Detection of Buried Dielectric Cavities Using the Finite-Difference Time-Domain Method in Conjunction with Signal Processing Techniques

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***Abstract*—In this paper, we address the problem of detecting low-dielectric contrast cavities buried deep in lossy ground by using the finite-difference time-domain (FDTD) method in conjunction with signal processing techniques for extrapolation and object identification. It is well known that very low frequency probing is needed for deep penetration into the lossy ground, owing to a rapid decay of electromagnetic (EM) waves at higher frequencies. It is also recognized that numerical modeling using the FDTD method becomes very difficult, if not impossible, when the operating frequency becomes as low as 1 Hz. To circumvent this difficulty, we propose a hybrid approach in this paper that combines the FDTD method with signal processing techniques, e.g., rational function approximation and neural networks (NNs). Apart from the forward problem of modeling buried cavities, we also study the inverse scattering problem—that of estimating the depth of a buried object from the measured field values at the surface of the earth or above. Numerical results for a buried prism are given to illustrate the application of the proposed technique.**

***Index Terms*—Electromagnetic detection of buried objects, finite-difference time-domain (FDTD) methods, underground radar.**

I. Introduction

LOCATION of inhomogeneities buried within a stratified medium is a very important problem because it finds widespread applications in remote sensing, geophysical prospecting and nondestructive testing [1]. This has prompted the development of techniques for the analysis of the electromagnetic scattering by buried objects, a problem that is complicated by the presence of the air–earth interface [2], [3]. In this paper, we address the specific problem of electromagnetic sounding of a buried cavity filled with material such as air ($\varepsilon_r \approx 1$), whose conductivity is negligibly small. To detect such targets, which may be buried tens of meters underground, it is necessary to use fairly low frequencies (on the order of 1 Hz) for the sensing, because the penetration of the electromagnetic waves is extremely shallow at higher frequencies [1], [4], [5] in the kilo or megahertz range. Although the finite-difference time-domain (FDTD) method has been successfully used for simulating the probing of underground metallic targets at ultrahigh frequencies (UHF) [6], the algorithm runs into formidable difficulties when the frequency is lowered to 1 Hz. These difficulties stem from the fact that, at lower frequencies, it becomes necessary to run the FDTD code for a very large number of time steps that consume an inordinately long central processing unit (CPU) time. This is because the choice of the time step in FDTD is governed by the spatial resolution required to accurately model the given geometry. Consequently, for objects that have dimensions in the range of tens of meters, i.e., are very small ($\sim 0^{-7}$) compared to the wavelength, the time step dictated by the Courant condition is a very small fraction of the period of the sensing signal.

To circumvent this difficulty, we propose a hybrid approach in this paper that combines the FDTD method with signal processing techniques as explained below. We begin by carrying out the FDTD simulation at relatively high frequencies, somewhere in the range of 1–10 kHz, for which the run times are 10^3 to 10^4 times smaller than that for 1 Hz. Next, we employ extrapolation techniques based on neural networks (NNs) and rational function approximations to derive the fields at the desired operating low frequency, greatly reducing the FDTD computation time in the process.

Following the solution of the forward problem, we move on to investigate the companion and practical problem of inverse scattering by buried objects, *viz.* that of estimating the depth of the object from the observed scattered field components above the ground [7]. The inverse scattering problem usually requires the solution of a nonlinear equation that relates the sensed fields with the electromagnetic parameters of the buried object and the host medium. In this paper, we use a neural network to map this nonlinear relationship between the depth and observed field values and then use it to predict the depth of the buried object.

The NN approach has recently gained attention as a fast and flexible technique for microwave circuit modeling, simulation and optimization [8]–[10]. For function approximation, most of its applications have been limited to interpolation. Other recent applications include the estimation of the conductivity of a prism buried under seawater in sedimentary layers by sensing the electric field just above the layer [11], and the calculation of equivalent circuit parameters in microwave computer-aided design (CAD) applications [9], [10]. In this paper, we apply the NN not only for both function interpolation and extrapolation, but for addressing the inverse scattering problem as well.

