Laboratory Models Used in the Investigation of Radio Wave Propagation in Irregular Structures

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***Abstract*—A laboratory model first conceived by Wait to investigate radio wave propagation in the perturbed environment of the earth-ionosphere waveguide is described here. The construction of the laboratory model involves the exploitation of the following fundamental invariant properties of Maxwell's equations and the ability to physically simulate the appropriate boundary conditions including the earth's curvature: 1) invariance of Maxwell's equations to size-wavelength transformations permits the scaling down in size of the earth-ionosphere waveguide; 2) duality relationships between the electric and magnetic fields permits the representation of the azimuthally independent TM$_{n,o}$ modes excited by vertical dipoles in the earth ionosphere waveguide by the TE$_{n,o}$ modes in rectangular waveguides; 3) a perfectly conducting magnetic wall (where the tangential component of the magnetic field vanishes) is simulated through the use of imaging techniques; 4) to account for dissipation in the ionosphere, an equivalent surface impedance boundary is simulated using a wall loading material with a specific thickness and complex permittivity; and 5) to simulate the earth's curvature in the rectangular waveguide, an especially fabricated dielectric material with a prescribed permittivity height profile is used as the medium of propagation in the interior of the waveguide. All five of the above artifices have been employed in order to construct a scaled model of the earth-ionosphere waveguide. However, one or a combination of them can be employed by researchers today to construct laboratory models from which controlled experimental data can be obtained to validate analytical and numerical solutions as well as to provide insights for novel approaches to solve difficult propagation problems.**

***Index Terms*—Earth-ionosphere waveguide, electromagnetic (EM) wave propagation, nonhomogeneous electromagnetic media.**

I. Introduction

N SITU experimental data of the transmitted waves in the earth-ionosphere waveguide may be obtained in the straightforward manner at the surface of the earth. But in order to analyze these experimental results and to compare them with theoretical solutions, it is necessary to know the effective height of the ionosphere, its vertical profile, etc., as functions of distance along the path of propagation. Such information is usually not known so that it is difficult to make comparisons between experiment and theory. Hence, *in situ* measurements can only provide a partial check on the theoretical investigations.

A very useful method for conducting an experimental study of propagation in the earth-ionosphere waveguide is by means of a frequency scaled model, that can be placed inside a carefully controlled laboratory. Several classes of waveguide boundary perturbations, can be modeled. The controlled experimental data obtained from these precisely defined models has been successfully used to check the validity of the approximate theoretical solutions. To make such use of the experimental data conducted in the laboratory model, it is not even necessary that the model correspond exactly to the actual physical conditions in the earth-ionosphere waveguide. It need only be devised to prove or disprove an analytical solution.

The specific objective of this paper is to describe the development of electromagnetic (EM) models. It is shown here how they are used in the investigation of propagation in the earth ionosphere waveguide for cases in which the effective height of the ionosphere and or its physical parameters (characterized by a surface impedance) change along the path of propagation. The models also provide insights for the derivation of novel solutions to these problems.

The construction of the laboratory model of the earth-ionosphere waveguide involves the exploitation of the following fundamental invariant properties of Maxwells equations and the ability to simulate the boundary conditions in a physically realizable manner.

1) Scaling: Invariance of Maxwell's equation to size/wavelength transformations.
2) Duality relationships between the electric and magnetic fields. The dual model was constructed because the TM$_{n,o}$ modes do not exist in waveguides with rectangular cross sections and perfectly conducting boundaries.
3) Use of imaging techniques in EM theory to simulate a magnetic conducting wall upon which the tangential components of the magnetic field vanish.
4) Physical simulation of impedance boundary conditions to account for dissipation in the ionosphere.
5) Simulation of the earth's curvature through the use of inhomogeneous dielectric materials.

Each of the topics listed above, associated with the construction of the laboratory model, is considered here. The experiments conducted in the model are described briefly in this paper (Section IV). A detailed discussion is beyond the scope of this paper. The involvement of Wait in this work is referred to in his last published manuscript [1].

II. Overview of the Problem

Following Marconi's discoveries that radio waves decay more slowly than was predicted on the basis of diffraction theory (ground waves), the concept that radio waves at low frequencies propagate between the earth and the ionosphere as in a spherical waveguide has been extensively developed.

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Several approaches have been employed to obtain solutions for the propagation of EM waves in such an earth-ionosphere waveguide, with their respective usefulness and limitations depending upon the frequency of propagation and the distance between the transmitter and the receiver. The exact harmonic series solution for EM waves in the spherical waveguide lacked good convergence. This series was converted by Watson [2] into a more rapidly converging series. The new series representation corresponds to a sum of residue contributions at poles in the complex plane. The terms of the series associated with these poles are the waveguide modes. Watson treated in particular the case of low frequencies and assumed very highly conducting spherical boundaries.

At higher frequencies and at short distances from the transmitter, the ray theory (multiple hop) involving geometrical optics seems to be the most efficient solution. Adaptations of the ray theory to radio propagation have been presented by Wait and Murphy [3], [4], Budden [5], Bremmer [6], and others. At lower frequencies and at larger distances from the transmitter these solutions are less suitable. A self-contained treatment of the waveguide mode theory with special reference to VLF radio waves is given by Wait [7], who lists extensive references to contributions in this field.

