A Recursive Single-Source Surface Integral Equation Analysis for Wave Scattering by Heterogeneous Dielectric Bodies

David R. Swatek, *Member, IEEE*, and Ioan R. Ciric, *Fellow, IEEE*

***Abstract*—The problem of electromagnetic wave scattering by heterogeneous dielectric bodies is formulated in a recursive manner by organizing their homogeneous subregions into hierarchical multiply-nested structures. The inner details of each multiply-nested body are completely accounted for by an equivalent surface representation that yields the electric and magnetic fields tangent to the body only in terms of a single unknown electric surface current density distributed on its outer surface. In this manner, the problem of wave scattering by heterogeneous dielectric bodies is reduced to a scattering problem over their outermost surfaces in terms of only a single unknown current density. For a large number N of different homogeneous dielectric subregions within such a heterogeneous body, the proposed method has a computational complexity of $O(N^{1.5})$ and storage requirements that increase in proportion to $O(N)$. Furthermore, the equivalent surface representation derived for a particular subregion is invariant under rotation and translation and may, therefore, be applied to identical subregions without repeating the computation. The fields at any interior points are calculated by a fast backward recursion.**

***Index Terms*—Heterogeneous dielectrics, integral equations, recursive methods, wave scattering.**

I. Introduction

THE numerical solution of electromagnetic wave scattering problems involving complex heterogeneous dielectric bodies can become intractable if the associated algorithm requires the direct simultaneous solution of all unknown quantities. Recently, iterative and also recursive solutions using the volume integral equation formulations have received much attention [1]–[7]. These so-called fast algorithms typically enjoy computational complexities in the order of $O(N_v^{1.5})$ or even $O(N_v \log N_v)$, where N_v is the number of unknowns distributed throughout the cross section of a two-dimensional (2-D) structure. An exhaustive review of such algorithms can be found in [3]. Coupled surface integral equation formulations—such as the electric fields integral equation (EFIE), magnetic field integral equation, and the combined field integral equation—replace the volume distribution of the unknown equivalent sources within a homogeneous subregion with coupled electric and magnetic surface currents distributed on the enclosing surface and, thus, greatly reduce the number of unknowns (and the necessary amount of computation) for problems where the regions can be partitioned into a small number of homogeneous subregions [8]–[10]. However, as the concentration of these homogeneous subregions increases, the number of unknowns attributed to the coupled surface integral equation formulation approaches that of the volume integral equation methods and the computational advantage is lost. A recent region-by-region solution of the coupled surface integral equations succeeded in reducing the associated computation time and storage requirements, yet still required the simultaneous solution of a linear system involving nearly all the interfaces [10].

An alternative to the typical coupled surface integral equation formulations is to model the fields in terms of single source surface integral equations satisfied by only a single unknown current density. This can be done in different ways, but always such that the fields in a particular homogeneous region are represented in terms of a single unknown surface current density, in agreement with the Kirchhoff integral representation of the fields in an adjacent homogeneous region [11]–[32]. Recursive formulations of these single-source surface integral equations have been derived for the problems of wave scattering by layered periodic gratings [12], [13], by layered dielectric cylinders [28], and by systems of multiply-nested dielectric cylinders [29], [30]. The term “multiply-nested” describes a body composed of a set of inhomogeneous inclusions embedded within a homogeneous medium, each of these inclusions consisting itself, in general, of a homogeneous dielectric body enclosing its own set of inhomogeneous inclusions. These previous recursive formulations retained a common necessity: that each inclusion be completely surrounded by a homogeneous medium in which the fields are to be modeled. This requirement has limited their application to a narrow class of layered or multiply-nested structure problems in which the inclusions touch neither each other nor the enclosing surface.

In this paper, we treat the general problem of wave scattering by structures where different enveloping surfaces and inclusion surfaces have sections in common such that the recursive multiply-nested algorithm previously developed by the authors [29], [30] is extended to the problem of electromagnetic wave scattering by a system of general heterogeneous bodies. The internal structure of each inclusion is completely accounted for by a pair of surface integral operators, which we refer to as the *exclusive operators*, that yield the components of the electric and magnetic fields tangent to the scattering surface only in terms of a single unknown current density distributed on that same surface. Progressing outwardly, in a recursive manner, we derive similar exclusive operators for the successive enveloping surfaces, until,

Manuscript received March 30, 1999; revised December 8, 1999.

The authors are with the Department of Electrical and Computer Engineering, The University of Manitoba, Winnipeg, MB R3T 5V6, Canada.

Publisher Item Identifier S 0018-926X(00)07710-3.

finally, the wave scattering problem is solved in terms of a single unknown electric current density distributed only on the outermost surfaces. Furthermore, the exclusive operators derived for a particular scattering feature enjoy the property to be invariant under rotation and translation and, therefore, provide an equivalent surface representation that may be reused without repeating the associated computation for identical features located elsewhere, while permitting the calculation of fields at interior points by way of a fast backward recursion. The associated algorithm has a computational complexity that grows asymptotically as $O(N^{1.5})$, where N is the number of different homogeneous subregions within the heterogeneous structure. While the reduction procedure is generally valid, in this paper, we only illustrate the formulation and computational aspects for the wave scattering by systems of 2-D heterogeneous dielectric structures.

II. Formulation

A. Physical Problem and Objective

Consider the TM $(H_z = 0)$ illumination of a system of heterogeneous dielectric cylinders V_i, $i = a, b, \ldots, m$, arranged parallel to the z-axis, as shown in a cross section in Fig. 1. The electric field has only a z-directed component E_z that satisfies the homogeneous Helmholtz equation

$$(\nabla^2 + k^2)E_z = 0 \tag{1}$$

with the continuity conditions of the tangential components of the electric and magnetic field intensities across each interface, i.e.,

$$\Delta E_z(\boldsymbol{r}) = 0, \qquad \boldsymbol{r} \in \text{any interface} \tag{2}$$

$$\Delta H_t(\boldsymbol{r}) = 0, \qquad \boldsymbol{r} \in \text{any interface} \tag{3}$$

where:

$k = \omega\sqrt{\varepsilon\mu}$	relevant wave number that varies from region to region;
$H_t = (1/j\omega\mu)\partial E_z/\partial n$	magnetic field component tangent to the interface;
$\partial/\partial n$	derivative in the direction of the normal unit vector $\hat{\boldsymbol{n}}$;
Δ	step discontinuity in the direction of $\hat{\boldsymbol{n}}$.

A time dependence of $e^{j\omega t}$ has been assumed and suppressed throughout. The actual field E_z in the free-space region V_0 is decomposed into the sum of the incident field E_z^{inc} and a scattered field E_z^{sc}.

A necessary feature of our formulation is the organization of the homogeneous subregions into a nested hierarchical structure. We first examine the heterogeneous body V_a that contains a collection of different heterogeneous inclusions V_q, $q = 1, 2, \ldots, n$, within an enveloping surface S_a, which may conform to portions of the included surfaces S_q. The inclusions V_q may be either in contact or separated within a linear and homogeneous region V_a' of permittivity ε_a and permeability μ_a. Each inclusion V_q may itself be comprised of a linear and homogeneous region V_q' enclosing its own set of inclusions.

The objective of our formulation is to completely account for the internal structure of the heterogeneous body V_a by deriving a pair of exclusive operators $\mathcal{E}_a^{ex}$ and $\mathcal{H}_a^{ex}$ that yield the field

