A Viable Model for Power Focusing in a Lossy Cylinder

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***Abstract*—In this paper, we are interested in controlling the power dissipation within a homogeneous lossy cylinder of finite length when a field is applied to the surface of the cylinder. The fields are assumed to be independent of the azimuthal angle. To begin, a field which satisfies the required dissipation inside the cylinder is assumed on the axis of the lossy cylinder. An analytical evaluation of the continuous source on the surface of the cylinder can then be carried out. This is an inverse problem where the response is known and the source is to be determined. A realization of the continuous source (surface field) in terms of a discrete array is also given. Results will be presented that show an excellent agreement between the actual continuous sources and the discrete array in producing the field on the axis of the lossy cylinder. In fact, a small number of slots (fifteen or less) produce accurate agreement for the required field on the axis of the cylinder. Bioelectromagnetics and hyperthermia treatment of cancer in cylindrical objects such as limbs and torsos is one potential application. Nondestructive testing of manufactured cylindrical products is another one where the energy is focused in a given region in the cylinder.**

***Index Terms*—Absorbing media, biomedical applications of electromagnetic (EM) radiation, cylinders.**

I. Introduction

As electromagnetic (EM) waves travel through any lossy medium, they suffer distortion that increases as distance or conductivity increases. Thus, in general, it is not straightforward to relate the fields at a given location in a lossy medium with their required surface sources. Our task here is classified as inverse EMs.

In many situations, we desire to control the spatial field distribution in a lossy medium. For example, in bioEMs hyperthermic cancer therapy [1] it is important to focus the field at the tumor location, so that it is exposed to the proper amount of power radiation. In this paper, we consider a finite length lossy cylinder which may model a human limb or torso. The field (or power) distribution on the axis of the lossy cylinder may be assumed to have any desired form and their sources can then be calculated. The approach presented here may be viewed as an alternative to phase and compensated focusing [2]–[4], which assumes a given relationship between the phases and amplitudes of the distributions on the array elements. Therefore, the idea here widens the class of focusing possibilities. In nondestructive testing [5]–[7], it is often required to specify field levels and regions of application for testing a specific component, element, or product. Our analysis should also be relevant to this subject.

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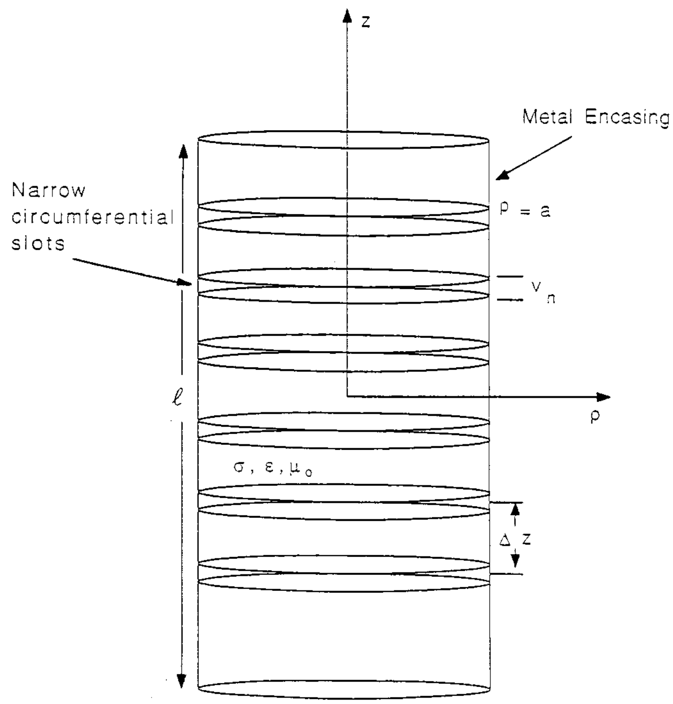

Fig. 1. Geometry for the lossy cylinder in cylindrical coordinates system. The array of cylindrical slots system is also shown with only six slots.

The continuous source on the surface of the lossy cylinder can be realized with an array of thin circumferential slots cut in a metal encasing. The agreement between the fields produced by continuous source and that of the array is excellent.

