On the Convergence of a Perturbation Series Solution for Reflection from Periodic Rough Surfaces

Rayner K. Rosich, *Senior Member, IEEE* and James R. Wait

***Abstract*—In 1977, Rosich and Wait presented a general expression for a perturbation series solution to electromagnetic (EM) reflection from an imperfectly conducting periodic rough surface. This is the first known publication of such a general result. Although this expression was shown to be consistent with earlier published results, all questions concerning the convergence of the series were deferred. This paper takes a first step in addressing this oversight.**

***Index Terms*—Electromagnetic (EM) scattering, perturbation series, perturbation solution, rough surfaces, rough surface scattering.**

I. Introduction

IN 1997, Rosich and Wait [1] and Rosich [2] considered reflection of a vertically polarized electromagnetic (EM) plane wave from an imperfectly conducting periodic rough surface. The surface profile was decomposed into a Fourier expansion and was characterized by a local boundary impedance. Using a perturbation analysis that was an extension of Wait's [3], a general expression was obtained for the scattering amplitudes that is valid for all spectral and all perturbation orders. This is the first known publication of such a general perturbation result [4].

Although it was shown that this expression is consistent with the earlier results of Rayleigh [5], [6], Rice [7], Barrick [8], and Wait [3], all questions concerning the convergence and conditions for the convergence of this series were deferred in [1]. This paper takes a first step toward addressing this oversight by providing some numerical insight into the convergence properties of this general perturbation series solution.

II. Formulating the Problem

The details of the derivation of the general expression for the scattering amplitudes $a_m^{(p)}$, of specular order m and perturbation order p, can be found in Rosich and Wait [1] and in Rosich [2]. Here it will suffice to reproduce the expression for $a_m^{(p)}$ and enough explanatory information to allow the result to be understood.

Assume a vertically polarized plane EM wave, of free-space wavelength λ_0, is incident at an angle θ_0 on a periodic rough surface. Let the surface profile be given by $z = s(x) = s(x+L)$, where L is the spatial period of the surface. This means the incident magnetic field $\boldsymbol{H}_0$ is antiparallel to the y-axis and the incident electric field $\boldsymbol{E}_0$ and the wave vector $\boldsymbol{k}_0$ are in the xz plane at an angle θ_0 to the x- and the z-axes, respectively. Since the surface is periodic, the total field can be written as a sum of incident and scattered fields with the latter expressed as a discrete angular spectrum of plane waves. For an implied time factor $\exp(i\omega t)$, the appropriate form of the total magnetic field is

$$H_y = H_0 \left[\exp(ik_0(C_0 z - S_0 x)) + \sum_{m=-\infty}^{+\infty} a_m(-ik_0(C_m z + S_m x)) \right]. \quad (1)$$

In (1) $S_0 = \sin\theta_0$ and $C_0 = \cos\theta_0$ characterize the incident field while S_m, C_m and the scattering amplitudes a_m characterize the scattered field. k_0 has the value shown in (2).

Clearly, (1) has the required periodicity, so if we impose the Helmholtz equation $(\nabla^2 + k_0^2)H_y = 0$, we find

$$S_m = S_0 + m\alpha/k_0,\ C_m = (1 - S_m^2)^{1/2}$$
$$\alpha = 2\pi/L,\ k_0 = 2\pi/\lambda_0. \quad (2)$$

The sign of the radical for C_m in (2) is chosen when $S_m^2 > 1$ so the appropriate radiation condition is satisfied as $z \to +\infty \cdot S_m$ and C_m can be interpreted as $\sin\theta_m$ and $\cos\theta_m$, respectively, for an appropriately defined scattering angle θ_m. Therefore, the radiation condition implies for C_m positive real $(S_m^2 < 1)$, the waves are outwardly propagating at an angle $\theta_m = \arcsin S_m$. When C_m is negative imaginary $(S_m^2 > 1)$, however, the waves are evanescent (damped in the $+z$ direction) with $\theta_m = \pi/2 + i\mathrm{arccosh} S_m$. At the transition point $(S_m^2 = 1)$, on the other hand, $C_m = 0$, $\theta_m = \pi/2$, and the waves travel undamped laterally over the surface. As is discussed in [2], these correspond to the well-known Rayleigh wavelengths [5], [9] and Wood's anomalies [10]. In writing (1), we have invoked the Rayleigh hypothesis, which states that only outgoing $(+z)$ reflected waves are needed—even in the troughs of the surface. According to Millar [11]–[13], this is justified when the slope of the surface is everywhere less than 0.448—a criterion satisfied herein. This will be discussed further in Section XI, however.