The paper is organized as follows. The FDTD simulation and the problem of extrapolating the time signature are presented in Section II. Numerical validations of the extrapolation scheme and the simulation results for an underground buried

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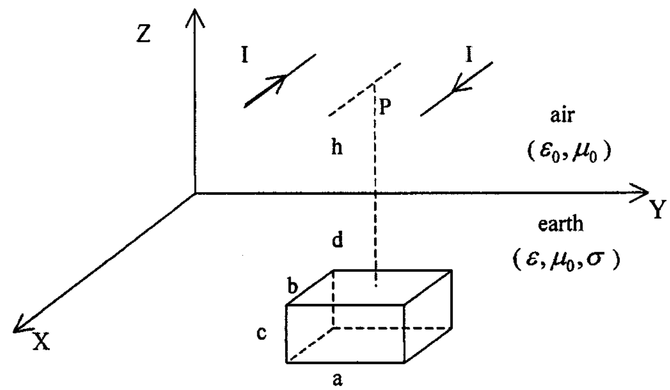

Fig. 1. Buried cavity excited by a pair of dipoles fed 180° out of phase.

prism cavity appear in Sections III and IV, respectively. The inverse scattering problem, pertaining to the buried cavity, is discussed in Section V. Finally, a summary and some conclusions are included in Section VI.

II. FDTD Simulation and Low-Frequency Extrapolation

A. FDTD Model

We use the total field type of FDTD formulation to model the buried cavity problem shown in Fig. 1. The cavity, whose dimensions are $a \times b \times c$, is buried at a depth d underneath the ground. Two dipole antennas, that are fed 180° out of phase, are positioned parallel to the ground–air interface, and are located at a height of $h$above the ground. The parameters of the air, ground and cavity interiors are (ε_0, μ_0), $(\varepsilon, \mu_0, \sigma)$ and (ε_0, μ_0), respectively. The advantage of the two-dipole arrangement is that there is a null field at the center of the two-dipole when fed in opposition [3], [12]. By moving the antennas along the y-direction, we can obtain the sweep response of the electric field E_x at the observation point P (see Fig. 6). The null point of this sweep response helps us to locate the cavity buried underneath the ground.

B. Extrapolation using Function Approximation Technique

In this section, we examine the problem of using the function approximation technique to extrapolate the frequency-domain response to low frequencies. The function approximation technique is described as follows.

Let there be a functional relationship f between X and Y where $X \subset R^n$, $Y \subset R^m$, R^n/R^m is n/m-dimensional real or complex space. Then, given a finite data set $[D = \{[x_i, t(x_i), x_i \in X, t(x_i) \in Y, i \in [1, N]\}$ we assume that $t(x_i) = f(x_i)$. The objective of the function approximation is to approximate the unknown function f from the set of samples from the data set D.

1) Rational Function Approximation: The rational function approximation attempts [13], [14] to approximate the unknown function $f(x)$ in the form

$$f(x) = N(x)/D(x) \tag{1}$$

where

$$N(x) = N_0 + N_1 x + N_2 x^2 + \cdots + N_n x^n, \tag{2}$$

and

$$D(x) = D_0 + D_1 x + D_2 x^2 + \cdots + D_d x^d. \tag{3}$$

The coefficients Ns and Ds in (2) and (3) can be obtained from the sampled values of the function f, which, in our work, represents the electric field values obtained from FDTD calculation and x is the frequency variable.

We use the Neville type of extrapolation algorithm [13], [15] in this work. It is a recurrence scheme that starts at the nearest point, and then adds a sequence of decreasing corrections as information from other points is incorporated. When the function is well behaved, the last correction is the smallest one and it can be used as an informal bound on the error. The advantages of this algorithm are that the extrapolation process provides an improved estimate in a step-by-step manner and that it returns an estimate of its own error, which help us assess the accuracy of the estimated results.

