Electromagnetic Induction and Surface Impedance in a Half-Space from an Overhead Moving Current System

Hsueh-Yuan Pao, *Senior Member, IEEE*, and James R. Wait

***Abstract*—How does an overhead current system having a velocity v influence the electromagnetic (EM) fields and the surface impedance as measured on the ground? We attempt to answer this question in this paper. A formulation is given for the fields produced by a current system having a uniform translational velocity. The numerical results that reflect the influence of velocity on the surface impedance at the air/earth boundary are presented. The relativity effect is not considered.**

***Index Terms*—Electromagnetic (EM) induction, geological measurements, geophysics, surface impedance.**

I. Introduction

THE nondestructive testing of underlying media often utilizes the interaction of electromagnetic (EM) fields with the specimen. However, the source of the fields may vary both in space and time so that the interaction mechanism between EM waves and the underlying media cannot always be simply described. A particularly good example is an aircraft carrying probe instruments, which flies at a finite height with a uniform velocity v over the earth. Another example is the magnetotelluric method used in geophysics [1]. It is the purpose of this paper to examine the effect of the motional velocity on the nature of the EM fields excited by a uniform moving current system.

A very convenient and physically meaningful concept in radio propagation theory is the surface impedance [2]. In its simplest form, it is the ratio of the tangential electric field and the corresponding orthogonal tangential magnetic field at the air/earth boundary. The surface impedance depends only on the properties of the underlying media and is only weakly influenced by the source field configuration [3].

Here we present a fairly general formulation of the EM fields on the air/earth surface due to a moving current system. This current system can be thought of as an airborne platform, flying at a finite height over a horizontal medium that can be considered at rest and the sources of the natural currents located in the earth, which generate magetotelluric fields [1]. The surface impedance is defined by the EM fields on the ground. According to the derived formula, the numerical results give the influence of the velocity of the moving current system on the surface impedance at the air/earth boundary.

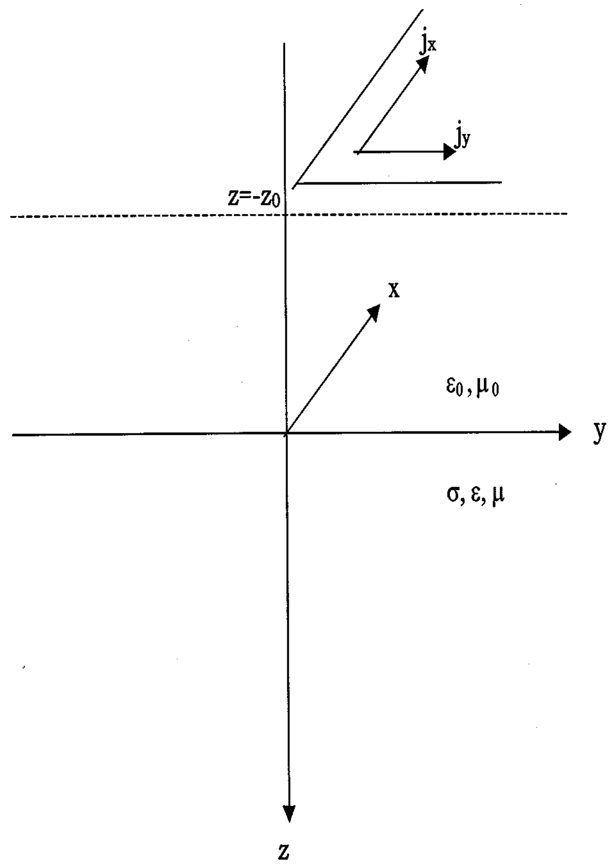

Fig. 1. An overhead uniform moving current excitation of a half-space.

II. Formulation

We shall formulate the problem in as simple a manner as possible, but without restricting its utility. We neglect earth curvature and choose a rectangular coordinate system (x, y, z) with the earth's surface to be the plane $z = 0$ and with the z-axis pointing downward as indicated in Fig. 1. The region $z < 0$ is assumed to be free-space with permittivity ϵ_0 and permeability μ_0 except for a source region at the level $z = -z_0$ (i.e., at a height z_0 above the earth's surface).