The electric field components for $z > s(x)$ can be obtained from (1) through the use of Maxwell's equation

$$\boldsymbol{E} = -(i\eta_0/k_0)\nabla \times \boldsymbol{H} \quad (3)$$

where η_0 is the impedance of free-space. Given both the magnetic and the electric fields, all that is needed for a solution to exist to the problem posed here is a boundary condition. As stated earlier, we will require the tangential fields satisfy a local

Manuscript received August 31, 1999; revised November 17, 1999.
R. K. Rosich is at Littleton, CO 80128-5558 USA.
J. R. Wait, deceased.
Publisher Item Identifier S 0018-926X(00)09356-X.

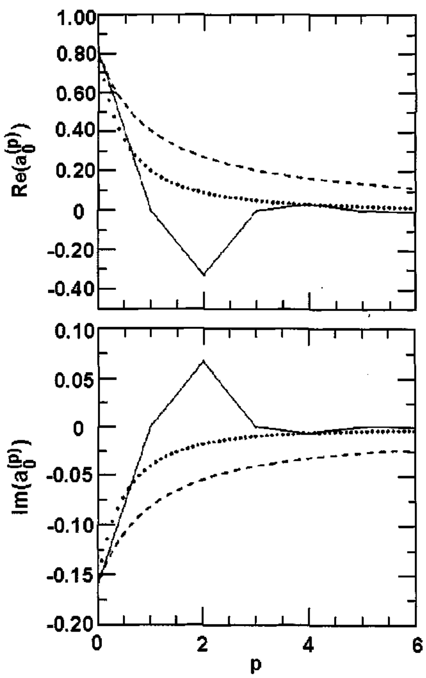

Fig. 1. $a_0^{(p)}$ versus p for $\Delta = 0.10 + i0.10$ at $\theta_0 = 0°$.

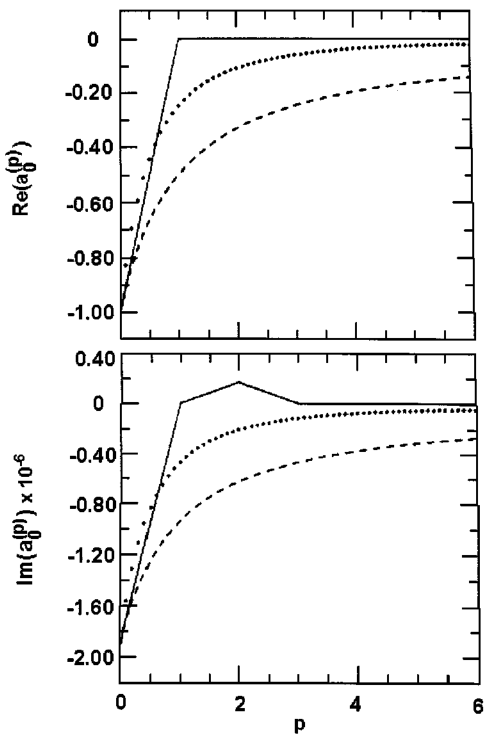

Fig. 2. $a_0^{(p)}$ versus p for $\Delta = 0.10 + i0.10$ at $\theta_0 = 90°$.

boundary impedance. As is shown in [1] and [2], this boundary condition can be expressed at $z = s(x)$ as

$$E_x + \gamma(x)E_z = -\eta_0[1+\gamma^2(x)]^{1/2}\Delta H_y \tag{4}$$

where

$$\gamma(x) = \tan\theta(x) = ds/dx, \quad \Delta = Z/\eta_0, \quad \eta_0 = 120\pi \tag{5}$$

and where Z is the surface impedance and Δ is the normalized surface impedance.