2) Neural Network Approximation: An alternative approach to accomplishing the same goal is to use a multilayer NN. Generally speaking, the two-layer network (a nonlinear transfer function in the hidden layer and a linear transfer function in the output layer) with appropriate biases is capable of approximating functions with a finite number of discontinuities if the hidden layer has a sufficient number of neurons [8], [16].

Let the network-approximated function be f_w. It maps the relationship of two sets of variables in space R^n and space R^m, i.e.,

$$f_w\colon R^n -> R^m.$$

The function f_w depends on the network parameters called weight vector w and bias vector v. These are determined from the sampled data through a process called training, during which the network learns the relationship between R^n and R^m.

The procedure for applying the NN approach is as follows. We prepare a training set L, which is a finite subset of the set D, select an appropriate adaptation algorithm or a learning rule and begin training the NN. During this learning phase, the weight vector w and the bias vector v of the network are changed by using the adaptation algorithm such that $[x_i, f_w(x_i)]$ approximately equals $[x_i, t(x_i)]$, implying that the network has learned the relationship between the function variables. Usually a performance test with the test set $T = D - L$ is carried out after the training. If the performance is not good enough to be acceptable, it is necessary to go back to check if the sample data is insufficient or the network has less hidden neurons. If this is the case, then we implement appropriate changes and train it again. Finally, when this exercise is over, the network is deemed ready for generalization so that it can yield the output $f_w(x_j)$ for an input x_j, which was not included in the known data set D.

Appropriate architecture of the NN and the choice of a suitable training algorithm are two of the major issues that need to be addressed in developing NN models for approximating a function. In our application, we have used a two-layer feed forward NN architecture and the Levenberg–Marquardt backpropagation training algorithm in conjunction with Bayesian regula-

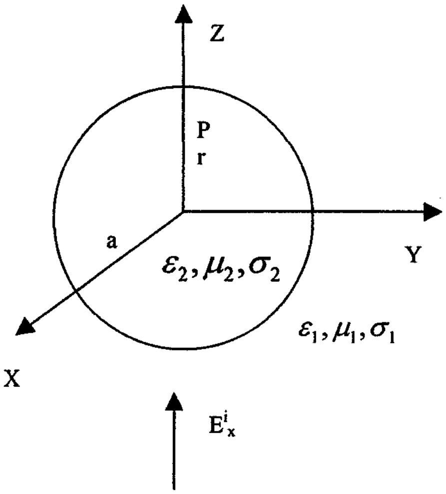

Fig. 2. Air-filled sphere surrounded by uniform medium and illuminated by a plane wave.

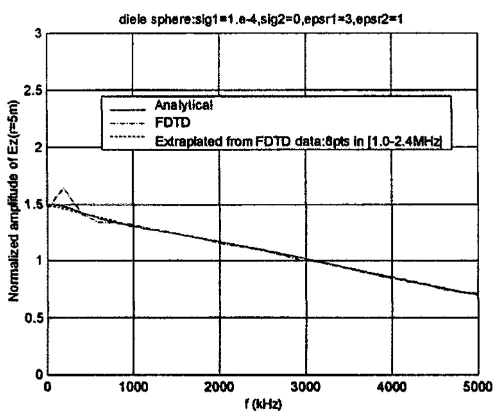

Fig. 3. Electric field inside the air-filled sphere versus frequency ($r = 5$ m).

tion [16]. We have found that the network, thus developed, possesses good generalization characteristics, which is obviously very desirable.

III. Validation of Extrapolation Algorithms

To validate the function approximation algorithms proposed above, we choose two simple examples that have analytical solutions. The first of these examples demonstrates the effectiveness of combined FDTD and function approximation approach. We compute the frequency behavior of the electromagnetic (EM) field in the interior of an air-filled sphere of 10-m radius, which is surrounded by an uniform lossy dielectric medium, and illuminated by a plane wave (see Fig. 2). The observation point is located inside the sphere at 5 m from the center as shown in the figure. The surrounding medium has a relative permittivity of three, its conductivity is 1.0e-004, and its relative permeability is one. We compute the field at the observation point as a function of frequency, using both the analytical solutions expressed in a series form and the FDTD code. We observe from Fig. 3 that the FDTD result deviates from the analytical one at low frequencies. Next, we turn to extrapolating the FDTD results via the rational function