The modal equation for $\mathrm{TM}_{n,o}$ (azimetrically syametric transverse magnetic) waves propagating between concentric (uniform) spherical surfaces bounded by a homogeneous earth of radius a_o and a homogeneous ionosphere of radius $a_o + h$ (Fig. 1) to a first-order approximation for $|(ka_o/2)^{1/3}C| \gg 1$ is given by [7]

$$R_g(C)R_i(C')\exp\left\{-i2k\int_o^h \left[C^2+\frac{2y}{a}S^2\right]^{1/2} dy\right\} = \exp\{-i2\pi n\} \qquad n=1,2,3,\cdots \tag{1}$$

in which an $\exp(i\omega t)$ time dependence is assumed and the reflection coefficients for waves incident on the earth and the ionosphere are, respectively

$$R_g(C)=\frac{N_gC-C_g}{N_gC+C_g} \quad \text{and} \quad R_i(C')=\frac{N_iC'-C_i'}{N_iC'+C_i'}. \tag{2}$$

In (2), C and C' are the cosines of the angles of incidence in the air (at the earth and ionosphere boundaries) and C_g and C_i' are the cosines of the angles of incidence in the earth and in the ionosphere respectively. They are related through Snell's law. The corresponding sines of the angles S and S' satisfy $(a_o + h)S' = a_oS$. It should be noted that, in general, C and S can be complex as in the case when the modes are attenuated.

The respective refractive indexes are

$$N_g=\left[\frac{\sigma_g+i\varepsilon_g\omega}{i\varepsilon\omega}\right]^{1/2}, \qquad N_i=\left[\frac{\sigma_i+i\varepsilon_i\omega}{i\varepsilon\omega}\right]^{1/2} \tag{3}$$

where σ_g, ε_g and σ_i, ε_i are the electrical conductivity and permittivity of the earth and the ionosphere while the permeability is assumed to be that of free space everywhere.

Consideration has been restricted to the propagation of $\mathrm{TM}_{n,o}$ modes in the earth-ionosphere waveguide, corresponding to radiation from vertical electric dipoles.

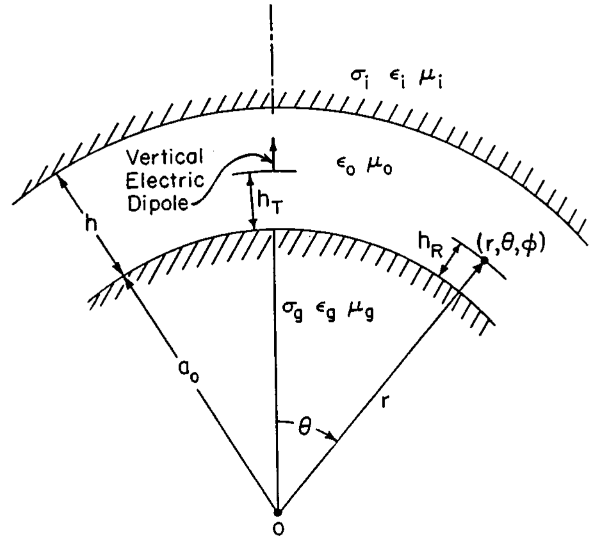

Fig. 1. Vertical electric dipole in spherical earth-ionosphere waveguide.

The zero-order approximation of (1) (flat earth approximation for $h/a_o \ll |C^2|$) is

$$R_g(C)R_i(C)\exp[-i2khC]=\exp(i2\pi n) \qquad n=1,2,3,\cdots. \tag{4}$$

For the cases in which the above restriction on C is violated, the effects of the earth's curvature cannot be neglected. Unless curvature effects are introduced into the model, it cannot be expected to yield the correct results. This aspect of the problem is discussed in Section III-E.

In many cases, such as for propagation over the sea, the earth can be considered a good conductor for TM modes in the VLF range. Also for the ionosphere, it may be assumed that $|N_iC/C_i| \ll 1$. This is particularly the case for the lower order modes of grazing incidence. Hence, (2) reduces to

$$R_g(C)\approx 1,\ R_i(C)\approx -[1-2N_iC/C_i] \approx -\exp\{-2N_iC/C_i\} \tag{5}$$

Furthermore, in special idealized cases the approximation $R_i(C) \approx -1$ can be used. In this case, the tagential component of the magnetic field at the boundary of the ionosphere is negligible, and for $\mathrm{TM}_{n,o}$ modes the boundary of the ionosphere can be considered a "magnetic conducting wall" (as compared with an "electric conducting wall" on which the tagential components of the electric field vanishes). This assumption would neglect absorption at the ionosphere boundary and indeed for the lower order modes the attenuation constant is very small.

Initially the experimental investigations were conducted assuming these idealized conditions [flat earth, with $R_g(C) = 1$ and a sharply bounded ionosphere with $R_i(C) = -1$]. By simulating an impedance boundary (Section III-D), a better approximation for $R_i(C)$ was obtained in which absorption at the ionosphere boundary was not disregarded.

With $R_g(C) = 1$ and $R_i(C) = -1$, the modal equation (4) reduces to

$$\exp\{-i2khC_p\} = -\exp\{-i2\pi p\} = \exp\{-i\pi(2p-1)\};$$

$$p = 1, 2, 3, \cdots$$

from which it follows that

$$C_p = \frac{\pi(2p-1)}{2kh} \tag{6}$$

is the solution for the cosine of angle of incidence.

In this model the ionosphere is assumed to be isotropic. In terms of the specific parameters of the ionosphere for the region of the ionosphere where reflection is assumed to take place, it is necessary that $\omega_H < \nu$ (where ω_H is the gyrofrequency and ν is the collision frequency). Hence, the system is considered reciprocal.