II. Formulation

As indicated in Fig. 1, the lossy homogeneous circular cylinder is defined by $0 < \rho \leq a$ and $0 \leq z \leq \ell$ with conductivity σ and permittivity ϵ. The region defined by $a < \rho < \infty$ is free-space, with permittivity ϵ_o. The whole region has a magnetic permeability μ_o. On the surface of the cylinder, defined by $\rho = a$, we now specify the source as a surface magnetic source or equivalent surface electric field $E_z(a, z)$ V/m. The source and the fields are assumed azimuthally invariant, but the source can be any function of z. It is to be noted that the source here is TM (to cylinder axis) and that it produces

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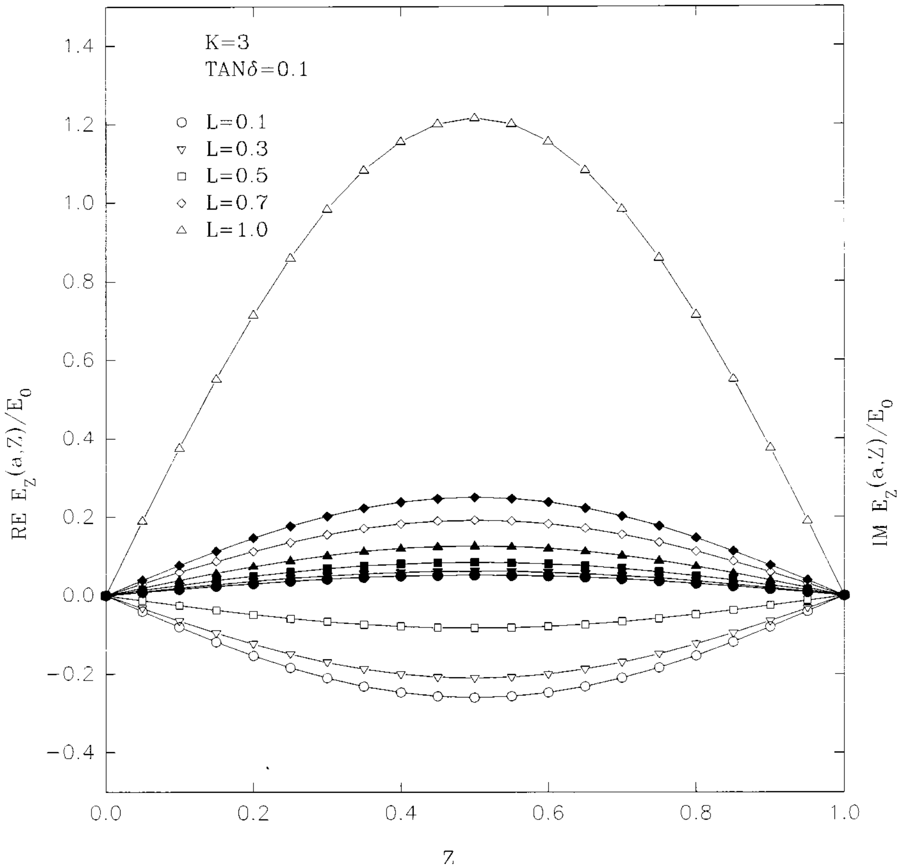

Fig. 2. Real part (hollow symbols) and imaginary part (filled symbols) of the required surface field to produce the prescribed field on the axis for $\tan\delta = 0.1$

purely TM fields. The end caps for the cylinder are perfectly insulating which results in zero current flow ($J_Z = 0$) across the ends of the cylinder. A time harmonic factor $\exp(j\omega t)$ with angular frequency ω. Thus, the desired form of the solution for $0 < \rho < a$ and $0 \le z \le \ell$ is clearly seen to be [8]

$$E_z(\rho, z) = \sum_{m=1}^{\infty} A_m I_o(u_m \rho) \sin\left(\frac{m\pi z}{\ell}\right) \tag{1}$$