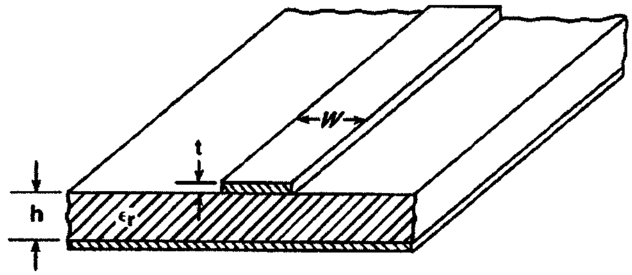

Fig. 4. Mircostrip line structure.

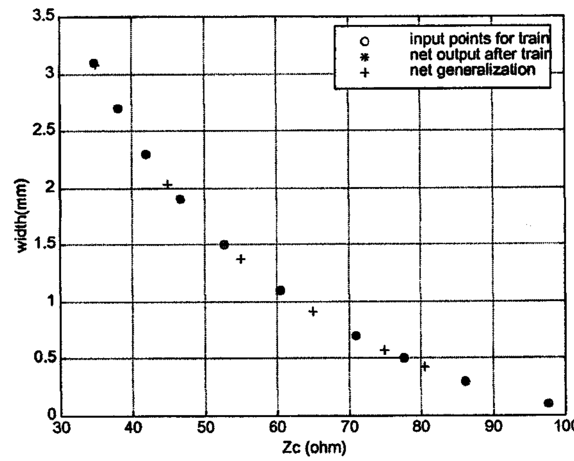

Fig. 5. Width of microstrip line determined by the NN to yield desired characteristic impedance; $f = 15$ GHz, $\varepsilon_r = 10$, $h = 1.35$ mm, $t = 0.002$ mm.

approximation to derive the field values at low frequencies. Fig. 3 shows that the extrapolated FDTD values are in good agreement with the analytical results.

The second example, though it has little to do with the buried cavity problem, nevertheless serves to demonstrate the effectiveness of the NN type of mapping in solving an inverse problem. The problem at hand is to find the width of a microstrip line (see Fig. 4) that would yield a specified characteristic impedance (when the substrate height, dielectric permittivity, strip thickness, and the frequency are held constant). The result of the exercise (see Fig. 5) shows that the NN output agrees well with the training data and that generalization achieved is good when new input data are presented to the network.

IV. Scattering from Buried Cavity—Forward Problem

In this section, we return to the buried cavity problem, whose geometry was displayed in Fig. 1. The dimensions of the cavity are $a = 30$ m, $b = c = 10$ m, $d = 10$ m, $h = 5$ m. The material parameters are $\varepsilon_r = 2$, $\sigma = 0.001$ for the earth, and $\varepsilon_r = 1$, $\sigma = 0$ for the cavity. When we move the antenna in the y-direction, we obtain a curve of the type shown in Fig. 6, computed directly with the FDTD, with frequency parameter ranging from 1 to 10 kHz. Fig. 7 shows how the observed field varies as a function of the frequency and it is evident from this figure that this function has a very smooth behavior. Next, we employ the rational function approximation scheme to derive

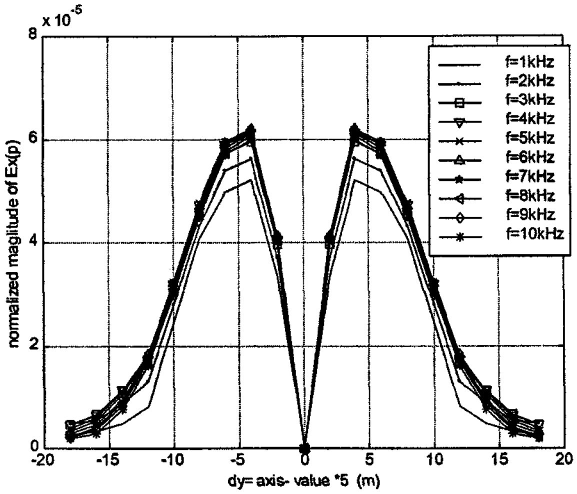

Fig. 6. Variation of E_x at point P (in between the dipoles) as a function of the location when the sensing dipole moves along the longitudinal (y) direction.