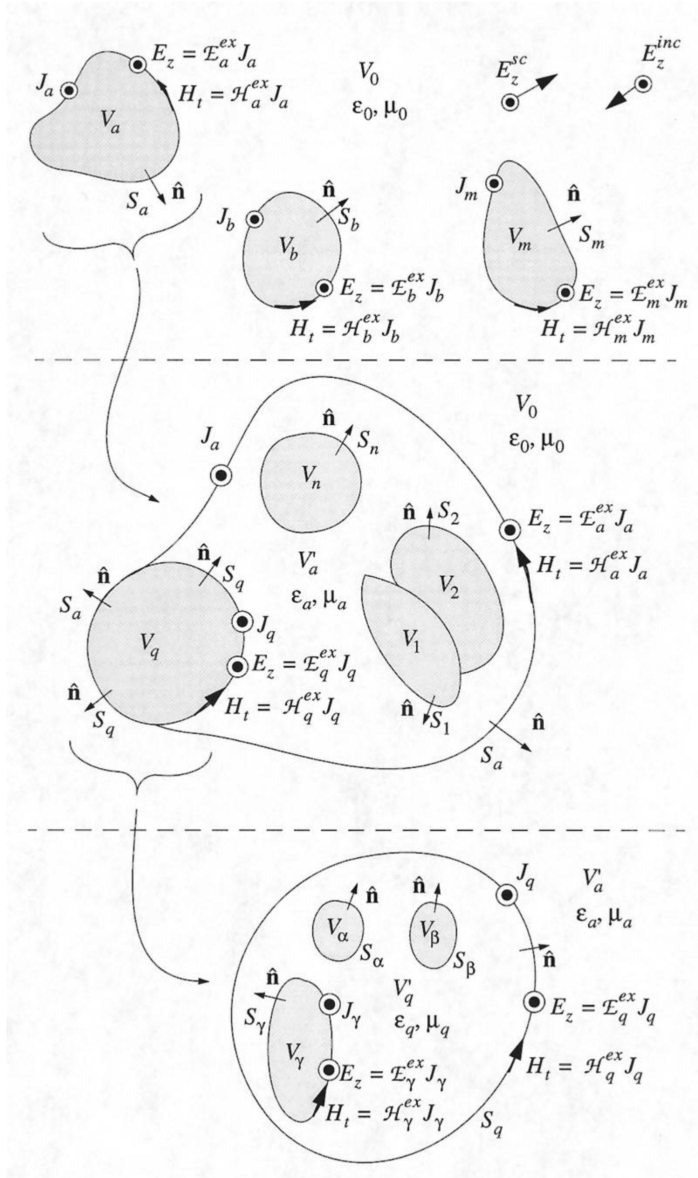


Fig. 1. TM illumination of a system of heterogeneous dielectric cylinders whose cross sections are organized in a hierarchical multiply-nested structure.

components tangent to the surface S_a exclusively in terms of a single electric current density J_a distributed on that same surface

$$E_z(\boldsymbol{r}) = \mathcal{E}_a^{ex} J_a, \qquad \boldsymbol{r} \in S_a \tag{4}$$

$$H_t(\boldsymbol{r}) = \mathcal{H}_a^{ex} J_a, \qquad \boldsymbol{r} \in S_a. \tag{5}$$

These exclusive operators are to be derived in a manner that is independent of external material and illumination so as to be invariant under rotation and translation.

B. The Local Scattering Problem

The framework of a *local scattering problem* is prescribed in order to provide an insightful physical interpretation of the recursive single integral equation formalism. Consider the heterogeneous body V_a described above and let us assume for the moment that the inclusions V_q touch neither each other nor the

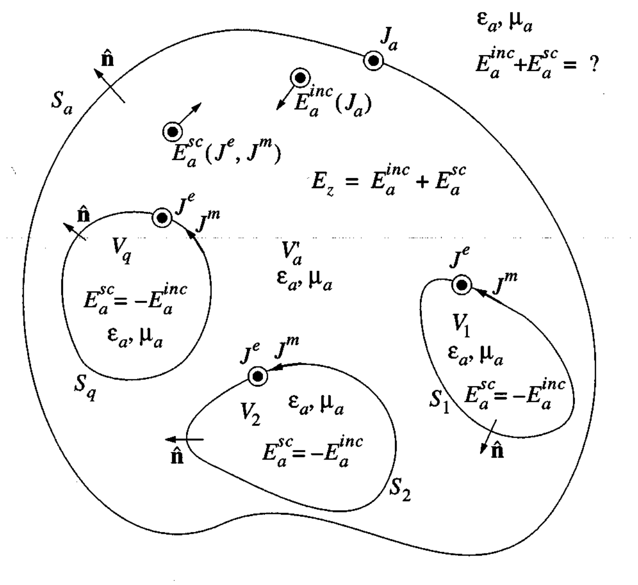


Fig. 2. The local scattering problem. All material is replaced with that of the homogeneous multiply-connected dielectric; the fields are made to vanish within the included surfaces by means of equivalent electric and magnetic surface currents; the incident field is generated by a single layer of electric surface current distributed on the enveloping surface.

enveloping surface S_a. This restriction will soon be lifted but, for the present, it aids in developing the field model. We formulate the field problem within V'_a as a local scattering problem in the following manner (see Fig. 2). All material internal and external to S_a is replaced with that of V'_a; the actual electric field E_z in V'_a is decomposed into the sum of a local incident field E_a^{inc} and a local scattered field E_a^{sc}; the local incident field is assumed to be generated by a single layer of electric current $\hat{\boldsymbol{z}} J_a$ distributed on the closed surface S_a, where $\hat{\boldsymbol{z}}$ is the unit vector in the positive z-axis direction, while the local scattered field is assumed to be generated by equivalent single layers of electric and magnetic current $\hat{\boldsymbol{z}} J^e$ and $\hat{\boldsymbol{t}} J^m$, respectively, distributed over the surfaces S_q of the inclusions V_q, $q = 1, 2, \ldots, n$, where $\hat{\boldsymbol{t}} = \hat{\boldsymbol{z}} \times \hat{\boldsymbol{n}}$ such that the sum of the local incident field and the local scattered field vanishes within each inclusion V_q. Imposing only one current density (i.e., J_a) on S_a implies that the sum of the local incident and the local scattered fields is unconstrained beyond S_a. Thus, an integral representation of the actual field E_z in the linear homogeneous subregion V'_a is obtained in terms of the local incident and the local scattered fields

$$E_a^{\text{inc}}(\boldsymbol{r}) + E_a^{sc}(\boldsymbol{r}) = \begin{cases} E_z & \text{for } \boldsymbol{r} \in V'_a \\ 0 & \text{for } \boldsymbol{r} \in V_q,\ q = 1, 2, \ldots, n \\ ? & \text{for } \boldsymbol{r} \notin V_a \end{cases} \tag{6}$$

with

$$E_a^{\text{inc}}(\boldsymbol{r}) = -\frac{\omega\mu_a}{4} \int_{S_a} J_a(\boldsymbol{r}') H_0^{(2)}(k_a R)\, dl' \tag{7}$$

$$E_a^{sc}(\boldsymbol{r}) = \sum_{q=1}^{n} \int_{S_q} \left(-\frac{\omega\mu_a}{4} J^e(\boldsymbol{r}') H_0^{(2)}(k_a R) - \frac{j}{4} J^m(\boldsymbol{r}') \frac{\partial}{\partial n'} H_0^{(2)}(k_a R) \right) dl' \tag{8}$$

where:

$k_a = \omega\sqrt{\varepsilon_a \mu_a}$	wave number in the homogeneous subregion V'_a;
$H_0^{(2)}$	Hankel function of the second kind and order zero;
R	$= \lvert \boldsymbol{r} - \boldsymbol{r}' \rvert$, with $\boldsymbol{r}$ and $\boldsymbol{r}'$ the position vectors of the field point and the source point, respectively;
$\partial/\partial n'$	derivative in the direction of the unit vector normal to the surface S_q at $\boldsymbol{r}'$.

The equivalent surface densities $\hat{\boldsymbol{z}} J^e$ and $\hat{\boldsymbol{t}} J^m$ at any interface point are expressed in terms of the actual magnetic and electric fields tangent to the scattering surfaces

$$\hat{\boldsymbol{z}} J^e(\boldsymbol{r}) = \hat{\boldsymbol{n}} \times \Delta \boldsymbol{H}(\boldsymbol{r}) = \hat{\boldsymbol{z}} H_t(\boldsymbol{r}) \tag{9}$$

$$\hat{\boldsymbol{t}} J^m(\boldsymbol{r}) = -\hat{\boldsymbol{n}} \times \hat{\boldsymbol{z}} \Delta E_z(\boldsymbol{r}) = \hat{\boldsymbol{t}} E_z(\boldsymbol{r}) \tag{10}$$

where:

$\boldsymbol{r}$	$\in S_q$;
q	$= 1, 2, \ldots, n$;
$\hat{\boldsymbol{n}}$ and $\hat{\boldsymbol{t}}$	functions of $\boldsymbol{r}$.

With (9) and (10), (8) becomes

$$E_a^{sc}(\boldsymbol{r}) = \sum_{q=1}^{n} \int_{S_q} \left(-\frac{\omega\mu_a}{4} H_t(\boldsymbol{r}') H_0^{(2)}(k_a R) - \frac{j}{4} E_z(\boldsymbol{r}') \frac{\partial}{\partial n'} H_0^{(2)}(k_a R) \right) dl' \tag{11}$$

where E_z and H_t are the tangential components of the actual electric and magnetic fields on the included surfaces. Thus, the local scattering model allows us to derive [via (6)] a relationship between the single unknown current density J_a and the two unknown field components E_z and H_t on the surface of each inclusion. This relationship provides the ground work for the following recursive reduction algorithm.