Finally, combining (1)–(5), yields (6). The expression in (6), if solved, provides the solution to the problem posed here. Unfortunately, this problem is intractable by direct mathematical techniques. As a result, (6) must either be solved numerically or by recourse to the Rayleigh–Rice perturbation technique. We will address both techniques below

$$\begin{aligned}\sum_{m=-\infty}^{+\infty} a_m &\left(\frac{[1-(S_0+m\lambda_0/L)^2]^{1/2}}{[1+\gamma^2(x)]^{1/2}} - \frac{(S_0+m\lambda_0/L)\gamma(x)}{[1+\gamma^2(x)]^{1/2}} + \Delta\right)\\ &\cdot \exp\Big(-ik_0[1-(S_0+m\lambda_0/L)^2]^{1/2}s(x)\Big)\\ &\cdot \exp(-i2\pi mx/L)\\ =&\left(\frac{[1-S_0^2]^{1/2}}{[1+\gamma^2(x)]^{1/2}} + \frac{S_0\gamma(x)}{[1+\gamma^2(x)]^{1/2}} - \Delta\right)\\ &\cdot \exp(+ik_0[1-S_0^2]^{1/2}s(x)).\end{aligned} \tag{6}$$

III. Numerical Point-Matching Solution

If the modal expansions in m in (1) and (6) converge, then for every $\varepsilon > 0$, there exists an M, such that if we truncate the modal expansions at $\pm M$, the truncated sum will be within ε of the sum of the full series. Such a truncation leaves the series with $2M + 1$ scattering coefficients $(a_{-M}, \ldots, a_{-1}, a_0, a_{+1}, \ldots, a_{+M})$ to be determined. To do this numerically, we choose $2M + 1$ values of x randomly distributed on the open interval $(0, L)$. We then require (6) be satisfied at each of these values of x and solve the resulting equations for a_m. This is known as the "point-matching" or "collocation" technique.

IV. Perturbation Solution

In the spirit of Rayleigh–Rice, we next proceed to identify quantities in (6) of various orders of smallness. For example, the surface height $k_0 s(x)$ and slope $\gamma(x)$ are of first-order smallness, whereas squares or products of these are of second order and so on for higher orders. In contrast to Barrick [8], however, we do not need to also assume C_0 and Δ are of first-order smallness. To facilitate the analysis, we expand the scattering coefficients a_m in terms of perturbation coefficients $a_m^{(p)}$ of order p smallness. Products of $a_m^{(p)}$ with $k_0 s(x)$ and $\gamma(x)$ are of $p+1$ smallness, while higher order products are of higher order smallness. Furthermore, since $k_0 s(x)$ and $\gamma(x)$ are of first-order smallness, then $\exp[\pm iC_m k_0 s(x)]$ and $[1+\gamma^2(x)]^{1/2}$ can be expressed as Taylor series (see [1], [2]).

Although the Rayleigh–Rice process uncouples all of the perturbation orders in p, it does not uncouple the individual modes (specular orders) in m. To uncouple the modes, $s(x)$, $\gamma(x)$, and their various products must be decomposed into Fourier expansions in terms of the functions $\exp(-iq\alpha x)$, where q (the mode

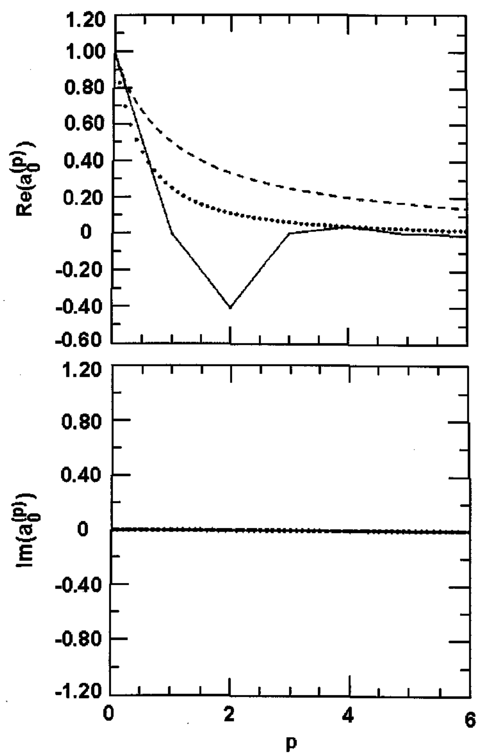

Fig. 3. $a_0^{(p)}$ versus p for $\Delta = 0 + i0$ at $\theta_0 = 0^\circ$.

number) goes from $-\infty$ to $+\infty$ and where $\alpha = 2\pi/L$. Since the surface $s(x)$ is periodic, we can write

$$s(x) = \sum_{q=-\infty}^{+\infty} P_q \exp(-iq\alpha x)$$
$$\gamma(x) = \sum_{q=-\infty}^{+\infty} (-iq\alpha) P_q \exp(-iq\alpha x). \tag{7}$$