III. Electromagnetic Models

A. Scaling of EM Models

General aspects of scaling theory are briefly discussed here. Maxwell's equation along with several auxiliary constitutive relations in the rationalized meters-kilograms-seconds (MKS) system are given by

$$\begin{aligned} \nabla x\overline{E} &= -i\omega\overline{B} \\ \nabla x\overline{H} &= \overline{J} + i\omega\overline{D} \\ \nabla \bullet \overline{D} &= \rho \\ \nabla \bullet \overline{B} &= 0 \\ \overline{D} &= \varepsilon\overline{E} \\ \overline{B} &= \mu\overline{H} \\ \overline{J} &= \sigma\overline{E} \end{aligned} \tag{7}$$

where standard notations have been used for the field quantities and an $\exp(i\omega t)$ time dependence is assumed. It can readily be shown that these equations are invariant to the following four-parameter set of transformations [8]:

$$\begin{aligned} &\overline{E} = \alpha\overline{E}' \quad &&\overline{H} = \beta\overline{H}' \quad &&\ell = \gamma\ell' \quad &&\omega = \frac{1}{\delta}\omega' \\ &\overline{D} = \frac{\beta\delta}{\gamma}\overline{D}' \quad &&\overline{B} = \frac{\alpha\delta}{\gamma}\overline{B}' \quad &&\overline{J} = \frac{\beta}{\gamma}\overline{J}' \quad &&\rho = \frac{\beta\delta}{\gamma^2}\rho' \\ &\varepsilon = \frac{\beta\delta}{\alpha\gamma}\varepsilon' \quad &&\sigma = \frac{\beta}{\gamma\alpha}\sigma' \quad &&\mu = \frac{\alpha\delta}{\beta\gamma}\mu' \end{aligned} \tag{8}$$

where the symbol ℓ represents length and the symbols $\alpha, \beta, \gamma, \delta$ are scaling constants that may take any values, real or complex. However, for practical applications of the above transformation to modeling theory, length, frequency, and conductivity should transform into positive real quantities, hence, γ, δ and β/α should be positive real quantities, (7), (8). Thus, it is possible to make measurements in one system (the model) and to interpret them in terms of another system (the actual system under investigation) with different size and EM parameters and excited at a different frequency. The relationship between them being given by the transformations (8). Usually, as in the case treated in this work, these models are scaled down in size and hence upward in frequency, but it is also conceivable that a very small system could be investigated by scaling it upward in size and downward in frequency.

B. Dual Models

The theory of modeling can be extended to include what is called dual models [9]. The concept of the dual model is based upon observation that Maxwell's equations in free-space are asymmetric with respect to the electric and magnetic fields. Hence,

$$\begin{aligned} \nabla x\overline{E} &= -i\omega\mu\overline{H} \\ \nabla x\overline{H} &= i\omega\varepsilon\overline{E} \\ \nabla \bullet \varepsilon\overline{E} &= 0 \\ \nabla \bullet \mu H &= 0 \end{aligned} \tag{9}$$

are also invariant (as a group) to the following interchange in physical quantities:

$$H \to E, \qquad E \to -H \quad \mu \to \varepsilon, \qquad \varepsilon \to \mu \tag{10}$$

where ε and μ are, in general, the complex permittivity and permeability.

The above transformations could be combined with the four-parameter transformations (8) to give the following generalized dual-model transformations:

$$\begin{aligned} &\overline{E}_2 = -\alpha\overline{H}_1, \quad \overline{H}_2 = \beta\overline{E}_1, \quad \ell_2 = \gamma\ell_1, \quad \omega_2 = \frac{1}{\delta}\omega_1 \\ &\varepsilon_2 = \frac{\beta\delta}{\alpha\gamma}\mu_1, \quad \mu_2 = \frac{\alpha\delta}{\beta\gamma}\varepsilon_1. \end{aligned} \tag{11}$$

Hence, if in a given systems whose EM constants ε_1 and μ_1 (in general complex and functions of position) operating at a frequency ω_1, the electric and magnetic fields are E_1 and H_1, respectively; in another system whose EM constants ε_2 and μ_2 are numerically equal to $(\beta\delta/\alpha\gamma)\mu_1$ and $(\alpha\delta/\beta\gamma)\varepsilon_1$, respectively, operating at a frequency $\omega_2 = (1/\delta)\omega_1$ and whose linear dimensions are γ times those of the first system, the electric and magnetic fields will be found to be $-\alpha H_1$ and βE_1, respectively. Obviously either of these systems may be considered to be the dual model of the other.

It also follows from the above (dual) set of transformations, that an "electric conducting wall" transforms into a "magnetic conducting wall," a TM mode in a waveguide transforms into a TE mode and the surface impedance for a TM mode will transform into a surface admittance of a TE mode. Similarly, the reflection coefficient for a vertically polarized wave will transform into the reflection coefficient for a horizontally polarized wave. These relationships are also indicated by the dimensionless form of the TM model equation (1)

C. Simulation of the Earth's Boundary in an Ideal Two-Dimensional (2-D) Model Using Imaging Techniques

Under the special conditions discussed in Section II the initial flat earth approximation of the earth-ionosphere waveguide with perfectly reflecting boundaries is represented by the region between an "electric conducting" wall (earth) and a parallel "magnetic conducting" wall (ionosphere), separated by the distance h equal to the effective height of the ionosphere. In Section III-D, it is shown how the physical characteristics of the ionosphere boundary are more closely simulated. The earth's curvature is shown to be simulated by means of an inhomogeneous medium within the earth-ionosphere waveguide in Section III-E.

Considering the size and the complexity involved in the construction and operation of a three-dimensional (3-D) model, it was decided to construct a two-dimensional (2-D) model. The principal difference in the field variations for the case of the vertical electric dipole would be that the amplitude of the propagating waves in the 2-D model would not have the inverse radial dependence one finds in the 3-D model. However, the 2-D model can be made to be subject to the same modal equation as for the 3-D modal equation (4). This 2-D treatment of mode theory has been considered in detail by Wait [10]. In this 2-D scheme, the broad walls perpendicular to the earth and the ionosphere boundaries must be "magnetic conductors" and less than half a wavelength apart so that there would be no magnetic field variations across the width of the waveguide (see Fig. 2).