C. The Recursive Reduction Algorithm

Let the actual fields E_z and H_t tangent to each included surface S_q be expressed in terms of a single unknown electric current density J_q distributed on that same surface by means of the pair of exclusive operators $\mathcal{E}_q^{ex}$ and $\mathcal{H}_q^{ex}$ in the same manner as in (4) and (5)

$$E_z(\boldsymbol{r}) = \mathcal{E}_q^{ex} J_q, \qquad \boldsymbol{r} \in S_q, \quad q = 1, 2, \ldots, n \tag{12}$$

$$H_t(\boldsymbol{r}) = \mathcal{H}_q^{ex} J_q, \qquad \boldsymbol{r} \in S_q, \quad q = 1, 2, \ldots, n. \tag{13}$$

By this assertion, the equivalent electric and magnetic surface current densities J^e and J^m [see (9) and (10)] are replaced, in the local scattering problem, in favor of the single electric current density J_q. Substituting (12) and (13) in (11), and imposing the null-field boundary condition in (6) just inside each inclusion yields a system of n integral equations in terms of $n+1$ unknowns, namely the n unknown electric current densities J_q and the unknown source of the local incident field, i.e., the electric current density J_a. By isolating J_a from the J_qs and moving it to the right-hand side of each integral equation, we can write the n integral equations in the form

$$\sum_{q=1}^{n}\left[{}_{q}^{p}\mathcal{E}_{a}^{e}\mathcal{H}_{q}^{ex}+\left(-\frac{\delta_{pq}}{2}\,I+{}_{q}^{p}\mathcal{E}_{a}^{m}\right)\mathcal{E}_{q}^{ex}\right]J_q=-{}_{a}^{p}\mathcal{E}_{a}^{e}J_a \qquad p=1,\,2,\,\ldots,\,n \tag{14}$$

where:

δ_{pq}	Kronecker delta;
I	identity operator;
${}_{q}^{p}\mathcal{E}_{a}^{e}$ and ${}_{q}^{p}\mathcal{E}_{a}^{m}$	surface integral operators derived as follows:

$$ {}_{q}^{p}\mathcal{E}_{a}^{e}x=-\frac{\omega\mu_a}{4}\int_{S_q}x(\boldsymbol{r}')H_0^{(2)}(k_aR)\,dl', \qquad \boldsymbol{r}\in S_p \tag{15}$$

$$ {}_{q}^{p}\mathcal{E}_{a}^{m}x=-\frac{j}{4}\,⨍_{S_q}x(\boldsymbol{r}')\frac{\partial}{\partial n'}\,H_0^{(2)}(k_aR)\,dl', \qquad \boldsymbol{r}\in S_p. \tag{16}$$

The notation adopted for these operators is the following: the left-hand side superscript and subscript indicate the field surface and the source surface, respectively; the right-hand side subscript shows the corresponding material constants, for instance k_a and μ_a for V_a'; the right-hand side superscripts e and m indicate the nature of the source current involved (i.e., e: electric, m: magnetic) and (16) contains the principal value of the integral. Solution of the system of equations (14) in terms of the current density J_a yields a linear mapping from J_a to the single-source current density on each included surface

$$J_q=\mathcal{L}_qJ_a, \qquad q=1,\,2,\,\ldots,\,n. \tag{17}$$

The components of the actual fields tangent to the outer surface S_a can now be expressed exclusively in terms of the unknown electric current density J_a as in (4) and (5), i.e., $E_z(\boldsymbol{r})=\mathcal{E}_a^{ex}J_a$ and $H_t(\boldsymbol{r})=\mathcal{H}_a^{ex}J_a$, $\boldsymbol{r}\in S_a$, where the exclusive operators $\mathcal{E}_a^{ex}$ and $\mathcal{H}_a^{ex}$ for the surface S_a are obtained from (6) to (11) by expressing the electric and magnetic field intensities tangent to the scattering surfaces in (12) and (13), respectively, in terms of J_a via the source mapping (17). This finally gives

$$\mathcal{E}_a^{ex}={}_{a}^{a}\mathcal{E}_{a}^{e}+\sum_{q=1}^{n}[{}_{q}^{a}\mathcal{E}_{a}^{e}\mathcal{H}_{q}^{ex}+{}_{q}^{a}\mathcal{E}_{a}^{m}\mathcal{E}_{q}^{ex}]\mathcal{L}_q \tag{18}$$

$$\mathcal{H}_a^{ex}=-\tfrac{1}{2}\,I+{}_{a}^{a}\mathcal{H}_{a}^{e}+\sum_{q=1}^{n}[{}_{q}^{a}\mathcal{H}_{a}^{e}\mathcal{H}_{q}^{ex}+{}_{q}^{a}\mathcal{H}_{a}^{m}\mathcal{E}_{q}^{ex}]\mathcal{L}_q \tag{19}$$

where ${}_{q}^{p}\mathcal{H}_{a}^{e}$ and ${}_{q}^{p}\mathcal{H}_{a}^{m}$ are the integral operators that provide the contributions to H_t on S_p due to the electric and magnetic current densities on the surface S_q, respectively, in the form

$${}_{q}^{p}\mathcal{H}_{a}^{e}x=\frac{j}{4}\,⨍_{S_q}x(\boldsymbol{r}')\frac{\partial}{\partial n}\,H_0^{(2)}(k_aR)\,dl', \qquad \boldsymbol{r}\in S_p \tag{20}$$

$${}_{q}^{p}\mathcal{H}_{a}^{m}x=-\frac{1}{4\omega\mu_a}\int_{S_q}x(\boldsymbol{r}')\frac{\partial^2}{\partial n\partial n'}\,H_0^{(2)}(k_aR)\,dl', \qquad \boldsymbol{r}\in S_p \tag{21}$$

with the integral in (20) taken in principal value. The body V_a is now said to be reduced from the point of view of the wave scattering analysis by single-source surface integral equations. The exclusive operators (18) and (19) together with the source mapping (17) constitute an equivalent surface representation of the multiply-nested body V_a, which is derived independently of external material and illumination and is, therefore, invariant under rotation and translation. This means that the equivalent surface representation derived for a particular body can be simply copied to identical bodies that appear elsewhere in the wave scattering problem without repeating the relevant calculations and without transformation.

Obviously, the reduction algorithm derived for the body V_a may be applied to any multiply-nested body in which each inclusion has been described by a pair of exclusive operators in the manner of (12) and (13). Since these inclusions are themselves multiply-nested bodies, the reduction algorithm is clearly recursive. Consider a set of inclusions nested within an arbitrary host body. Solving (14) over the surface of these inclusions yields the linear mapping (17), which permits the calculation via (18) and (19), of the exclusive operators for the enveloping surface in terms of those previously determined for each inclusion surface. The body thus reduced may now be treated as an inclusion within a greater host body; in this manner, the algorithm progresses recursively until the exclusive operators have been determined over the outermost surface of the composite scattering body. This recursive procedure begins at the innermost level of nesting where the exclusive operators for the homogeneous inclusions are generated only in terms of the respective local incident fields, i.e., $n=0$ in (18) and (19).

D. Coincident Surfaces

From the perspective of the local scattering problem, it was convenient to visualize the inclusions (i.e., the local scatterers) as being completely surrounded by the homogeneous region V_a' in which we had defined the local incident and scattered fields. However, for practical heterogeneous problems it is necessary to place the inclusions in contact with each other and/or with the enveloping surface S_a (as shown in Fig. 1). Such practical arrangements can be accommodated by allowing the portion of V_a' that separates the respective surfaces to become infinitesimally thin such that in the limit the contact is obtained. We refer to these surfaces that touch in the limit as *coincident surfaces*.

Successful application of (14)–(21) to problems involving such coincident surfaces requires a proper accounting of the step discontinuities of E_z and H_t [see (9) and (10)] when crossing the corresponding surface distributions of magnetic and electric current, respectively. In the analysis in the preceding sections, these step discontinuities had been extracted from the Cauchy principal valued integrals of (16) and (20) and represented in (14) and (19) by an appropriately weighted identity operator. In order to accommodate coincident surfaces, we must broaden the interpretation of the Cauchy principal valued integral to allow not only for the extraction of the self-patch integral, but also for the extraction of the coincident-patch integral. In order to incorporate both the extracted self- and coincident-patch integrals into the integral equation formulation, we derive a specialized coincidence operator to take the place of the weighted identity operator used in (14) and (19).