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A uniformly moving current source, which flows in the x-direction with a velocity v in the positive y-direction, is located at $z = -z_0$. It is expressed as

$$j_x(x, y, t) = \Re[I_x \delta(x)\delta(y - vt)e^{i\omega t}] \quad (1)$$

where I_x is a constant amplitude of the current source and ω is the angular frequency as seen by an observer in the rest frame (x, y, z). The rest frame can be thought of a coordinate system fixed to the ground. Similarly, in the y direction the spatial component of the current is

$$j_y(x, y, t) = \Re[I_y \delta(x)\delta(y - vt)e^{i\omega t}] \quad (2)$$

where I_y is a constant amplitude of the current source. j_x and j_y are current densities with dimension of amperes/meter and, for simplicity, are always on the $x = 0$ plane. $\Re$ is the real part operator. For mathematical convenience we will only pay attention to the content inside of the brackets of equations (1) and (2).

In the spectral domain

$$\begin{aligned}\tilde{j}_x(\lambda, \omega) &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} I_x \delta(x)\delta(y - vt)e^{i\omega t}e^{-i\beta x}e^{-i\lambda y}\, dx\, dy \\ &= I_x e^{i(\omega - \lambda v)t}\end{aligned} \quad (3)$$

and

$$\tilde{j}_y(\lambda, \omega) = I_y e^{i(\omega - \lambda v)t} \quad (4)$$

where β and λ are wave numbers that describe the spatial variation of the source field. Equations (3) and (4) imply spectral broadening of temporal variations from moving sources relative to their stationary counterpart. This broadening demonstrates temporal contraction of ground-based observations relative to the true morphology. It also introduces significant phase lags or leads between local field maxima observed on the ground. Consequently, the geophysical measurements will be affected by this movement.

It is obvious that the angular frequency becomes $\omega - \lambda v$ instead of ω in the rest frame. Any field component f found in the homogeneous region should satisfy the D'Alembertian equation

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} - \sigma\mu\frac{\partial}{\partial t} - \epsilon\mu\frac{\partial^2}{\partial t^2}\right) f = 0 \quad (5)$$

therefore, the spectral counterpart $\tilde{f}$ satisfies the Helmhotz equation

$$\left[\frac{\partial^2}{\partial z^2} - \lambda^2 - \beta^2 - i(\omega - \lambda v)\sigma\mu + \epsilon\mu(\omega - \lambda v)^2\right]\tilde{f} = 0. \quad (6)$$

We rewrite (6) as

$$\left(\frac{\partial^2}{\partial z^2} - \lambda^2 - \beta^2 - \gamma^2\right)\tilde{f} = 0 \quad (7)$$

where $\gamma^2 = i(\omega - \lambda v)\mu[\sigma + i\epsilon(\omega - \lambda v)]$.

It is convenient to describe the fields in terms of the electric and magnetic Hertz vectors that have only vertical (or z) components. These are denoted by $\mathbf{\Pi}$ and $\mathbf{\Pi}^*$, respectively,

$$\left.\begin{aligned}\Pi_z &= \Pi_0(z)e^{-i\beta x}e^{-i\lambda y}e^{i(\omega - \lambda v)t}, \\ \Pi_z^* &= \Pi_0^*(z)e^{-i\beta x}e^{-i\lambda y}e^{i(\omega - \lambda v)t}.\end{aligned}\right\}. \quad (8)$$

Within any of the homogeneous regions (say, a conductivity σ, permittivity ϵ, and permeability μ), we can write [4]

$$\left.\begin{aligned}\boldsymbol{E} &= \nabla \times \nabla \times \mathbf{\Pi} - i\mu(\omega - \lambda v)\nabla \times \mathbf{\Pi}^* \\ \boldsymbol{H} &= \nabla \times \nabla \times \mathbf{\Pi}^* + [\sigma + i\epsilon(\omega - \lambda v)]\nabla \times \mathbf{\Pi}\end{aligned}\right\} \quad (9)$$