It is this region that will be modeled for experimental investigations in the laboratory. But it is obvious that such a waveguide with "magnetic conducting walls" cannot be constructed without great difficulty and since the $\mathrm{TE}_{n,o}$ modes are the only modes with 2-D field variations within a waveguide with "electric conducting walls," it was necessary to make use of a dual model (Section III-B). In the dual of the region in Fig. 2 the $\mathrm{TE}_{n,o}$ modes would correspond to the $\mathrm{TM}_{n,o}$ modes in the earth-ionosphere and the ideal ionosphere boundary together with the broad walls will be represented by "electric conducting walls," but the earth boundary will be represented by a "magnetic conducting wall."

To overcome the difficulty in simulating the dual of the earth boundary (on which the tangential magnetic field must vanish) a "mirroring" or "imaging" technique was suggested by Wait [9]. This technique is illustrated by referring to Fig. 3 in which the dual-model waveguide, with twice the scaled height of the ionosphere and a width less than half a wavelength, is shown. All the walls of this waveguide are taken to be good electric conductors.

If the symmetry about the x-axis of the waveguide is always maintained (even when the height h varies as a function of x) and assuming that only waves with electric fields that are symmetric about the x-axis are launched into a waveguide, it is clear that only $\mathrm{TE}_{(2p-1),0}$ modes will exist in the model waveguide. This is true even if $h(x) \neq$ constant. Under these conditions it follows that the amplitude of the tangential magnetic-field on the plane of symmetry (x, z) is zero. Hence, this plane of symmetry in the dual model can represent the earth boundary. One should note, therefore, that only half the region in the model waveguide represents the earth-ionosphere waveguide, with either the top or bottom narrow conducting wall representing the ionosphere boundary. Hence, a $\mathrm{TM}_{p,o}$ mode in the earth-ionosphere waveguide is represented by a $\mathrm{TE}_{n,o}$ mode in the dual-model waveguide, with $n = 2p - 1$ and $p = 1, 2, 3, \ldots$.

Since the media within the model waveguide is air, with permittivity ε, permeability μ, and wave impedance η it follows from (9) that

$$\frac{\mu_1}{\varepsilon_2} = \frac{\mu}{\varepsilon} = \eta^2 = \frac{\alpha\gamma}{\beta\delta} \tag{12a}$$

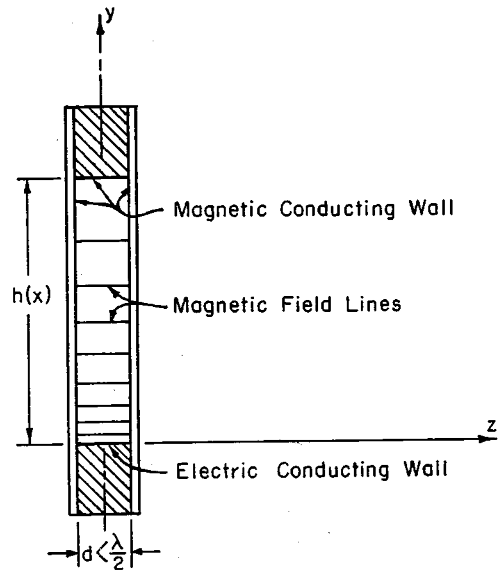

Fig. 2. Cross section of the 2-D model of the idealized earth-ionosphere waveguide.

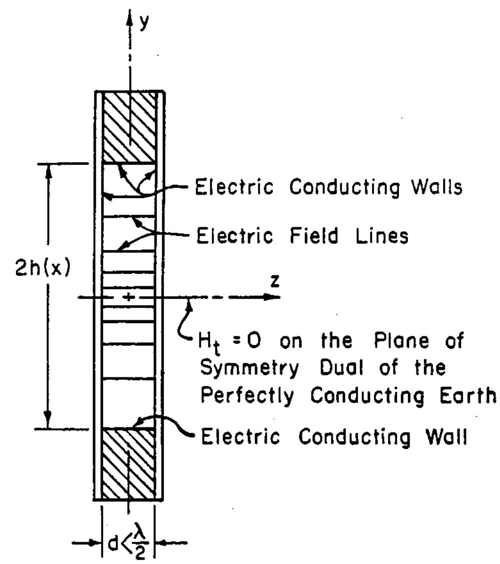

Fig. 3. Cross section of the 2-D dual model of the idealized earth-ionosphere waveguide used in the experimental investigation.

and

$$\frac{\mu_2}{\varepsilon_1} = \frac{\mu}{\varepsilon} = \eta^2 = \frac{\alpha\delta}{\beta\gamma}. \tag{12b}$$

On the basis that γ and δ must be positive real quantities

$$\gamma = \delta \tag{13}$$

from which it follows that

$$\alpha = \eta^2\beta. \tag{14}$$

The frequencies of interest (VLF) in the earth-ionosphere waveguide are about 15 kc/s, hence, in order to scale the model down to a convenient laboratory size, it was decided to operate at 9 GHz (X band). This determines the values of γ and δ since

$$\gamma = \delta = \omega_1/\omega_2 = \frac{9 \cdot 10^9}{15 \cdot 10^3} = 6 \cdot 10^5. \qquad (15)$$

The subscripts 1 and 2 refer to physical quantities in the model and in the earth-ionosphere waveguide, respectively.

The transformations (9) are, therefore, in this case

$$\begin{aligned} \overline{E}_2 &= -\alpha \overline{H}_1 \qquad \overline{H}_2 = (\eta)^{-2}\alpha \overline{E}_1 \qquad \ell_2 = 6 \cdot 10^5 \ell_1 \\ \omega_2 &= \omega_1/6 \cdot 10^5 \qquad \varepsilon_2 = (\eta)^{-2}\mu_1 \qquad \mu_2 = \eta^2 \varepsilon_1. \end{aligned} \qquad (16)$$

Provided that α has the dimension of impedance, all the quantities have their appropriate dimensions. Neglecting any nonlinear effects, the parameter α need not be specified since only relative amplitudes will have significance.