As portions of the enveloping surface S_a are made coincident with the included surface S_q [see Fig. 3(a)], the electric and magnetic fields at points immediately inside S_a are equivalent

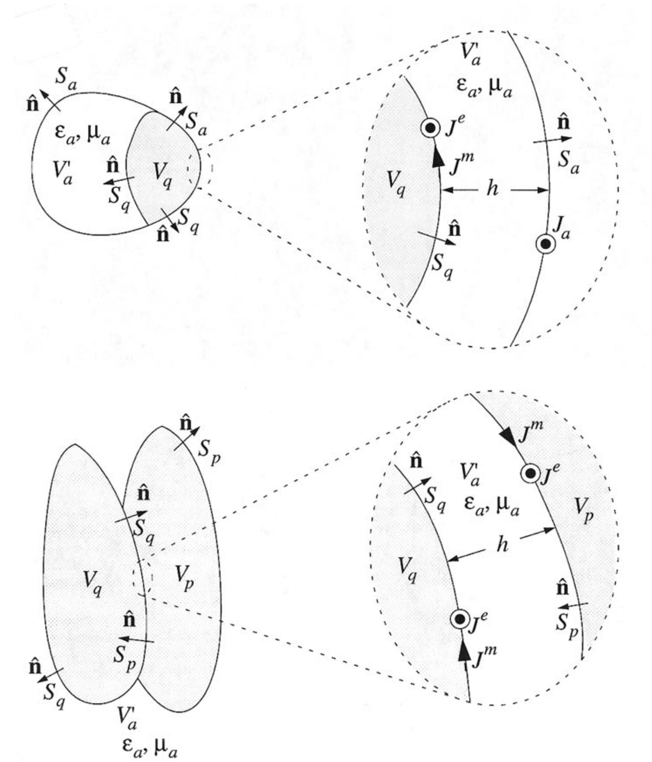


Fig. 3. Treatment of coincident surfaces in the limit as $h \to 0$. (a) The included surface is made coincident with the enveloping surface. (b) Two included surfaces are brought into contact.

to those immediately outside S_q. Thus, the electric field intensity tangent to S_a in V_a will receive a contribution of $(1/2)J^m$ from integration over the coincident-patch, while the magnetic field intensity tangent to S_a in V_a will receive contributions of $(1/2)J^e$ and $-(1/2)J_a$ from the coincident- and from the self-patch integrations, respectively. Similarly, as the neighboring inclusions V_q and V_p are brought together in the limit, the electric field tangent to S_p in V_p will receive a contribution of $-(1/2)J^m$ from the self-patch integration over S_p and $(1/2)J^m$ from the coincident-patch integration over S_q [see Fig. 3(b)]. These contributions, which are omitted correspondingly from the principal value integrals of (16) and (20), are accounted for by the *coincidence operator* ${}^p_q\mathcal{C}$ which is defined in terms of the Dirac delta function as

$$ {}^p_q\mathcal{C}x = \pm\tfrac{1}{2}\int_{S_q} x(\boldsymbol{r}')\delta(\boldsymbol{r}-\boldsymbol{r}')\,dl' $$
$$ \boldsymbol{r}\in S_p, \quad \begin{cases} +, & \text{if } p\neq q \\ -, & \text{if } p=q \end{cases} \tag{22} $$

such that the exclusive operators in (18) and (19) become now

$$ \mathcal{E}_a^{ex} = {}^a_a\mathcal{E}_a^e + \sum_{q=1}^{n}[{}^a_q\mathcal{E}_a^e\mathcal{H}_q^{ex} + ({}^a_q\mathcal{C} + {}^a_q\mathcal{E}_a^m)\mathcal{E}_q^{ex}]\mathcal{L}_q \tag{23} $$

$$ \mathcal{H}_a^{ex} = {}^a_a\mathcal{C} + {}^a_a\mathcal{H}_a^e + \sum_{q=1}^{n}[({}^a_q\mathcal{C} + {}^a_q\mathcal{H}_a^e)\mathcal{H}_q^{ex} + {}^a_q\mathcal{H}_a^m\mathcal{E}_q^{ex}]\mathcal{L}_q \tag{24} $$

and the null condition of the local scattering problem in (14) is obtained now in the form

$$ \sum_{q=1}^{n}[{}^p_q\mathcal{E}_a^e\mathcal{H}_q^{ex} + ({}^p_q\mathcal{C} + {}^p_q\mathcal{E}_a^m)\mathcal{E}_q^{ex}]J_q = -{}^p_a\mathcal{E}_a^e J_a $$
$$ p = 1, 2, \ldots, n \tag{25} $$

which yields the source mapping $J_q = \mathcal{L}_q J_a$, $q = 1, 2, \ldots, n$. Thus, the recursive single-source surface integral equation formulation can be applied to general heterogeneous dielectric bodies, once the homogeneous subregions have been organized into hierarchical multiply-nested structures. As a special case, if no surfaces are in contact, then (23)–(25) revert to (18), (19), and (14), respectively, via the coincidence operator in (22).

E. Solving for the Free-Space Scattered Field

Consider again the system of composite scattering cylinders $V_a, V_b, \ldots, V_m$ shown in a cross section in Fig. 1. Having reduced, independently, each of these heterogeneous bodies to an equivalent surface representation, we proceed to solve only for the single unknown electric current density on the surface of each of them, i.e., J_i, $i = a, b, \ldots, m$, in terms of the known incident electric field E_z^{inc}. The free-space scattering problem is treated in the same manner as each local scattering problem except that the enveloping surface now recedes to infinity and the null field condition yields the linear system

$$ \sum_{i=a,b\ldots}^{m} [{}^p_i\mathcal{E}_0^e\mathcal{H}_i^{ex} + ({}^p_i\mathcal{C} + {}^p_i\mathcal{E}_0^m)\mathcal{E}_i^{ex}]J_i = -E_z^{\text{inc}}|_{Sp} $$
$$ p = a, b, \ldots, m. \tag{26} $$

Once the surface current densities J_i, $i = a, b, \ldots, m$ are determined, the free-space scattered field E_z^{sc} can be expressed by using the Kirchhoff integral representation in terms of the fields tangent to each of the outermost scattering surfaces $E_z(\boldsymbol{r}) = \mathcal{E}_i^{ex}J_i$, $H_t(\boldsymbol{r}) = \mathcal{H}_i^{ex}J_i$, $\boldsymbol{r}\in S_i$, as

$$ E_z^{sc}(\boldsymbol{r}) = \sum_{i=a,b\ldots}^{m} ({}_i\mathcal{E}_0^e\mathcal{H}_i^{ex} + {}_i\mathcal{E}_0^m\mathcal{E}_i^{ex})J_i, \qquad \boldsymbol{r}\in V_0 \tag{27} $$

where the left-hand side superscripts have been omitted in the corresponding equations (15) and (16) in order to indicate that the field is expressed at an arbitrary point in V_0. For computing this free-space field only the current distributions on the outer surfaces of the heterogeneous bodies $a, b, \ldots, m$ need be determined.

F. Internal Fields by Backward Recursion

In the formulation presented, the field within any homogeneous region is expressed exclusively in terms of the source of the local incident field in that region, i.e., the unknown electric current density distributed on the respective enveloping surface. For example the electric field in region V_q' of Fig. 1 is expressed in terms of J_q as

$$ E_r(\boldsymbol{r}) = \left({}_q\mathcal{E}_q^e + \sum_{i=\alpha,\beta,\ldots}({}_i\mathcal{E}_q^e\mathcal{H}_i^{ex} + {}_i\mathcal{E}_q^m\mathcal{E}_i^{ex})\mathcal{L}_i\right)J_q $$
$$ \boldsymbol{r}\in V_q'. \tag{28} $$