which yield the fields

$$\left.\begin{aligned}E_x &= -i\beta\frac{\partial \Pi_z}{\partial z} - \lambda\mu(\omega - \lambda v)\Pi_z^*, \\ E_y &= -i\lambda\frac{\partial \Pi_z}{\partial z} + \beta\mu(\omega - \lambda v)\Pi_z^* \\ E_z &= \left(\frac{\partial^2}{\partial z^2} - \gamma^2\right)\Pi_z \\ H_x &= -i\beta\frac{\partial \Pi_z^*}{\partial z} - i\lambda[\sigma + i\epsilon(\omega - \lambda v)]\Pi_z \\ H_y &= -i\lambda\frac{\partial \Pi_z^*}{\partial z} + i\beta[\sigma + i\epsilon(\omega - \lambda v)]\Pi_z \\ H_z &= \left(\frac{\partial^2}{\partial z^2} - \gamma^2\right)\Pi_z^*\end{aligned}\right\}. \quad (10)$$

The partial fields associated with the $\mathbf{\Pi}$ potential are usually designated TM and those with the $\mathbf{\Pi}^*$ potential as TE.

In order to determine the fields, we need to first determine $\Pi_0(z)$ and $\Pi_0^*(z)$. For the source region $-z_0 \leq z \leq 0$

$$\left.\begin{aligned}\Pi_0(z) &= ae^{u_0 z} + be^{-u_0 z} \\ \Pi_0^*(z) &= a^*e^{u_0 z} + b^*e^{-u_0 z}.\end{aligned}\right\}. \quad (11)$$

For the source-free region $z < -z_0$

$$\left.\begin{aligned}\Pi_0(z) &= ce^{u_0 z} \\ \Pi_0^*(z) &= c^*e^{u_0 z}\end{aligned}\right\} \quad (12)$$

where $u_0 = \sqrt{\lambda^2 + \beta^2 + \gamma_0^2}$ is the eigenvalue and $\gamma_0^2 = -\epsilon_0\mu_0(\omega - \lambda v)^2$.

Applying boundary conditions at $z = -z_0$ and $x = 0$

$$\left.\begin{aligned}E_x^{(1)}\Big|_{z=-z_0} - E_x^{(2)}\Big|_{z=-z_0} &= 0 \\ E_y^{(1)}\Big|_{z=-z_0} - E_y^{(2)}\Big|_{z=-z_0} &= 0 \\ H_x^{(1)}\Big|_{z=-z_0} - H_x^{(2)}\Big|_{z=-z_0} &= -j_y \\ H_y^{(1)}\Big|_{z=-z_0} - H_y^{(2)}\Big|_{z=-z_0} &= j_x\end{aligned}\right\} \quad (13)$$

where $E_x^{(1)}$, $E_y^{(1)}$, $H_x^{(1)}$, $H_y^{(1)}$, and $E_x^{(2)}$, $E_y^{(2)}$, $H_x^{(2)}$, $H_y^{(2)}$ are the electric and magnetic field components above and below