The equivalent height of the ionosphere (h_2) under normal conditions at night is about 90 km [11], hence, h_1 was chosen to be 12.7 cm (5 in), about 3.81 λ at 9 GHz. The width of the waveguide d_1 was chosen to be 0.9525 cm. (3/8 in), such that $d_1 < \lambda_1/2 = 1.67$ cm. Hence, the full cross section of the model waveguide was $(2h_1 x\, d) = (25.4x.9525)$ cm.

A rather detailed account on the modeling technique involved, including a survey of the problems that can be modeled in this fashion can be found in an ARPA Report [9].

D. Simulation of the Nonideal Ionosphere Boundary in the Model Waveguide

For the lower order modes, the reflection coefficient at the ionosphere boundary is (5)

$$R_i(C) \cong -\left(1 - \frac{2N_i C}{C_i}\right) \cong -\exp\left\{-\frac{2N_i C}{C_i}\right\} \qquad (17)$$

provided that $2N_i C/C_i \ll 1$. Strictly speaking, this equation holds only if the ionosphere can be regarded as sharply bounded and homogeneous. From experimental data available [12], [13] it appears that the effective permittivity of the ionosphere can be well approximated by an exponential profile. However, in view of the long wavelength (at VLF) and a relatively rapid change of the electron density in the D region, there is particularly good agreement with the sharply bonded ionosphere model at highly oblique incidence (which is the case for the lower order modes) [14], [15]. Under these conditions the waves in the region below the ionosphere can be considered to be reflected off a sharply bounded homogeneous isotropic boundary with a reflection coefficient $R(C)$ (referred to the effective height of the "sharply bounded" ionosphere). For highly oblique incidence it has been shown [15] that for vertically polarized waves (TM modes) the general behavior of the reflection coefficient for highly grazing incidence is given by

$$R_v(C) \cong -[1 + \alpha C] \cong -\exp \alpha C \qquad (18)$$

provided that $\alpha C \ll 1$ and $\mathrm{Re}\alpha < 0$. The complex parameter α is seen to be almost independent of the cosine of the angle of incidence C. These results were based mainly on studying a permittivity profile of the form

$$K(y) = K_o\left[1 - \frac{i}{L}\exp \beta y\right] \qquad (19)$$

where K_o (the reference permittivity) is taken to be unity and $L \cong \omega\nu/\omega_0^2$ [14] and β (a constant) is a measure of the sharpness of gradient of the permittivity profile. Here, ν and ω_o are the collision frequency and plasma frequency at the reference height $y = 0$.

Complex values of α have been plotted [15] for $\lambda_o = 15$ km and $L = 1/2$ as a function of β. It has been shown that for $\beta = 1/2$ [km]$^{-1}$ (appropriate value for quiet daytime conditions), the real part of α is approximately -2.85. The imaginary part of α has been disregarded since its effects can be taken into consideration by the choice of an appropriate effective height of reflection in the case of flat earth model.

For a sharply bounded ionosphere whose normalized surface impedance is z, the reflection coefficient for vertically polarized waves is given by

$$R_v(C) = \frac{C - z}{C + z} \approx -\left[1 - \frac{2C}{z}\right] \approx -\exp\left\{-\frac{2C}{z}\right\} \qquad (20)$$

provided that $2C/z \ll 1$.

Comparing (20) with (18), one gets

$$z \equiv \frac{Z_{\mathrm{TM}}}{\eta} \cong \frac{-2}{\alpha}$$

and

$$Z_{\mathrm{TM}} \cong 0.702\eta. \qquad (21)$$

It is interesting to note that this value for Z_{TM} corresponds very well with one derived by assuming a sharply bounded homogeneous ionosphere with a complex permittivity $\varepsilon_i = 1 - i(1/L) = 1 - i$. The surface impedance at the interface between such a homogeneous, isotropic media and free-space for vertically polarized waves (TM modes) can be shown to be [16]

$$Z_{\mathrm{TM};\,n,o} = \eta\frac{(\varepsilon_i - S_n^2)^{1/2}}{\varepsilon_i} \qquad (22)$$

where S_n is the sine of the angle of incidence on the surface of the ionosphere boundary. The principal mode $(n = 1)$ is at grazing incidence $(S_1 \rightarrow 1)$, hence

$$Z_{\mathrm{TM},1,0} = \eta\frac{(-i)^{1/2}}{1 - i} = 0.707\eta. \qquad (23)$$

To simulate this more realistic model of the ionosphere boundary in the dual-model waveguide, it is therefore necessary to replace both the upper and lower narrow conducting walls by a material whose relative EM constants ε_w and μ_w are

$$\varepsilon_w = \frac{\mu_i}{\mu} = 1 \quad \text{and} \quad \mu_w = \frac{\varepsilon_i}{\varepsilon} = 1 - i. \qquad (24)$$

Since the ionosphere is assumed homogeneous in this model, this material must be at least several skin depths thick so that it could be assumed to be a semi-infinite media. Although a few ferrite materials exhibit a relative permeability $\mu_r \approx 1 - i$, they all have rather high relative permittivities ($\varepsilon_r \approx 10$) together with some other undesirable properties (such as nonlinearity).

Because the exact dual model cannot be obtained, it was decided to simplify the duality requirements such that the dual relationship would hold exactly for the principal mode and only approximately for the higher order modes. This dual relationship is expressed as

$$[Z_{\mathrm{TM};\,1,o}] \text{ ionosphere} = \eta^2 [Y_{\mathrm{TE};\,1,0}] \tag{25}$$

dual model. For the purposes of verifying analytical results it is not necessary to simulate the earth ionosphere waveguide exactly.

The surface impedance corresponding to the nth mode is defined as the ratio E_{nt}/H_{nt}, where E_{nt} and H_{nt} are the tangential components of the electric and magnetic fields (associated with the nth mode), respectively.