where $\gamma^2 = j\mu_o\omega(\sigma + j\epsilon\omega)$. Here, $I_o(u_m\rho)$ is the modified Bessel function of order zero and argument $u_m\rho$ and $u_m = (\gamma^2 + (m\pi/\ell)^2)^{1/2}$. The unknown A_m is related to the aperture distribution $E_z(a, z)$ by

$$A_m = \frac{2}{\ell}\int_0^{\ell} E_z(a, z) \sin\left(\frac{m\pi z}{\ell}\right) dz \tag{2}$$

for any integer $m = 1, 2, 3 \cdots$. Thus, $E_z(\rho, z)$ is determined for any specified aperture distribution. Conversely, we might wish to specify $E_z(\rho, z)$ at, say, $\rho = 0$ and then determine the required form of $E_z(a, z)$. For example if the axial field $E_z(0, z)$ is specified or given, then A_m is given by (2) with $E_z(a, z)$ replaced by $E_z(0, z)$.

Of course, the problem of interest is to specify the power dissipation $\sigma|E_z(0, z)|^2$ without restricting the phase. As a test example we may specify for $0 \le z \le \ell$ that

$$E_z(0, z) = A_o \sin\left(\frac{\pi z}{\ell}\right) \tag{3}$$

where A_o is a constant. Then, clearly, $A_m = A_o$ if $m = 1$ and zero otherwise. Thus

$$E_z(a, z) = A_o I_o(u_1 a) \sin\left(\frac{\pi z}{\ell}\right) \tag{4}$$

where $u_1 = (\gamma^2 + (\pi/\ell)^2)^{1/2}$. In this case, the aperture distribution harmonic content has the same form as the axial field, but enhanced by the complex factor $I_o(u_m a)$.

A numerical study of this problem should give much needed insight into the question of relating desirable axial fields (and power) at the center of the cylinder and the requisite aperture distribution function $E_z(a, z)$.

III. Discretizing the Surface Magnetic Source

As a check on the validity of our results, we would like to realize the surface source by a reasonable model. Thus, we choose a discrete number of narrow circumferential slots (cut in an encasing metal sheath of radius a), which are spaced equally by $\Delta z = z_n - z_{n-1}$ along the length of the cylinder on the z-axis and where the first and last slots are at half spacing from the ends of the cylinder. The slots are fed by applying a voltage across