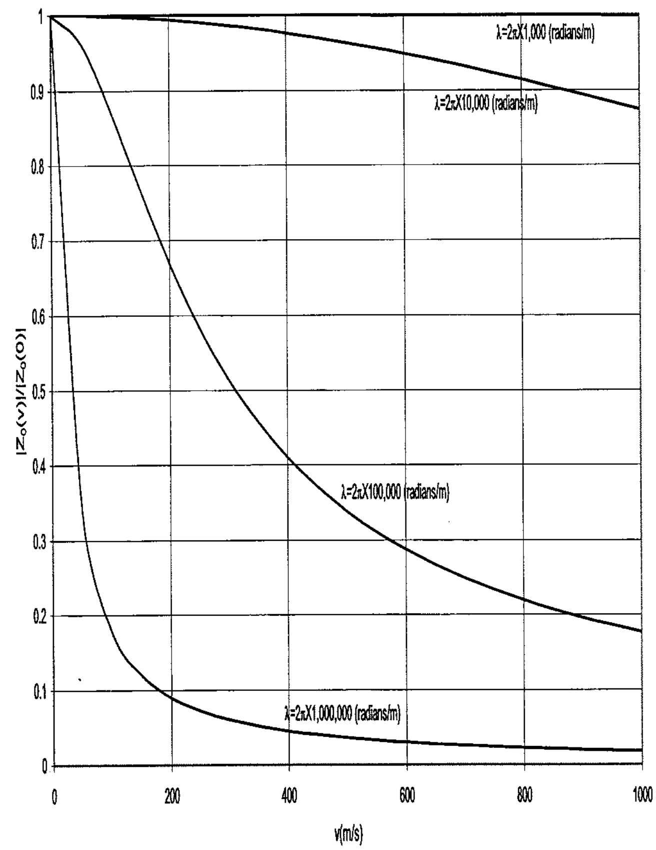

Fig. 2. Normalized surface impedance as a function of velocity for frequency $f = 0.1$ Hz.

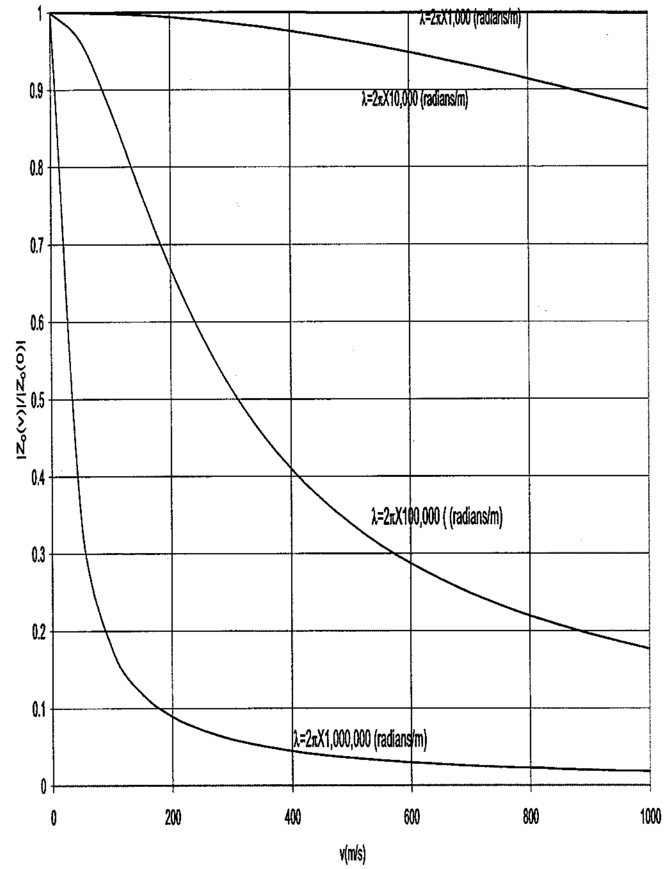

Fig. 3. Normalized surface impedance as a function of velocity for frequency $f = 1$ kHz.

the source at $z = -z_0$. To solve these equations, the "source" coefficients b and b^* are obtained

$$\left.\begin{aligned} b &= \frac{\beta j_x + \lambda j_y}{2\epsilon_0(\omega - \lambda v)(\lambda^2+\beta^2)} e^{-u_0 z_0} \\ b^* &= -\frac{\lambda j_x - \beta j_y}{2iu_0(\lambda^2+\beta^2)} e^{-u_0 z_0} \end{aligned}\right\}. \tag{14}$$

The other coefficients are related to the source coefficients b and b^* as follows:

$$\left.\begin{aligned} a &= Rb \\ a^* &= R^* b^* \\ c &= (R - e^{2u_0 z_0})b \\ c^* &= (R^* + e^{2u_0 z_0})b^* \end{aligned}\right\} \tag{15}$$

where R and R^* are the reflection coefficients

$$\left.\begin{aligned} R &= \frac{K_0 - Z}{K_0 + Z} \\ R^* &= \frac{N_0 - Y}{N_0 + Y} \end{aligned}\right\} \tag{16}$$

where

$$K_0 = \frac{u_0}{i\epsilon_0(\omega - \lambda v)}, \quad \text{and} \quad N_0 = \frac{u_0}{i\mu_0(\omega - \lambda v)}$$

are the TM wave impedance and TE wave admittance, respectively. If the earth were homogeneous with electrical constants σ, ϵ, and μ, we would have

$$Z_0 = \frac{u}{\sigma + i\epsilon(\omega - \lambda v)} \tag{17}$$

and

$$Y_0 = \frac{u}{i\mu(\omega - \lambda v)} \tag{18}$$

respectively, where $u = \sqrt{\lambda^2 + \beta^2 + \gamma^2}$.