Rather than replace the narrow conducting walls of the "ideal" model waveguide of Fig. 3, it was found to be preferable to "load" the conducting walls with a lossy dielectric material as illustrated in Fig. 4.

It is obvious that from the point of view of construction of the model waveguide it would be very desirable to leave the conducting narrow walls in place, but this has further advantages. If the lossy slab is to be considered semi-infinite physically (several skin depth s thick), it turns out that this material must be very thick [9]. Also a system completely enclosed in a conducting boundary is easier to analyze than one bounded by dielectric walls.

It now remains to determine the relative complex permittivity of the lossy dielectric slab ε_w (μ_w is assumed to be unity) and its thickness $t = g - h$ such that (25) is satisfied. Since this equation involves complex quantities, a range of values for (complex) ε_w can be found that satisfy the equation as a function of the parameter t.

Considering only $\mathrm{TE}_{n,0}$ modes (with $n = 2p - 1$ $p = 1, 2, 3, \ldots$ such that $H_{nt} = 0$ at the plane of symmetry $y = 0$); the electric field for the nth mode can be shown to be given by

$$\left.\begin{array}{ll} E_{Inz} = a_{In} \cos k_{In} y e^{-\gamma_{In} x}, & |y| \le h \\ E_{IInz} = a_{IIn} \sin k_{IIn}(g - |y|) e^{-\gamma_{IIn} x}, & h \le |y| \le g \end{array}\right\} \tag{26}$$

with

$$\left.\begin{array}{ll} \gamma_{In} = (k_{In}^2 - k_{Io}^2)^{1/2} & k_{Io}^2 = \omega^2 \varepsilon_I \mu_I \\ \gamma_{IIn} = (k_{IIn}^2 - k_{IIo}^2)^{1/2} & k_{IIo}^2 = \omega^2 \varepsilon_{II} \mu_{II} \end{array}\right\} \tag{27}$$

where the subscripts I and II refer to physical quantities in region I $|y| \le h$ and region II $h \le |y| \le g$, respectively.

The x component of the magnetic field (parallel to the surface of the lossy dielectric) is

$$\left.\begin{array}{ll} H_{Inx} = \dfrac{k_{In}}{i\omega\mu_I} a_{In} \sin k_{In} y e^{-\gamma_{In} x} & h \ge |y| \\ H_{IInx} = \pm \dfrac{k_{IIn}}{i\omega\mu_{II}} a_{IIn} \cos k_{IIn}(a - |y|) & \\ \quad \cdot e^{-\gamma_{IIn} x} & g \ge |y| \ge h \end{array}\right\} \tag{28}$$

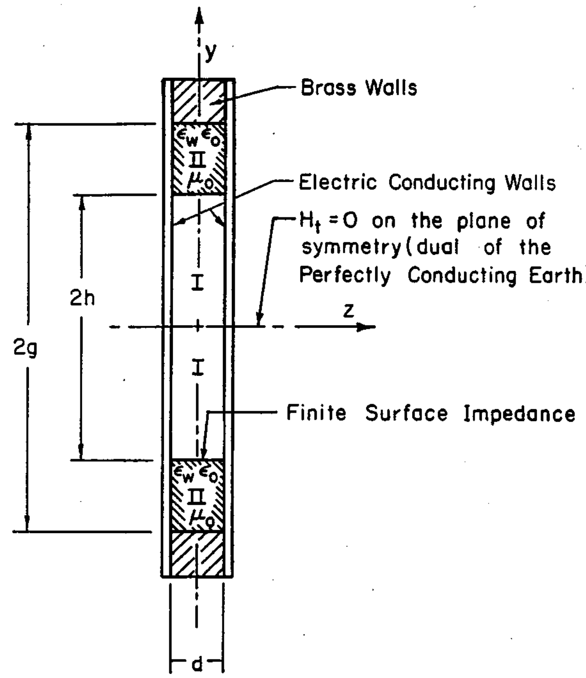

Fig. 4. Cross section of the dual-model waveguide with lossy dielectric slabs "loading" narrow walls to simulate finite surface impedance of the ionosphere boundary.

where the plus sign is for $y > h$ and the minus sign is for $y < -h$. Boundary conditions at $|y| = h$ yield

$$\begin{aligned} \gamma_{In} &= \gamma_{IIn} \\ a_{In} \cos k_{In} h &= a_{IIn} \sin k_{IIn}(g - h) \\ z_{In} &= z_{IIn} = z_{\mathrm{TE};\,n,o} \end{aligned} \tag{29}$$

where Z_{In} and Z_{IIn} are the ratios E_{Inz}/H_{Inx} and E_{IInz}/H_{IInx}, respectively, which are each equal to the surface impedance for the $\mathrm{TE}_{n,o}$ mode $Z_{\mathrm{TE};\,n,o}$.

The surface impedance for grazing incidence ($S_{In} \to 1$) is

$$Z_{\mathrm{TE};\,n,o} = \frac{\eta}{[\varepsilon_w - 1]^{1/2}} \tanh[ikt(\varepsilon_w - 1)^{1/2}]. \tag{30}$$

Substituting (23) and (25) into (30) yields

$$\exp\{-ikt(\varepsilon_w - 1)^{1/2}\} = \frac{1 - \sqrt{2}(\varepsilon_w - 1)^{1/2}}{1 + \sqrt{2}(\varepsilon_w - 1)^{1/2}} \tag{31}$$

giving the required relationship between the relative complex dielectric constant ε_w and the thickness of the slab t.