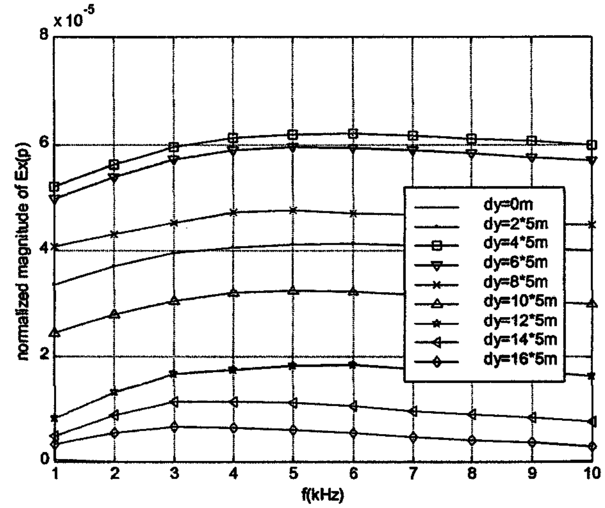

Fig. 7. Variation of $E_x(p)$ as a function of frequency.

the fields at the desired frequencies that are in the range of 1 Hz. Fig. 8 shows the extrapolated results when the offset in the y-direction (dy) equals 30 m, with the overlap region of 1–2 kHz for the original and the rational approximation results. A similar extrapolation procedure can be employed for other offset values along the y-axis, to obtain the spatial variation of the field at low frequency at the corresponding location.

Next, we carry out the extrapolation routine by using the NN approach and find that the results, shown in Fig. 9, compare very well with those in Fig. 8 that were derived by using the rational function approximation.

However, a closer examination reveals that the NN extrapolation is much less sensitive than the rational function approximation to the increments of the input variable and the number of sample points. One consequence of this is that the results derived by the rational function approximation are not always very smooth everywhere and may generate unstable results with discontinuous behaviors. In contrast, a well-trained NN always generated smooth and stable approximations in our numerical experiments. Fig. 10 serves to illustrate these performance characteristics of the above two schemes.

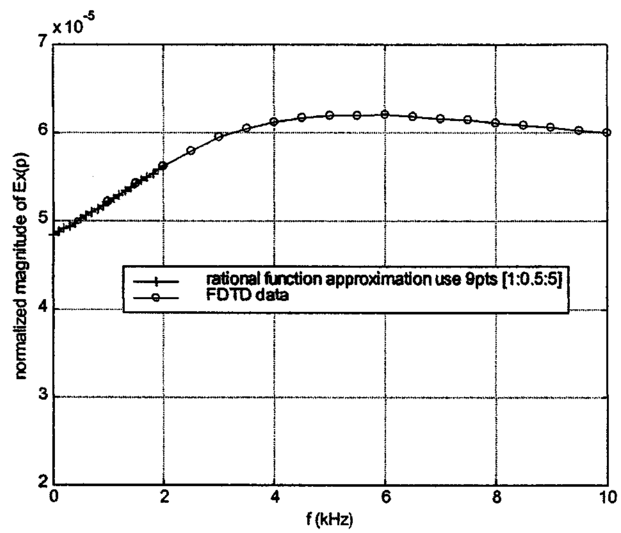

Fig. 8. Numerical extrapolation to a low frequency of 1 Hz. Rational function approach.