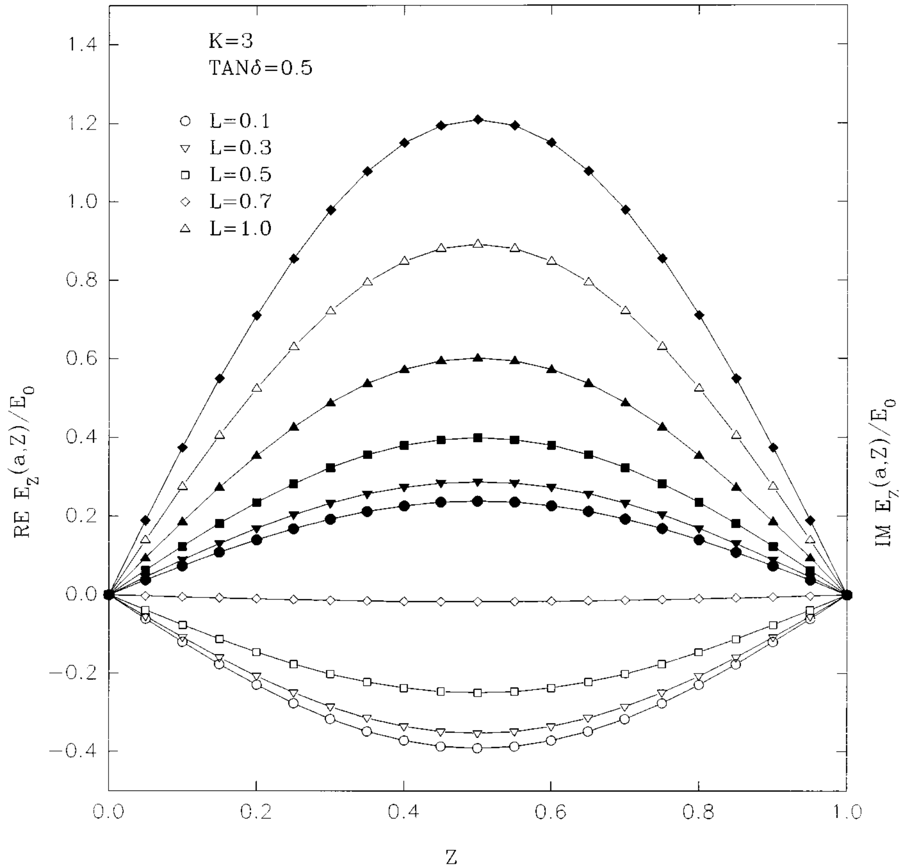

Fig. 3. Real part (hollow symbols) and imaginary part (filled symbols) of the required surface field to produce the prescribed field on the axis for tan$\delta = 0.5$

each slot as shown in Fig. 1. The voltages correspond to the actual continuous source on the surface of the cylinder. Thus, we can approximate

$$E_z(a, z) \simeq \sum_{n=1}^{N} V_n \, \delta(z - z_n) \tag{5}$$

where V_n is the complex voltage at the nth gap at $z = z_n$. The general form in (1) still holds for the internal field, and as Δz becomes small enough we have

$$E_z(\rho, z) \simeq \frac{2}{\ell} \sum_{n=1}^{N} V_n \sum_{m=1}^{\infty} \frac{I_o(u_m \rho)}{I_o(u_m a)} \cdot \sin\left(\frac{m\pi z_n}{\ell}\right) \sin\left(\frac{m\pi z}{\ell}\right). \tag{6}$$

Finally, in order for us to determine how good the discrete array approximation is, in producing the field on the axis of the cylinder, we should compare the required axial field with that produced by the array in (6) when $\rho = 0$. This is done next.

IV. Numerical Results

For the purpose of performing numerical calculations and presenting results, we use dimensionless parameters. We define $Z = z/\ell$, $Z_n = z_n/\ell$, $L = a/\ell$, $K = (\omega/c)(\epsilon/\epsilon_o)^{1/2}$ (c denotes the speed of light in free-space and ϵ_o is the permittivity of free-space), tan$\delta = \sigma/\epsilon\omega$. As a numerical test case, we considered that the required axial field is given by (4) and, for this case, the real and imaginary parts of the normalized surface field $E_Z(a, Z) = E_z(a, z)/E_o$ (with $E_o = A_o$) are plotted as a function of Z over the length of the cylinder, where the total normalized length of the cylinder is equal to one. The parameters used are indicated on the figures. In Fig. 2, the real and imaginary parts of the normalized surface field are shown for the case tan$\delta = 0.1$, while Fig. 3 show the case for tan$\delta = 0.5$. As is expected, the low loss case in Fig. 2 shows that the real part of the surface field is dominant, while its imaginary part is small. This is due to the fact that the required axial field is real and with slightly lossy medium, the surface field propagates into the cylinder with little change. For the lossy case in Fig. 3, tan$\delta = 0.5$, both real and imaginary parts are significant and are both required to produce real axial field.

The comparison of the required axial field with that produced by the discrete array is shown in Fig. 4. This is a forward confirming calculation in which we assume that the slots are fed by the voltages computed from the continuous source distribution. Then the axial field is computed as a response from these slots. The number of slots chosen is five and are equally spaced. The imaginary part is clearly zero (around 10^{-15}) while the real part is point by point coincident with the required axial field.