An exceptional feature of the method developed consists in the fact that it is not necessary to determine this single-source density on all interfaces simultaneously. The exclusive operators and the mapping operators generated and archived during the course of the reduction can be used, in a backward recursion procedure, to obtain the single electric current density and, hence, the components of the electric and magnetic field intensities tangent to any internal surface. The single-source density on any internal surface is readily obtainable by following a chain of hierarchical mappings from the enveloping surface to a particular included surface. For example, J_γ in Fig. 1 can be obtained as $J_\gamma = \mathcal{L}_\gamma J_q \leftarrow J_q = \mathcal{L}_q J_a \leftarrow J_a$, such that $E_z(\boldsymbol{r}) = \mathcal{E}_\gamma^{ex} J_\gamma$ and $H_t(\boldsymbol{r}) = \mathcal{H}_\gamma^{ex} J_\gamma, \boldsymbol{r} \in S_\gamma$. To formalize this backward recursion, it is necessary to renumber the inclusions along the chosen path of the mapping. The subregion within which we require the electric and magnetic fields is renumbered V_{b_0}, having an outer surface S_{b_0} on which the unknown current density J_{b_0} is distributed. Our interior subregion V_{b_0} is embedded within the overall scattering system as $V_{b_0} \subset V_{b_1} \subset V_{b_2} \subset \ldots \subset V_{b_l}$, where V_{b_l} is a subregion on whose surface the single-source current density J_{b_l} is known. This makes the target subregion V_{b_0} to be uniquely identified even when it appears as an inclusion that is defined only as a translated or a translated and rotated copy of another inclusion. The distribution of J_{b_0} on the surface of the target body is determined recursively as

$$J_{b_i} = \mathcal{L}_{b_i} J_{b_{i+1}}, \qquad i = l-1, l-2, \ldots, 0 \tag{29}$$

where the mapping operators $\mathcal{L}_{b_i}$, $i = 0, 1, \ldots, l-1$, have been determined and saved during the course of the reduction algorithm. The field components tangent to S_{b_0} are obtained directly from $E_z(\boldsymbol{r}) = \mathcal{E}_{b_0}^{ex} J_{b_0}$ and $H_t(\boldsymbol{r}) = \mathcal{H}_{b_0}^{ex} J_{b_0}, \boldsymbol{r} \in S_{b_0}$.

III. Numerical Results

A. Moment Method Implementation

A simple pulse-expansion and point-matching moment-method implementation is used to transform the integral operators defined above into matrices whose number of rows and columns are given by the number of patches on the discretized field surface and source surface, respectively. Regular patch contributions are calculated by Gaussian quadrature. A nonvanishing self-patch contribution to the Cauchy principal valued integral due to the curvature and the finite length of the patch is accounted for in the manner of [22]. A self-patch formula for the integration of the weakly singular kernel of (15) can be found in [33].

The hypersingularity of the kernel of (21) can be relatively easily treated in the case of a pulse expansion implementation. Indeed, the magnetic field contribution due to magnetic current sources on a given surface S can be expressed as

$$H_t^m(\boldsymbol{r}) = -\frac{\omega\varepsilon}{4}\int_S J^m(\boldsymbol{r}')H_0^{(2)}(kR)(\hat{\boldsymbol{n}}\cdot\hat{\boldsymbol{n}}')\,dl' + \frac{j}{4\mu}\int_S \rho^m(\boldsymbol{r}')\hat{\boldsymbol{z}}\cdot(\hat{\boldsymbol{n}}\times\nabla H_0^{(2)}(kR))\,dl' \tag{30}$$

where ρ^m is the surface density of magnetic charge and is determined from the continuity equation for the magnetic current

$$\nabla\cdot\boldsymbol{J} = -\frac{\partial\rho^m}{\partial t} \tag{31}$$

as

$$\rho^m(\boldsymbol{r}) = \frac{j}{\omega}\frac{\partial}{\partial l}J^m(\boldsymbol{r}) \tag{32}$$

with the derivative taken along the cross-sectional contour in the counter-clockwise direction. Substituting (32) into (30) and integrating by parts returns (21). If the magnetic current density is assumed uniform over each patch (as it is for a pulse-expansion implementation), then (32) represents only the charge densities corresponding to the step discontinuity of the magnetic current density between adjacent patches (i.e., a train of weighted Dirac delta functions). Therefore, the magnetic current sources on an isolated patch are modeled as a uniform magnetic current originating from and terminating on equal but opposite magnetic charges such that the magnetic continuity equation is satisfied for each patch individually. A simple self-patch formula is thus obtained as

$$(H_t^m)_{ii} \approx -\omega\varepsilon\Delta_i J_i^m[(0.25+j0.287\,924)-j0.159\,154\,94\ln(k\Delta_i)] + \frac{1}{2}\sqrt{\frac{\varepsilon}{\mu}}J_i^m H_1^{(2)}(kR_{ii})\cos\left(\frac{\kappa_i\Delta_i}{r}\right) \tag{33}$$

where $H_1^{(2)}$ is the Hankel function of the second kind and order one and

$$R_{ii} = \begin{cases} \dfrac{2}{\kappa_i}\sin\left(\dfrac{\kappa_i\Delta_i}{4}\right), & \kappa_i \neq 0 \\ \Delta_i/2, & \kappa_i = 0 \end{cases} \tag{34}$$

is the distance from the end of the patch i to its center and is expressed in terms of the patch curvature κ_i and patch length Δ_i. This accurate self-patch formula does not require integration over the entire closed surface as in the method presented in [22].

B. Computed Examples

The validity of the recursion, the invariance of the exclusive operators under rotation and translation, and the treatment of coincident surfaces are each demonstrated by way of a simple numerical example. Consider a heterogeneous dielectric cylinder interacting with a TM plane wave propagating in the negative x-axis direction, as shown in Fig. 4. We organize the four homogeneous quadrants of this heterogeneous cross section by two methods: first, as four separate quadrants each illuminated directly by the incident wave [see Fig. 4(a)], and second, as a thrice-nested body formed by gathering the quadrants, one by one, in an ever increasing enveloping surface [see Fig. 4(b)].

Initially, we set the permittivity of each quadrant to $4\varepsilon_0$ and compare the numerical solution of the bistatic radar cross section (RCS) to that obtained analytically through series expansion (see Fig. 5). All surfaces are discretized to roughly ten patches per local wavelength which entails 20 patches on the radial surfaces and 32 patches on the arc of each quadrant. The

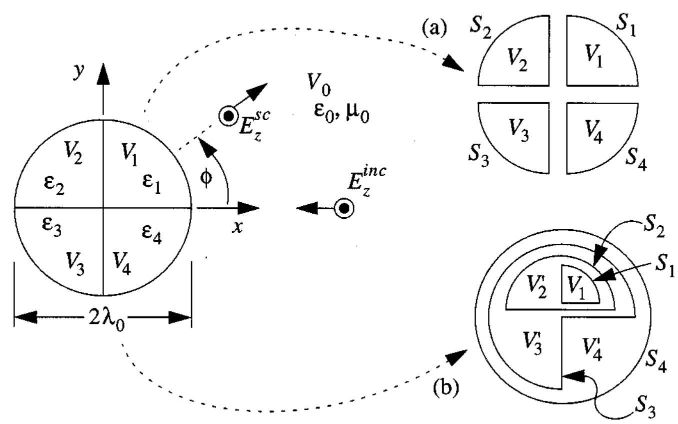


Fig. 4. Circular nonmagnetic ($\mu = \mu_0$) heterogeneous dielectric cylinder of radius λ_0 (i.e., the wavelength in free-space). The heterogeneous cross section is structured in two ways: (a) as four sectors and (b) as a thrice-nested body.

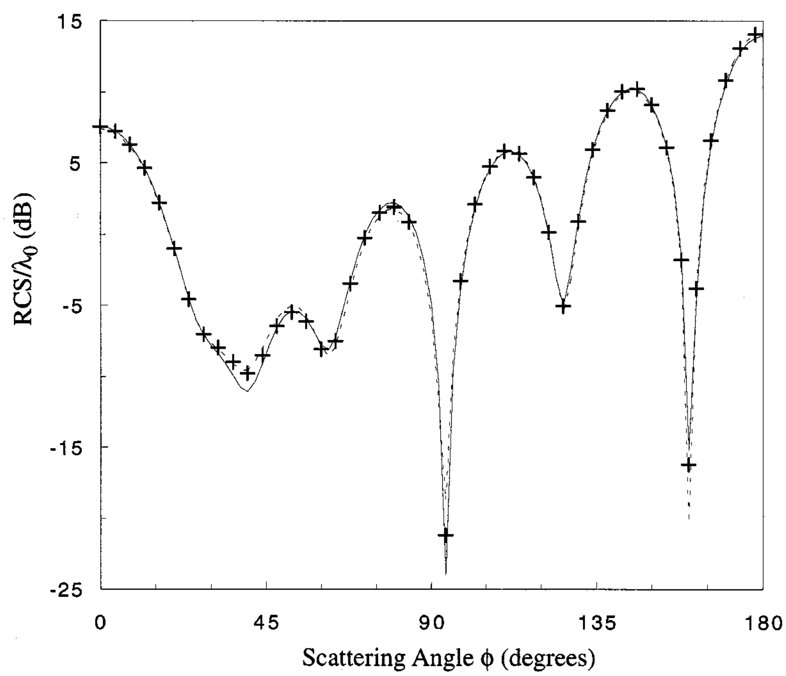


Fig. 5. Bistatic RCS of the cylinder shown in Fig. 4 with uniform permittivity $\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = \varepsilon_3 = 4\varepsilon_0$: analytical solution —, "four-sector" formulation - - -, and "thrice-nested" formulation $+++$.

solutions obtained by the "four-sector" and "thrice-nested" formulations are identical to each other and are in excellent agreement with both the analytical solution for the homogeneous circular cylinder and the numerical solution obtained by the EFIE assuming an identical surface discretization.