Based on the numerical solution of (31) $\varepsilon_w = \varepsilon_{wr} - i\varepsilon_{wi}$ as a function of t, it is seen that for thickness $0.5 < t < 1.5$ cm, the values for ε_{wr} and ε_{wi} are within a physically realizable range. A dielectric slab with $d = 1.585$ cm (5/8 in), $\varepsilon_{wr} = 1.33$, and a dissipation factor $\varepsilon_{wi}/\varepsilon_{wr} = 0.27$ was manufactured especially for this purpose with a deviation in the parameters of less than 10% from those required by (31) in the whole slab.

Inspection of (22) also shows that Z_{TM} is fairly independent of C for oblique incidence (lower order modes). Hence, the duality condition (25) is fairly well met for a few of the lower order modes. This is quite a significant property and it will also be found very useful in applying the surface impedance concept.

E. Simulation of Curvature Using an Inhomogeneous Dielectrical Material

The earth's curvature may be simulated artificially by modifying the medium of propagation in the straight model waveguide. In this case, it is necessary to fill the interior of the model waveguide with a material for which the square of the refractive index N^2, varies linearly with height (analogous to the earth-flattening technique given by Kerr [17]). It has been shown analytically that such a structure does support the Airy-integral-type modes as in the earth-ionosphere waveguide [18], with the restriction that $h/a \ll 1$, where h is the height of the ionosphere and a_o the radius of the earth.

Several methods have been suggested for achieving the desired variation of the magnetic or electric susceptibility of the material used to load the interior of the waveguide [19]. For instance, the inhomogeneous loading of the interior of the waveguide with thin metallic posts results in variation of the effective permeability of the propagating medium. Similarly, the effective dielectric coefficient of the propagating medium may be varied by loading the interior of the waveguide with thin polystyrene sheets of varying thickness. Since the model waveguide is approximately 25 ft long and 10 in wide, it would be rather difficult and expensive to fabricate such models, though probably cheaper than constructing curved surfaces. In addition, even if such models were fabricated, the effective susceptibility of the medium of propagation would not be independent of the axial coordinate in the waveguide. Moreover, the desired vertical variation of the electric susceptibility would only be crudely approximated. The requirement for axial symmetry in the model imposes a further restriction on these solutions since the dielectric material must fill the interior of the model symmetrically about its center line.

The variation of the dielectric coefficient of synthetic foam materials under compression has led to a physically realizable solution to the problem of synthesizing the desired medium of propagation [20]. This solution is based on the inhomogeneous compression of a slab of dielectric foam material. The material found to be most appropriate for the purpose of this work is an extruded polyethylene foam, which is made from blown-up polyethylene beads that trap large quantities of air when heated.

Using a logarithmic-mixing rule given by Lechtenecker and confirmed by Buchner [21], the relative permittivity of the polyethylene foam ε_f may be expressed in terms of the relative permittivities of the polyethylene and the trapped air ε_2, respectively. Hence

$$\log \varepsilon_f = \frac{1}{V_1 + V_2}\,(V_1 \log \varepsilon_1 + V_2 \log \varepsilon_2) \tag{32}$$

in which V_1 and V_2 are the volumes occupied by the polythene and air, respectively. Equation (32) yields a very good approximation for ε_f since $\varepsilon_1 \approx \varepsilon_2$. If the foam material is subjected to compression, V_2 decreases while V_1, ε_1 and ε_2 remain constant, and the resultant change in the relative permittivity of the foam is

$$\frac{1}{\varepsilon_f}\,\frac{\partial \varepsilon_f}{\partial V_2}\bigg|_{V, \varepsilon_1, \varepsilon_2} = \frac{1}{V_1 + V_2}\,(\log \varepsilon_2 - \log \varepsilon_f). \tag{33}$$

Since $\varepsilon_2 \approx 1$, $\log \varepsilon_2 \approx 0$. Also $\varepsilon_f = (1 + \chi_f)$ is approximately 1.03; hence, an expansion of log $(1 + \chi_f)$ in powers of susceptibility χ_f yields

$$\begin{aligned}\frac{\partial \chi_f}{\partial V_2} &= -\,\frac{(1+\chi_f)}{V_1 + V_2}\,(\chi_f - \chi_f^2/2)\\ &= -\,\frac{\chi_f}{V_1 + V_2}\,(1 + \chi_2/2) + 0(\chi_f^3).\end{aligned} \tag{34}$$

Consider now a slab of the foam material cut into a trapezoidal cross section. This material is homogeneous, has a relative permittivity equal to ε_f, and its thickness is

$$d(y) = d_0\left\{1 + \frac{y}{h}\left(\frac{d_h}{d_0} - 1\right)\right\} \equiv d_0\left(1 + \frac{\delta}{h}\,y\right) \tag{35}$$

in which $d_0 = d(0)$ and $d_h = d(h)$, and $\delta = d_h/d_0 - 1$.

Now, if this slab of dielectric material is compressed vertically so that its final cross section is rectangular, the height of the slab is d_0 for all values of y. Consider the strip of width dy, V_1 and V_2 are now the volumes occupied by the polythene and air, respectively, in the strip. Let $V_1 + V_2 = V$, hence, $(dV/dV_2)|_{V_1=\text{const.}} = 1$. The differential equation for the susceptibility of the form χ_f is, therefore

$$\frac{d\chi_f}{\chi(1+\chi_f/2)} = -\frac{dV}{V}. \tag{36}$$

Integrating both sides of the above equation yields

$$\frac{\chi_f}{1+\chi_f/2} = \frac{\chi_{f0}}{1+\chi_{f0/2}}\,\frac{V_0}{V} \tag{37}$$

in which χ_{f0} and V_0 are the corresponding values for χ_f and V, at $y = 0$. For the slab of foam material under consideration $V_0/V = d(y)/d_0$. Hence, solving (7) for χ_f yields

$$\varepsilon_f(y) = 1 + \chi_f(y) = 1 + \chi_{f0}\left(1 + \frac{\delta}{h}\,y\right)\left(1 + \frac{\chi_{f0}}{2}\,\frac{\delta}{h}\,y\right). \tag{38}$$

Note that since $\chi_{f0} \ll 1$, ε_f exhibits a near linear variation as a function of y.

