frac{2\pi\mu\sqrt{\varepsilon\mu}}{1+\left(\frac{\pi}{2}\,\frac{(2n+1)}{\text{Artanh } Z_0}\right)^2}. \quad \text{(A.3)}$$

Therefore, the sum incorporated in the right part of (7) has a form of series

$$b\sum_{k=-\infty}^{\infty}\frac{1}{(2k+1)^2+a^2} = 2b\sum_{k=0}^{\infty}\frac{1}{(2k+1)^2+a^2} \quad \text{(A.4)}$$

where $b = 8\pi\, d\sqrt{\varepsilon\mu}\text{Artanh } Z_0$, $a = (2/\pi)\text{Artanh } Z_0$. The summation in (A.4) produces $(b\pi/2a)\tanh(\pi a/2)$ [15]. Putting the above values for a and b we arrive at

$$2\pi\sum_{n=0}^{\infty}\lambda_n'' = 2\pi^2\mu\, d \quad \text{(A.5)}$$

in agreement with (7). The result in case of $|\mu| > |\varepsilon|$ is the same.

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