The invariance of the exclusive operators under rotation and translation is demonstrated by the "four-sector" formulation. Exclusive operators calculated for region V_1 are copied to regions V_2, V_3, and V_4 without further calculation since these regions are simply rotated and translated copies of V_1. The wave scattering problem is solved, for the "four-sector" formulation, in terms of a single unknown electric current density distributed on the surface of each of the four regions.

The validity of the recursive construction of the exclusive operators is demonstrated by the "thrice-nested" formulation [see Fig. 4(b)]. The region V_1 is embedded as an inclusion in the semicircular region V_2, which is an inclusion of the notched cylinder V_3, which itself is an inclusion of the circular cylinder V_4, that is, $V_1 \subset V_2 \subset V_3 \subset V_4$ (the subregions V_2', V_3' and V_4' correspond to the homogeneous subregion V_a' in Fig. 1). By the proposed recursion, the exclusive operators obtained for the homogeneous region V_1 are used to generate the exclusive operators on the surface S_2, which in turn contribute to the exclusive operators on S_3 and, finally, to those on S_4. Thus, the wave scattering problem is solved only in terms of the single unknown electric current density J_4 on S_4.

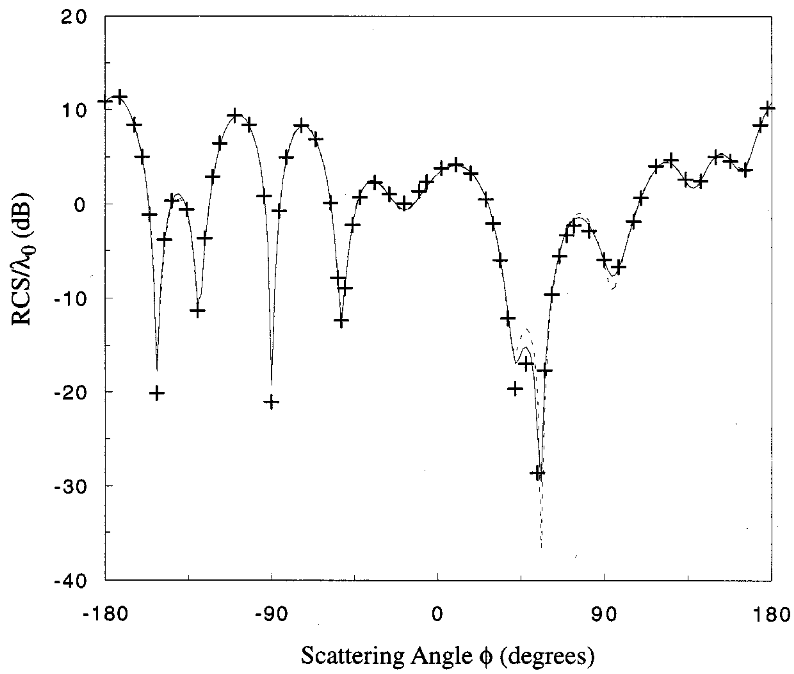


Fig. 6. Bistatic RCS of the cylinder shown in Fig. 4 with $\varepsilon_1 = 4\varepsilon_0$, $\varepsilon_2 = 2\varepsilon_0$, $\varepsilon_3 = 6\varepsilon_0$, and $\varepsilon_4 = 8\varepsilon_0$: "four-sector" formulation —, "thrice-nested" formulation $+++$, and EFIE method - - -.

As a second test, we create a genuine heterogeneous cylinder by setting the permittivities of the quadrants to $\varepsilon_1 = 4\varepsilon_0$, $\varepsilon_2 = 2\varepsilon_0$, $\varepsilon_3 = 6\varepsilon_0$, and $\varepsilon_4 = 8\varepsilon_0$. Surface discretization is set to 30 patches on the radial segments and 45 patches on the quadrant arcs in order to provide roughly ten patches per local wavelength in V_4. We note that the bistatic RCS calculated by way of the "four-sector" and "thrice-nested" models are both in excellent agreement with that calculated by way of the coupled EFIE (see Fig. 6). The same is true for the electric and magnetic fields tangent to the coincident surfaces along the positive x-axis (see Fig. 7). Having solved for the single electric current density on the surface of each homogeneous quadrant, the "four-sector" formulation yields these tangential components directly in terms of the single electric current surface density J_1 on the surface S_1 as $E_z = \mathcal{E}_1^{ex} J_1$ and $H_t = \mathcal{H}_1^{ex} J_1$. The "thrice-nested" formulation only requires a backward recursion in order to reconstruct these same fields since, having reduced the scattering obstacle to the equivalent circular cylinder V_4, we had only solved for the single electric current density J_4 on the surface S_4. The fields tangent to S_3 are constructed as $E_z = \mathcal{E}_3^{ex} J_3$ and $H_t = \mathcal{H}_3^{ex} J_3$, where we have applied the source mapping $J_3 = \mathcal{L}_3 J_4$. As a further confirmation of our backward recursion and our treatment of coincident surfaces, we continue this source mapping to obtain $J_2 = \mathcal{L}_2 J_3$ and $J_1 = \mathcal{L}_1 J_2$ and, thus, express the electric and magnetic fields at the same points along the positive x-axis in the form $E_z = \mathcal{E}_2^{ex} J_2$ and $H_t = \mathcal{H}_2^{ex} J_2$, as well as $E_z = \mathcal{E}_1^{ex} J_1$ and $H_t = \mathcal{H}_1^{ex} J_1$. Recovery of E_z and H_t on S_1 by backward recursion requires only 4.8% of the total execution time.

The question of accuracy and residual error is further investigated by considering the bistatic RCS of a "phantom" cylinder that is formed by setting the permittivity of each quadrant of the circular dielectric cylinder shown in Fig. 4 to that of free-space, i.e., ε_0. We first note that the residual error does not increase with increasing levels of nesting, moreover the RCS generated by the "thrice-nested" model is generally less than that of the "four-sector" model. Furthermore, the residual error of both formulations decreases with increasing discretization, thus indicating the convergence of both formulations.

C. Computational Complexity

We begin by studying the number of operations required in order to form the equivalent surface representation of a single multiply-nested body V_a that contains the inclusions V_q, $q = 1, 2, \ldots, n$. The total number of operations consists of four components: the two matrix–matrix multiplications required to form the left-hand side of (14), the LU decomposition of the same, the back substitutions required to generate the source mapping operators L_q, and the six matrix–matrix multiplications per inclusion required to generate the two exclusive operators $\mathcal{E}_a^{ex}$ and $\mathcal{H}_a^{ex}$. It should be noted that prior to equivalencing V_a, the operations listed above must be carried out for each included body V_q, with the exception of those bodies that are simply copies of previously equivalenced bodies (whose generated exclusive operators can be reused without repeating these operations). If V_a is a part of a larger system of scatterers in free-space, with $V_i, i = a, b, \ldots, m$, as in Fig. 1, then the wave scattering problem is solved in terms of the known incident field through an additional two matrix–matrix multiplication needed to form the left-hand side of (26) and the LU decomposition of the same. The fields on any interior surface are generated inexpensively by a small number of matrix–vector multiplications.