We define the simple scalar surface impedance and admittance as

$$Z = \left.\frac{E_x}{H_y}\right|_{TM} = -\left.\frac{E_y}{H_x}\right|_{TM} = -\left.\frac{\frac{\partial \Pi_0}{\partial z}}{i\epsilon_0(\omega - \lambda v)\Pi_0}\right|_{z=0} \tag{19}$$

and

$$Y = -\left.\frac{H_x}{E_y}\right|_{TE} = \left.\frac{H_y}{E_x}\right|_{TE} = -\left.\frac{\frac{\partial \Pi_0^*}{\partial z}}{i\mu_0(\omega - \lambda v)\Pi_0^*}\right|_{z=0} \tag{20}$$

respectively.

We introduce the "wave tilts" concept to get further insight of the basic wave impedance and admittance Z and Y. We first set $\lambda = 0$ so that the excitation is uniform in the y direction. It is not difficult to show that the electric wave tilt is given by

$$W = -\left.\frac{E_x}{E_z}\right|_{z=0} = \frac{u_0}{i\beta}\frac{1-R}{1+R} = \frac{\epsilon_0\omega}{\beta}Z. \tag{21}$$

Similarly, the magnetic wave tilt is

$$W^* = -\left.\frac{H_x}{H_z}\right|_{z=0} = \frac{u_0}{i\beta}\frac{1-R^*}{1+R^*} = \frac{\mu_0\omega}{\beta}Y. \tag{22}$$

These wave tilt ratios are measurable quantities, and they are simply related to the basic wave impedance and admittance. In general, however, the field ratios do not have a simple relationship to Z and Y.

Furthermore, the surface impedance, in general form, is postulated as

$$\left.\begin{aligned} E_x &= Z_{xx}H_x + Z_{xy}H_y \\ E_y &= Z_{yx}H_x + Z_{yy}H_y \end{aligned}\right\} \tag{23}$$

where Z_{xx}, etc., are the elements of the surface impedance matrix $[\boldsymbol{Z}]$ that relates the tangential electric field $\boldsymbol{E}$ and the tangential magnetic field $\boldsymbol{H}$.

As a further step, we need to exhibit the field components in the region $z > 0$ as follows:

$$\begin{aligned} \begin{Bmatrix} E_{0x} \\ E_{0y} \end{Bmatrix} =& \frac{(\beta j_x + \lambda j_y) i u_0}{2\epsilon_0(\omega - \lambda v)(\lambda^2+\beta^2)} \begin{Bmatrix} \beta \\ \lambda \end{Bmatrix} \\ &\cdot \left[e^{-u_0(z+z_0)} - Re^{-u_0(z+z_0)}\right] \\ &+ \frac{(\lambda j_x + \beta j_y)\mu_0(\omega - \lambda v)}{2iu_0(\lambda^2+\beta^2)} \begin{Bmatrix} \lambda \\ -\beta \end{Bmatrix} \\ &\cdot \left[e^{-u_0(z+z_0)} + R^* e^{-u_0(z+z_0)}\right] \end{aligned} \tag{24}$$

and

$$\begin{aligned} \begin{Bmatrix} H_{0x} \\ H_{0y} \end{Bmatrix} =& -\frac{(\lambda j_x - \beta j_y)}{2(\lambda^2+\beta^2)} \begin{Bmatrix} \beta \\ \lambda \end{Bmatrix} \\ &\cdot \left[e^{-u_0(z+z_0)} - R^* e^{-u_0(z+z_0)}\right] \\ &+ \frac{(\beta j_x + \lambda j_y)}{2(\lambda^2+\beta^2)} \begin{Bmatrix} \lambda \\ -\beta \end{Bmatrix} \\ &\cdot \left[e^{-u_0(z+z_0)} + Re^{-u_0(z+z_0)}\right]. \end{aligned} \tag{25}$$