For the curved model it is necessary that [18]

$$\varepsilon_f(y) = \varepsilon_{f0}(1 + \alpha y) \qquad \alpha = \frac{2}{a}. \tag{39}$$

Hence, $\alpha h \approx \delta \chi_{f0}$.

For the Dregion of the ionosphere αh is approximately 0.025 (corresponding to an 80 km effective ionosphere height). Hence, with $\chi_{f0} = 0.037$ cm and $d_0 = 0.375$ cm

$$d_h \cong d_0\left[1 + \frac{\alpha h}{\chi_f}\right] \cong 0.62 \text{ cm}. \tag{40}$$

The model waveguide with simulated curvature was fabricated and the basis functions can be expressed in terms of Airy-integral functions (see Section IV).

IV. CONCLUDING REMARKS

The dual-scaled model waveguide described in this paper was constructed and experimental data was obtained for various effective height profits along the propagation path. A microwave system was used to measure the horizontal component of the electric field intensity and phase both along the center line of the model waveguide (corresponding to a propagation path on the earth's surface) as well as at several cross sections of the scale model (see Section III-A) waveguide (corresponding to a vertical line between the earth's surface and the ionosphere effective height) [22]. Since the waveguide used in the experiments was a "dual" model (see Section III-B) of the earth-ionosphere waveguide, the horizontal component of the electric field in the model corresponds to the horizontal component of the magnetic field. The earth surface, which is assumed to be perfectly conducting, is simulated by the center plane of symmetry in the model waveguide. The fluctuations in the field intensity and phase due to a sinusoidal effective height perturbation was due to interference between waves travelling in opposite directions for paths before the height perturbation. For paths beyond the height perturbation the interference was due to waves traveling in the same direction (no reflected waves in this region). The relative intensities of the different waveguide modes can be obtained from the electric field (intensity and phase) measurements along a vertical path. The field along the centerline of the model waveguide was probed through holes about one eight of a wavelength apart. The field measurements along the cross section of the model waveguide was probed initially through similar holes along the cross section of the waveguide and later by an especially designed waveguide junction with a probe attached to a sliding side wall of the waveguide. Using this device the electric field could be measured continuously along a vertical path. Ideally it was only necessary to probe the field from the model centerline to one side. However, in order to estimate the field intensities of the even numbered modes,the fields were probed across the cross section of the model waveguide. These undesired modes (with relatively small intensities) were due to deviations from the centerline symmetry of the model waveguide (see Section III-C). They generated asymmetric electric fields along the cross section of the waveguide.

The fields generated by step variations in the effective heights of the model waveguide were measured in order to simulate a day to night or night to day transition [23]. These results compared very well with the analytical solutions that were derived. It was shown that judicious use of the reciprocity relations resulted in a significant reduction in the computations. This was due to the observation that the intensities of the reflected waves were significantly smaller for a day to night path compared to a night to day path.

Excellent agreement between experimental and analytical results was obtained for wedge shaped day to night transition regions for both vertical and axial paths in the model waveguide. Here, too judicious use of the reciprocity relations was very helpful in obtaining the analytical results for night to day propagation paths.

All the proceeding experiments were conducted in a model waveguide with perfectly reflecting (electric and magnetic) walls. These were followed by experiments in models with both perfectly reflecting boundaries [25] and surface impedance boundaries (see Section III-D) [26]. Sinusoidal day to night smooth transition regions as well as local sinusoidal depressions in the effective height were considered in these experiments. In these cases, excellent agreement between the experimental and analytical results (amplitude and phase) for both horizontal (centerline) as well as vertical paths were obtained. In the model with surface impedance boundaries [26], the higher order mode content was significantly smaller than for the model with perfectly conducting boundaries [25]. For the model with surface impedance boundaries the higher order modes attenuated over much shorter distances than the dominant principal mode launched in the model waveguide. This was not the case for the model with perfectly conducting boundaries. Analytical solutions, based on the representation of the fields as sums of modes in radial waveguides with continuously varying fare angles were also tested in this work.

Measurements in a model waveguide with an abruptly varying surface impedance were also compared with analytical solutions [27]. In these cases, very good comparisons were obtained when judicious use of reciprocity was made. It was shown that when the incident dominant mode was launched in the region where the surface impedance were very small compared to the surface impedance beyond the abrupt transition, the reflections at the transition were very small compared to the reflections in the reverse situation. The analytical solutions were first derived for the case in which the reflections were very small. On invoking reciprocity, the solution for the reverse situation was obtained. Thus, the experiments conducted in the model waveguide not only enable theorists to examine their analytical solutions, but also present them with observations that lead to simpler and more accurate solutions.

Curvature was also simulated in the model waveguide with the use of lossless dielectric material with varying susceptibility (see Section III-C) [19], [20]. For this model, the basis functions can be expressed in terms of Airy-integral functions [18]. The data obtained from this model waveguide is in agreement with the analytical results. Thus, for the dominant mode, field intensities initially increase (height gain) as a function of height above the earth's surface. For the flat earth model waveguide the basis functions are sinusoidal and the field intensities decrease monotonically as a function of height. It should be noted that in order to maintain symmetry about the centerline of the model (representing the earth's surface) the inhomogeneous dielectric material must fill the interior of the model waveguides symmetrically about the centerline of the model.

The model waveguide was excited by the dominant mode through a specially designed, gradually tapered waveguide that connects a standard X-band waveguide to the model waveguide [28].

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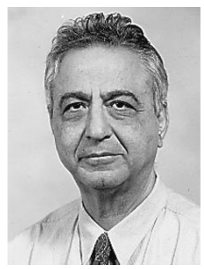

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