V. Concluding Remarks

A specified electric field on the axis of a lossy cylinder in space can be produced by a system of cylindrical slots cut in the

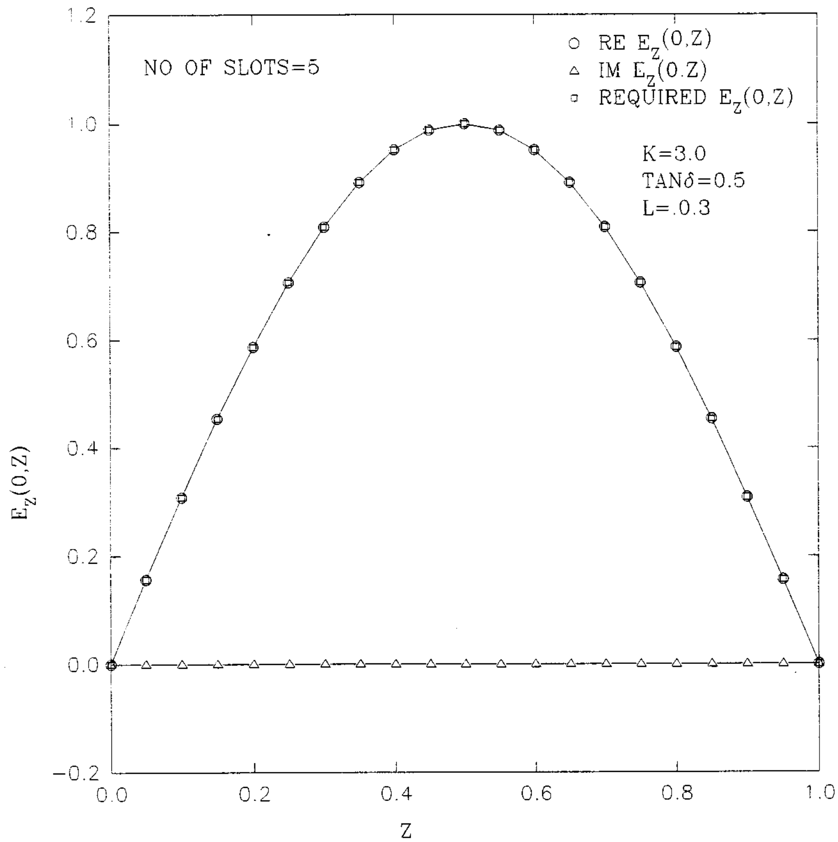

Fig. 4. Confirming forward calculation where the source distribution is approximated by five circumferential slots in a metal sheath.

encasing metal sheath around the cylinder, where each of the slots is fed by a certain voltage. Although the $E_z(0, Z)$ field was specified at the axis of the lossy cylinder, it is not difficult to apply the procedure to other locations inside the cylinder such as specifying $E_z(\rho_1, Z)$, where $0 < \rho_1 < a$. The study here can be extended to layered planar and cylindrical objects.

REFERENCES

[1] J. C. Lin (Ed.), "Special Issue on Phased Arrays for Hyperthermia Treatment of Cancer," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-34, no. 5, pp. 481–648, May 1986.

[2] K. A. Nabulsi and J. R. Wait, "On focusing into a lossy medium from a rectangular aperture," *J. Electromagn. Waves Applicat.*, vol. 6, no. 11, pp. ???–???, 1992.

[3] ——, "Focusing into a lossy medium from an annular slot array," *J. Electromagn. Waves Applicat.*, vol. 7, no. 3, pp. 345–353, 1993.

[4] J. R. Wait and K. A. Nabulsi, "Focusing electromagnetic waves into anisotropic conducting half-space," *J. Physics D—Appl. Phys.*, vol. 25, pp. 1395–1399, 1992.

[5] J. R. Wait, "The electromagnetic basis for nondestructive testing of cylindrical conductors," *IEEE Trans. Instrument. Meas.*, vol. IM-27, pp. 235–238, Sept. 1978.

[6] ——, "Review of electromagnetic methods in nondestructive testing of wire ropes," *Proc. IEEE*, vol. 67, pp. 892–903, June 1979.

[7] H. Lee and G. Wade, Eds., *Imaging Technology*. New York: IEEE Press, 1986.

[8] J. R. Wait, *Electromagnetic Wave Theory*. New York: Harper Row , 1991, ch. 6.

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Dr. Nabulsi was recognized and mentioned in *Who's Who In The World* (14th ed.) in 1997. He was also recognized in *Who's Who In Engineering And Science* in the millennium edition year 2000–2001.

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