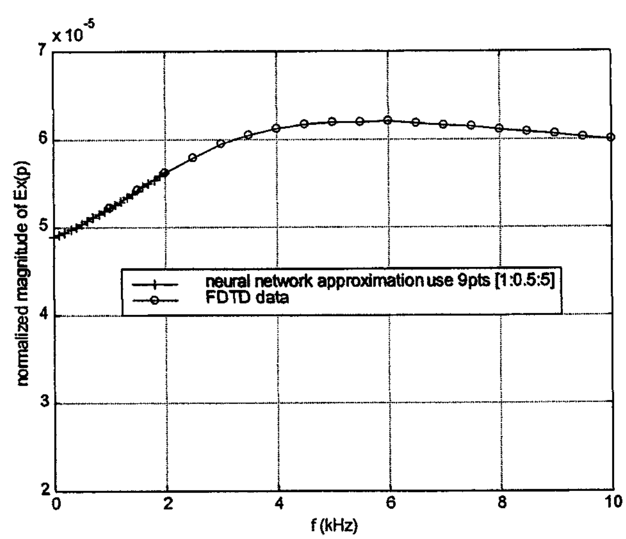

Fig. 9. Numerical extrapolation to a low frequency of 1 Hz. NN approach.

V. Depth Estimation of Buried Objects—Inverse Problem

We now discuss how the NN can be used to estimate the depth of a buried object by processing the electric field data above the ground. The network is used, as a first step, to establish the relationship between the sensed field and the depth of the buried object and to predict, subsequently, the depth of the object when new field values are presented to the network.

We begin by collecting sufficient data as sample points to train the network. Here, we regard the depth and the associated maximum sensed field value of $Ex(p)$ as one point in the context of the NN. Toward this end, we simulate the buried prism problem for a sequence of depths varying from 5 to 100 m. Fig. 11 shows the dependence of field values on the depth.

The next step is to establish a feedforward backpropagation NN, with one hidden layer of 22 neurons using nonlinear

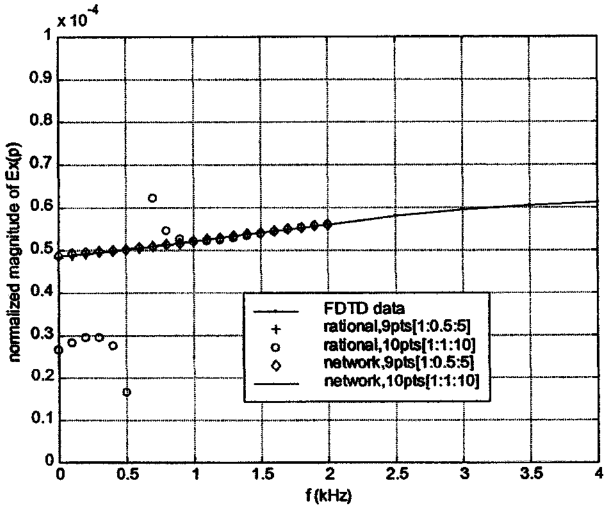

Fig. 10. Numerical extrapolation to a low frequency of 1 Hz. Comparison of NN and rational function approaches.

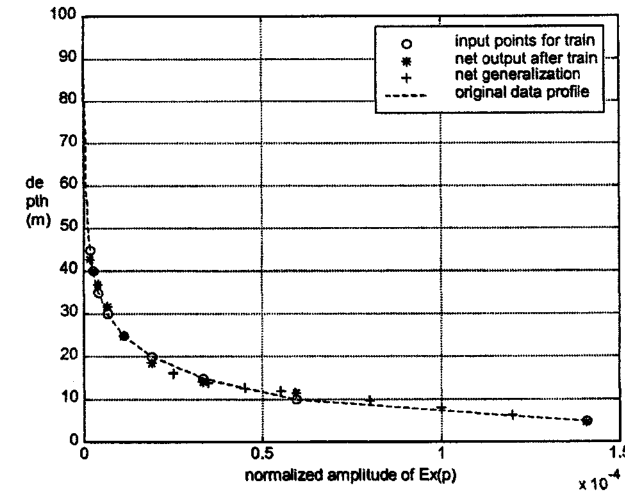

Fig. 12. Estimating the depth of the buried prism using a NN. The NN output is derived after training with nine points.