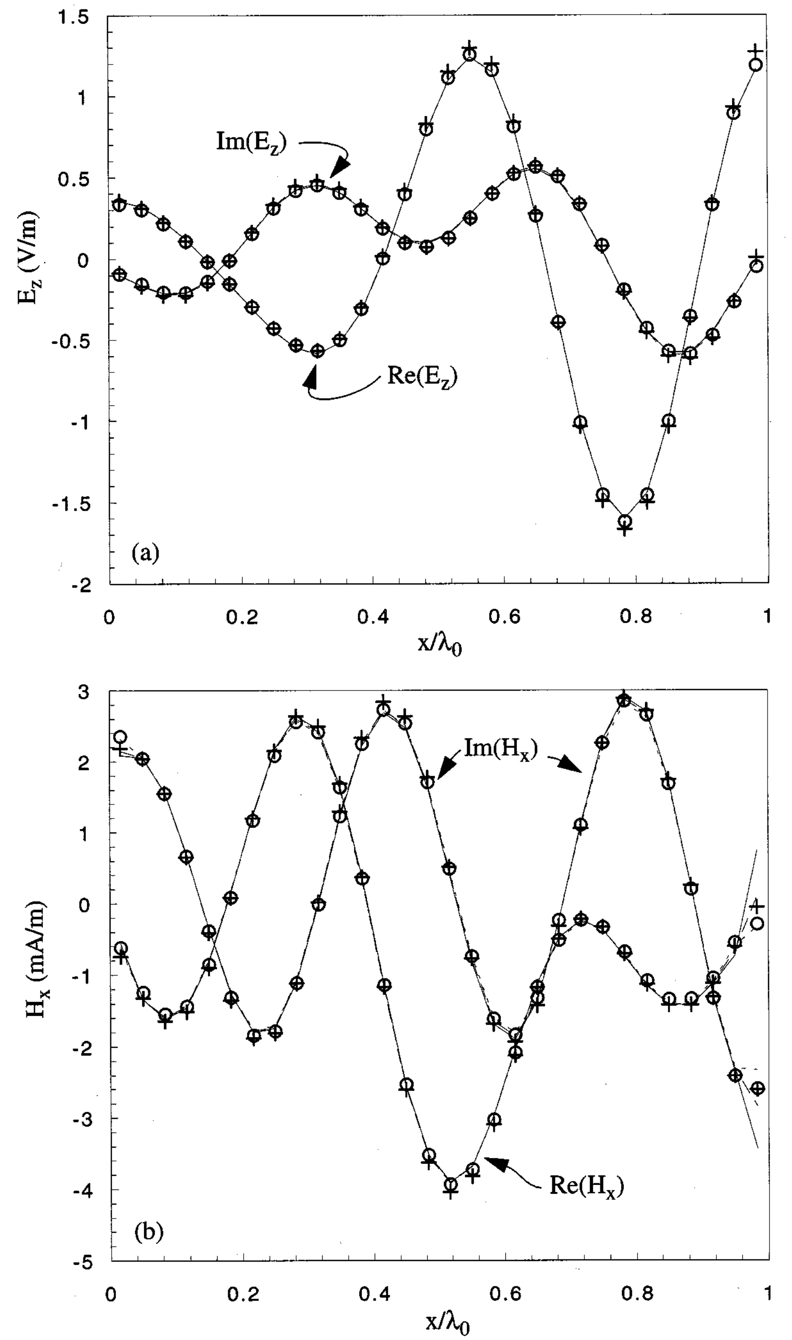


Fig. 7. Intensity of (a) electric and (b) magnetic fields tangent to coincident surfaces along the x-axis of Fig. 4, with $\varepsilon_1 = 4\varepsilon_0$, $\varepsilon_2 = 2\varepsilon_0$, $\varepsilon_3 = 6\varepsilon_0$, and $\varepsilon_4 = 8\varepsilon_0$: on S_1- - -, on S_2 − − −, and on S_3 — by the "thrice-nested" formulation; on S_1 by the "four-sector" formulation ∘ ∘ ∘; and by the EFIE $+++$.

The actual computational complexity depends greatly upon the hierarchical organization of the heterogeneous body. We consider two special cases for a heterogeneous body consisting of N equally sized homogeneous dielectric subregions, each of which being discretized by c surface patches: first, the proposed method is applied in the absence of any hierarchical organization (i.e., only one local scattering problem consisting of all N homogeneous dielectric subregions) and the computational complexity is found to increase as $O(N^3)$ and, second, the proposed method is applied within a nested hierarchical structure when the computational complexity is found to increase in proportion to $O(N^{1.5})$.

To illustrate the importance of the hierarchical organization, consider a heterogeneous cross section consisting of $N = r^2$ equally sized homogeneous dielectric subregions arranged on a $r \times r$ square grid. Let the cross-sectional contour surrounding each square subregion be discretized into c segments. Without imposing a hierarchical organization, the proposed method yields the single unknown electric current density over the surface of all subregions in $(1/3)c^3r^4(r^2 + 6)$ multiplications/divisions. Direct simultaneous solution of the electric and magnetic fields tangent to each interface by way of the EFIE would require $(1/3)c^3r^3(r^3+3r^2+3r+1)$ multiplications/divisions. Thus, the computational complexities of both methods for large r is of the order of $(1/3)(cN)^3$. Even in the case of a single homogeneous dielectric cylinder, where the single-source surface integral equation requires only one half the number of unknowns as compared to that required by the EFIE, the overall computational complexity is only reduced by one eighth.

Alternatively, consider a similar heterogeneous cross section consisting of $N = 4^s$ equally sized homogeneous dielectric subregions arranged on a $2^s \times 2^s$ square grid. As before, let the cross section of the surface surrounding each square region be discretized into c segments. We organize the heterogeneous cross section in a hierarchical manner as follows: the entire cross section is divided into four $2^{s-1} \times 2^{s-1}$ sized subregions, each of which contains four $2^{s-2} \times 2^{s-2}$ sized inclusions. Similarly, each subsequent inclusion contains four inclusions and this nesting process continues for a total of $s-1$ levels until the inclusions are of unit size. After some tedious algebra, the proposed method yields the solution of the wave scattering problem in a total number of multiplications/divisions

$$O_{tot} = \tfrac{2}{3}\, c^3 N^{1.5} \left(5 + 14 \sum_{i=1}^{s-1} 2^{-i}\right), \qquad s > 1 \tag{35}$$

which converges geometrically to $O_{tot} = (2/3)19c^3N^{1.5}$ for large s. Thus, the proposed recursive algorithm reduces the computational complexity from the order of $O(N^3)$ to the order of $O(N^{1.5})$ when the heterogeneous cross section is organized into a nested hierarchical structure. The total number of arithmetic operations for a given multibody structure is further substantially reduced when various subregions are translated and/or rotated copies of each other [29]–[32].

By recursive formulation of the single-source surface integral equation method, the overall storage requirements are limited to only those matrices needed to represent the single largest local scattering problem. For the case of the preceding example, the largest local scattering problem consists of four $2^{s-1} \times 2^{s-1}$ sized heterogeneous subregions; therefore, the storage requirements increase as $(1/4)c^2N$ where $N = 4^s$ is the total number of equally sized homogeneous subregions, and c is the number of surface patches over each of them. In comparison, the storage requirements for the simultaneous solution of all unknowns by way of the EFIE increase as $(cN)^2$.

Obviously, the proposed solution method does not require such a rigid grouping as that described above. In fact, in many applications, such as material optimization problems, it is more practical and efficient to group the dielectric regions into components that may be reused in subsequent problems without repetition of the equivalencing procedure.

IV. Conclusion

The inner structure of a general heterogeneous body is completely accounted for by a pair of surface integral operators that yield the electric and magnetic fields tangent to the outermost surface in terms of only a single unknown electric current density distributed on this same surface. These exclusive operators are derived by recursive application of a single-source surface integral equation formalism to a multiply-nested body, which is formed by organizing its distinct homogeneous subregions into hierarchically structured inclusions. Each inclusion is analyzed as a local scatterer embedded within a surrounding homogeneous medium in which the fields are formulated. A novel treatment of coincident surfaces permits this surrounding homogeneous region to become vanishingly thin as neighboring inclusions are brought in contact in the limit. In this way, any heterogeneous problem may be treated as multiply-nested. The resulting equivalent surface representation is formulated independently of external material and illumination and is, therefore, invariant under rotation and translation. Thus, a surface equivalent derived for a particular heterogeneous body may be archived, duplicated, and reused in an assortment of surface integral equation models without repeating the reduction procedure. This reduction algorithm does not preclude the calculation of fields at interior points. In fact, the exclusive operators and source mapping operators that were archived during the course of the reduction provide windows through which to observe the fields in any interior region through a fast backward recursion. The overall computational complexity and storage requirements of the associated algorithm only increase as $O(N^{1.5})$ and $O(N)$, respectively, with the number N of homogeneous subregions of the heterogeneous structure considered. This represents a substantial improvement over the simultaneous solution of all unknowns via the EFIE for which the computational complexity and storage requirements increase as $O(N^3)$ and $O(N^2)$ for the same N, respectively.

It should be noticed that in the special case where the inclusions do not touch each other or the enveloping surface, the proposed method reverts to that presented in [29] and [30].