In order to determine the elements of the surface impedance matrix, we examine the fields at the air/earth boundary $(z = 0)$ next, which are

$$\left.\begin{aligned} E_{0x}|_{z=0} &= Z_{xx}\ H_{0x}|_{z=0} + Z_{xy}\ H_{0y}|_{z=0} \\ E_{0y}|_{z=0} &= Z_{yx}\ H_{0x}|_{z=0} + Z_{yy}\ H_{0y}|_{z=0} \end{aligned}\right\}. \tag{26}$$

Equation (26) must hold for any linear combination of j_x and j_y and we can write a set of linear equations after a detailed mathematical manipulation

$$\left.\begin{aligned} &\frac{i\beta u_0}{\epsilon_0(\omega-\lambda v)}(1-R) = [Z_{xx}\lambda - Z_{xy}\beta](1+R) \\ &\frac{\mu_0(\omega-\lambda v)\lambda}{iu_0}(1+R^*) = -[Z_{xx}\beta + Z_{xy}\lambda](1-R^*) \\ &\frac{i\lambda u_0}{\epsilon_0(\omega-\lambda v)}(1-R) = [Z_{yx}\lambda - Z_{yy}\beta](1+R) \\ &-\frac{\mu_0(\omega-\lambda v)\beta}{iu_0}(1+R^*) = -[Z_{yx}\beta + Z_{yy}\lambda](1-R^*) \end{aligned}\right\}. \tag{27}$$

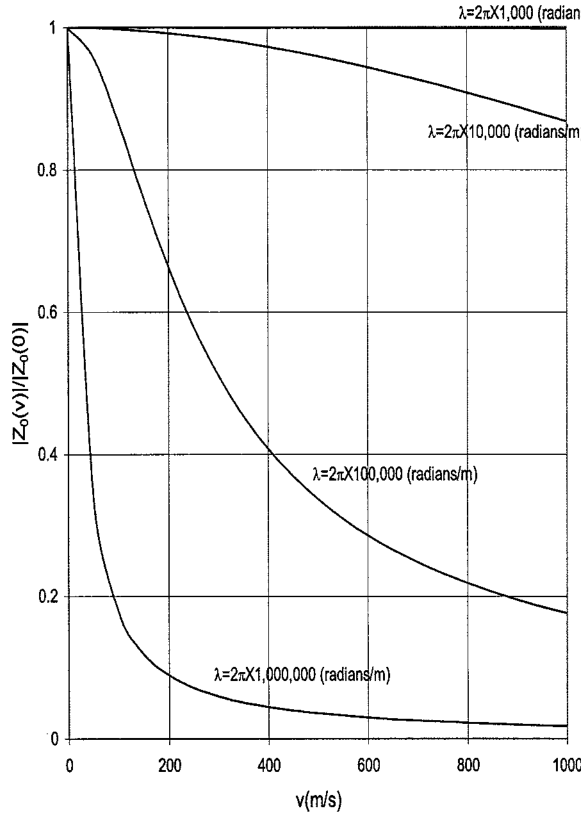

Fig. 4. Normalized surface impedance as a function of velocity for frequency $f = 300$ kHz.

Using wave tilt equations (21) and (22), we obtain the elements of the surface impedance matrix by solving (27)

$$\left.\begin{aligned} Z_{xx} &= -Z_{yy} = \frac{\lambda\beta}{\lambda^2+\beta^2}\left(Y^{-1} - Z\right) \\ Z_{xy} &= \frac{\beta^2 Z + \lambda^2 Y^{-1}}{\lambda^2+\beta^2} \\ Z_{yx} &= -\frac{\lambda^2 Z + \beta^2 Y^{-1}}{\lambda^2+\beta^2} \end{aligned}\right\}. \tag{28}$$

III. Numerical Results

It is interesting to examine the influence of velocity of an overhead moving current system on the surface impedance. We assume that the earth is homogeneous. The parameters we use are $\sigma = 0.01$ S/m, $\epsilon = 10\epsilon_0$, and $\mu = \mu_0$. Furthermore, we consider $\beta = 0$ for simplicity so that the excitation is uniform in the x-direction.