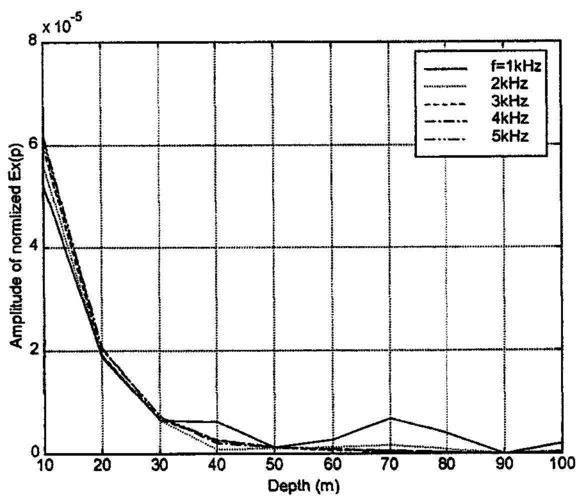

Fig. 11. Dependence of $E_x(p)$ on depth (1–5 kHz).

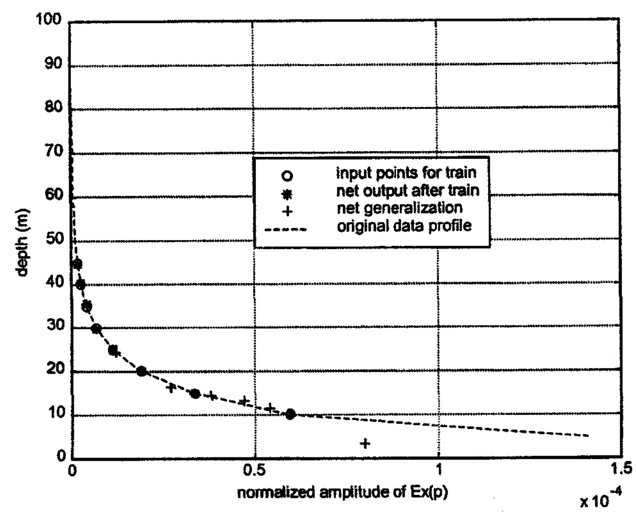

Fig. 13. Estimating the depth of the buried prism using a NN. The NN output is derived after training with eight points.

transfer functions and one layer of a single neuron using a linear transfer function. We regard the computed field values and the corresponding depths as the input and output variables, respectively. During the training phase, the depth is viewed as the target. The NN is trained with the known samples.

In Fig. 12, we show the training and generalization results when nine points are used as training samples. Levenberg–Marquardt backpropagation with Bayesian Regulation is used as the training algorithm, and the limit for the training is set to 2000 epochs. The Bayesian regulation minimizes a linear combination of squared errors and weights. In addition, it modifies the linear combination so that the resulting network has good generalization characteristics at the end of the training. Fig. 12 shows that the network generalization results compare reasonably well to the input data profile. This figure also shows that the training output does not fall exactly on the training target. The reason for this is that the sampled data exhibits a large change in the field value as the depth is varied from 5 to 10 m. This problem could be mitigated by including additional sample points for the depth between 5–10 m. In fact, when we discard the data point for the depth of 5 m and only use the remaining eight points to train the NN, the result improves as shown in Fig. 13. However, the above figure also shows that the prediction for the depth is not very satisfactory when it is less than 10 m, because this depth value is outside the range of the training data. Specifically, this type of prediction entails an extrapolation, which, in turn, requires additional sample points.

VI. Conclusion

We have shown that the FDTD method can be used to simulate scattering problems in the very low frequency regime by extrapolating the results derived at higher frequencies. We have investigated both the NN approach as well as the rational function approximation for the purpose of extrapolation and have found that, comparatively speaking, the former is smoother and more stable.

The inverse scattering problem associated with the buried object has also been investigated via the NN approach. We have found that the NN is good at approximating the relationship between the field scattered by the object and its depth. This is borne out by the fact that it can predict the depth quite accurately when new field values, not used previously for the training process, are presented to the network.

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