References

[1] M. A. Jensen and J. D. Freeze, "A recursive Green's function method for boundary integral analysis of inhomogeneous domains," *IEEE Trans. Antennas Propagat.*, vol. 46, pp. 1810–1816, Dec. 1998.

[2] A. Sahin and E. L. Miller, "Recursive T-matrix methods for scattering from multiple dielectric and metallic objects," *IEEE Trans. Antennas Propagat.*, vol. 46, pp. 672–678, May 1998.

[3] W. C. Chew, J.-M. Jin, C.-C. Lu, E. Michielssen, and J. M. Song, "Fast solution methods in electromagnetics," *IEEE Trans. Antennas Propagat.*, vol. 45, pp. 533–543, Mar. 1997.

[4] W. C. Chew and C.-C. Lu, "The recursive aggregate interaction matrix algorithm for multiple scatterers," *IEEE Trans. Antennas Propagat.*, vol. 43, pp. 1483–1486, Dec. 1995.

[5] D. Felbacq, G. Tayeb, and D. Maystre, "Scattering by a random set of parallel cylinders," *J. Opt. Soc. Amer. A*, vol. 11, pp. 2526–2538, Sept. 1994.

[6] Y. M. Wang and W. C. Chew, "A recursive T-matrix approach for the solution of electromagnetic scattering by many spheres," *IEEE Trans. Antennas Propagat.*, vol. 41, pp. 1633–1639, Dec. 1993.

[7] W. C. Chew and C.-C. Lu, "The use of Huygens' equivalence principle for solving the volume integral equation of scattering," *IEEE Trans. Antennas Propagat.*, vol. 41, pp. 897–904, July 1993.

[8] A. J. Poggio and E. K. Miller, "Integral equation solutions of three-dimensional scattering problems," in *Computer Techniques for Electromagnetics*, R. Mittra, Ed. New York: Pergamon, 1973, pp. 159–264.

[9] J. R. Mautz and R. F. Harrington, "Electromagnetic scattering from a homogeneous material body of revolution," *AEÜ*, vol. 33, no. 2, pp. 71–80, 1979.

[10] P. M. Goggans, A. A. Kisk, and A. W. Glisson, “Electromagnetic scattering from objects composed of multiple homogeneous regions using a region-by-region solution,” *IEEE Trans. Antennas Propagat.*, vol. 42, pp. 865–871, June 1994.

[11] D. Maystre and P. Vincent, “Diffraction d’une onde électromagnétique plane par un object cylindrique non infiniment conducteur de section arbitraire,” *Opt. Commun.*, vol. 5, pp. 327–330, Aug. 1972.

[12] D. Maystre, “A new general integral theory for dielectric coated gratings,” *J. Opt. Soc. Amer.*, vol. 68, pp. 490–495, Apr. 1978.

[13] ——, “Integral methods,” in *Electromagnetic Theory of Gratings*, R. Petit, Ed. Berlin, Germany: Springer-Verlag, 1980, pp. 63–100.

[14] E. Marx, “Single integral equation for wave scattering,” *J. Math. Phys.*, vol. 23, pp. 1057–1065, June 1982.

[15] ——, “Integral equation for scattering by a dielectric,” *IEEE Trans. Antennas Propagat.*, vol. AP-32, pp. 166–172, Feb. 1984.

[16] A. W. Glisson, “An integral equation for electromagnetic scattering by homogeneous dielectric bodies,” *IEEE Trans. Antennas Propagat.*, vol. AP-32, pp. 173–175, Feb. 1984.

[17] L. F. Knockaert and D. De Zutter, “Integral equation for the fields inside a dielectric cylinder immersed in an incident E-wave,” *IEEE Trans. Antennas Propagat.*, vol. AP-34, pp. 1065–1067, Aug. 1986.

[18] R. E. Kleinman and P. A. Martin, “On the single integral equation for the transmission problem of acoustics,” *SIAM J. Appl. Math*, vol. 48, pp. 307–325, Apr. 1988.

[19] P. A. Martin and P. Ola, “Boundary integral equations for the scattering of electromagnetic waves by a homogeneous dielectric obstacle,” *Proc. Roy. Soc. Edinburgh Sect. A*, vol. 123, pp. 185–208, 1993.

[20] J. Mautz, “A stable integral equation for electromagnetic scattering from homogeneous bodies,” *IEEE Trans. Antennas Propagat.*, vol. 37, pp. 1070–1071, Aug. 1989.

[21] E. Marx, “Scattering by an arbitrary cylinder at a plane interface: broad side incidence,” *IEEE Trans. Antennas Propagat.*, vol. 37, pp. 619–628, May 1989.

[22] ——, “Electromagnetic scattering from a dielectric wedge and the single hypersingular integral equation,” *IEEE Trans. Antennas Propagat.*, vol. 41, pp. 1001–1008, Aug. 1993.

[23] D. R. Swatek and I. R. Ciric, “Single integral equation for wave scattering by many-body systems,” in *Proc. 1995 URSI Int. Symp. Electromagn. Theory*, St. Petersburg, Russia, May 1995, pp. 286–288.

[24] ——, “Single source integral equation for wave scattering by multiply-connected dielectric cylinders,” *IEEE Trans. Magn.*, vol. 32, pp. 878–881, May 1996.

[25] E. Marx, “Integral equations for scattering by a particle in a layer,” in *Proc. 1997 IEEE Antennas Propagat. Soc. Int. Symp.*, Montréal, Canada, July 1997, pp. 878–881.

[26] D. R. Swatek and I. R. Ciric, “Eliminating spurious solutions in single integral equations for wave scattering by systems of dielectric cylinders,” in *Proc. 1997 PIERS Progress Electromagn. Res. Symp.*, Kowloon, Hong Kong, Jan. 1997, p. 406.

[27] G. Pelosi and G. Toso, “A boundary element approach to the scattering from inhomogeneous dielectric bodies,” *IEEE Trans. Antennas Propagat.*, vol. 46, pp. 602–603, Apr. 1998.

[28] D. R. Swatek and I. R. Ciric, “Single integral equation for wave scattering by a layered dielectric cylinder,” *IEEE Trans. Magn.*, vol. 34, pp. 2724–2727, Sept. 1998.

[29] ——, “Reduction of multiply-nested dielectric bodies for wave scattering analysis by single integral equations,” in *Proc. 1998 URSI Int. Symp. Electromagn. Theory*, Thessaloniki, Greece, May 1998, pp. 560–562.

[30] ——, “Reduction of multiply-nested dielectric bodies for wave scattering analysis by single source surface integral equations,” *J. Electromagn. Waves Applicat.*, vol. 14, pp. 405–422, Mar. 2000.

[31] ——, “An equivalent surface representation for wave scattering by inhomogeneous objects,” in *Proc. ANTEM’98*, Ottawa, Canada, Aug. 1998, pp. 255–258.

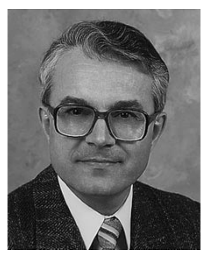
[32] ——, “Equivalent surface representation for heterogeneous dielectric cylinders,” *Electron. Lett.*, vol. 34, pp. 2321–2322, Nov. 1998.

[33] R. F. Harrington, *Field Computation by Moment Methods*. New York: Macmillan, 1968.

David R. Swatek (S’85–M’88) received the B.Sc. degree in electrical engineering from the University of Manitoba, Winnipeg, Canada, in 1988. He is currently working toward the Ph.D. degree in the field of applied electromagnetics at the same university.

In 1988, he joined Manitoba Hydro, Canada, where he has been involved in the modeling of the electromagnetic phenomena associated with high-voltage power transmission lines.

Mr. Swatek received a 1995 URSI Commission B (fields and waves) Young Scientist Award for his work in modeling electromagnetic scattering and diffraction. He is a registered Professional Engineer in the Province of Manitoba, Canada.



Ioan R. Ciric (SM’84–F’97) is a Professor of electrical engineering at the University of Manitoba, Winnipeg, Canada. His major research interests are in the mathematical modeling of stationary and quasi-stationary fields, electromagnetic fields in the presence of moving solid conductors and levitation, field theory of special electrical machines, dc corona-ionized fields, analytical and numerical methods for wave scattering and diffraction problems, propagation along waveguides with discontinuities, transients, and inverse problems. He has published about 200 technical papers and three electrical engineering textbooks and has been a contributor to chapters on magnetic field modeling and electromagnetic scattering by complex objects in four books.

Dr. Ciric is a member of the Electromagnetics Academy. He is listed in *Who’s Who in Electromagnetics*.