The numerical results derived from these assumptions show the normalized impedance $|Z_0(v)|/|Z_0(0)|$ varied by the velocity v, where $Z_0(0)$ is the surface impedance of the stationary case. The relativity effect is not considered here. Figs. 2 and Fig. 3 illustrate the normalized surface impedance at the frequencies 0.1 Hz and 1 kHz, which bound the frequencies used in most magnetotelluric surveys. Fig. 4 and Fig. 5 present the normalized impedances at frequencies 300 kHz and 300 MHz, respectively. These frequencies are adequate for shallow geophysical surveys performed on airborne platforms.

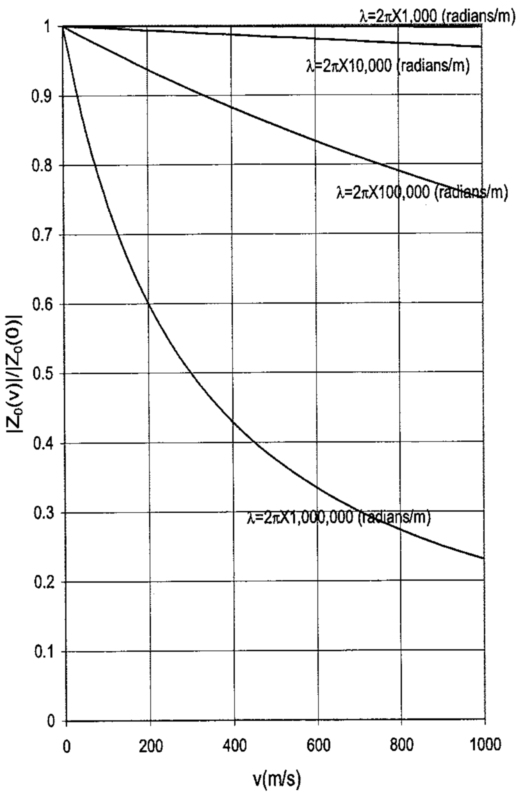

Fig. 5. Normalized surface impedance as a function of velocity for frequency $f = 300$ MHz.

The curves are drawn using different values of λ, which are ranged from $2\pi \times 1000$ radians/m to $2\pi \times 1\,000\,000$ radians/m. It is clear that the term λv is the frequency shift caused by the current source velocity. When λ is small, the frequency shift effect is negligible. Our results show that this effect starts to emerge as λ approaches $2\pi \times 1000$ radians/m. If the source fields change rapidly in space, we must take the source motion into account.

We also observed that the changes in normalized impedances are not easy to distinguish at the frequency range used for magnetotelluric surveys. We attribute this to the low operating frequencies. Since the velocity influence emerges as λ is greater than $2\pi \times 1\,000$ radians/m, the frequency shift factor λv is much bigger than the operating frequency (i.e., 0.1 Hz–1 kHz), therefore, the whole angular frequency terms $\omega - \lambda v$ for 0.1 Hz–1 kHz are not significant different. On the other hand, the frequency shift factor λv is comparable to the operating frequency as $f = 300$ MHz and, thus, we are able to observe the difference in normalized impedances between 300 kHz and 300 MHz.

IV. CONCLUSION

This paper uses classical EM theory to find the EM fields and the surface impedance and admittance on the ground when the source is moving with a finite horizontal velocity. The dependence of the surface impedance and admittance on the presence of the source velocity of the moving system is examined. The relativistic effects are not considered.

The numerical results demonstrate the influence of the velocity on an overhead moving current system to the simplest surface impedance on the air/earth boundary. Two frequency ranges are examined: the first is a very low frequency range as would be considered in magnetotelluric surveys; the second is a higher frequency range adequate for controlled source airborne surveys. The velocity of the moving source makes the source frequency shift. When the source fields change rapidly, this shift effect cannot be neglected. Since the velocity affects the measured value, the curves developed can be used to determine the effects of the velocity on the measured data.

These curves can, therefore, give the appropriate correction factors for the EM fields induced on the ground. From the curves obtained, it can be concluded that the surface impedance are more greatly influenced at high-source velocity.

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