For a real surface $P_q = (P_{-q})^*$ for $q \neq 0$ and for a surface with zero mean height $P_0 = 0$. All of the powers of $s(x)$ and $\gamma(x)$ and their products follow naturally from (7). Substituting this into (6) yields an equation in which all of the modes (specular orders) and all of the perturbation orders are uncoupled [1], [2]. Hence, we may solve for $a_m^{(p)}$ and obtain a general expression valid for all modes (spectral orders m) and for all perturbation orders (p). For $p = 0$, we have the following expression for $a_m^{(0)}$:

$$a_{\mathrm{m}}^{(0)} = \begin{Bmatrix} (C_0 - \Delta)/(C_0 + \Delta), & \text{for } m = 0 \\ 0, & \text{for } m \neq 0 \end{Bmatrix}, \quad \text{for } p = 0. \tag{8}$$

While for $p > 0$, solving for $a_m^{(p)}$ yields

$$\begin{aligned}
a_m^{(p)} = {} & \\
& -[C_m + \Delta]^{-1} \Bigg\{ -\frac{i^p C_0^{p+1} k_0^p}{p!} \sum_{q_1=-\infty}^{+\infty} \cdots \sum_{q_{p-1}=-\infty}^{+\infty} \\
& \cdot \left(\prod_{k=1}^{p-1} P_{q_k} \right) P_{m - \sum_{k=1}^{p-1} q_k} \\
& + \frac{i^p S_0 C_0^{p-1} k_0^{p-1} \alpha}{(p-1)!} \sum_{q_1=-\infty}^{+\infty} \cdots \sum_{q_{p-1}=-\infty}^{+\infty} \\
& \cdot \left(\prod_{k=1}^{p-1} P_{q_k} \right) \left(m - \sum_{k=1}^{p-1} q_k \right) P_{m - \sum_{k=1}^{p-1} q_k} \\
& + \sum_{n=0}^{+\infty} \sum_{l=0}^{+\infty} \delta_{n+2l,\, p} \frac{(-1)^{3l+1}(2l-3)!! i^{n+2l} \Delta C_0^n k_0^n \alpha^{2l}}{2^l l! n!} \\
& \cdot \sum_{q_1=-\infty}^{+\infty} \cdots \sum_{q_{p-1}=-\infty}^{+\infty} \\
& \cdot \left[\left(\prod_{k=1}^{n} P_{q_k} \right) \left(\prod_{k=n+1}^{n+2l} q_k P_{q_k} \right) \right]_{q_p = m - \sum_{k=1}^{p-1} q_k} \\
& + \sum_{n=0}^{+\infty} \sum_{j=0}^{p-1} \delta_{n+j,\, p} \frac{(-1)^n k_0^n}{n!} \sum_{q_1=-\infty}^{+\infty} \cdots \sum_{q_n=-\infty}^{+\infty} \\
& \cdot a^{(j)}_{m - \sum_{k=1}^{n} q_k} C^{n+1}_{m - \sum_{k=1}^{n} q_k} \left(\prod_{k=1}^{n} P_{q_k} \right) \\
& - \sum_{n=0}^{+\infty} \sum_{j=0}^{p-1} \delta_{n+j+1,\, p} \frac{(-1)^{n+1} k_0^n \alpha}{n!} \sum_{q_1=-\infty}^{+\infty} \cdots \sum_{q_{n+1}=-\infty}^{+\infty} \\
& \cdot a^{(j)}_{m - \sum_{k=1}^{n+1} q_k} S_{m - \sum_{k=1}^{n+1} q_k} C^n_{m - \sum_{k=1}^{n+1} q_k} \\
& \cdot \left(\prod_{k=1}^{n} P_{q_k} \right) (q_{n+1} P_{q_{n+1}}) + \sum_{n=0}^{+\infty} \sum_{j=0}^{p-1} \sum_{l=0}^{+\infty} \\
& \cdot \delta_{n+j+2l,\, p} \frac{(-1)^{l+1}(2l-3)!!(-i)^{n+2l} \Delta k_0^n \alpha^{2l}}{2^l l! n!} \\
& \cdot \sum_{q_1=-\infty}^{+\infty} \cdots \sum_{q_{n+2l}=-\infty}^{+\infty} a^{(j)}_{m - \sum_{k=1}^{n+2l} q_k} \\
& \cdot C^n_{m - \sum_{k=1}^{n+2l} q_k} \left(\prod_{k=1}^{n} P_{q_k} \right) \left(\prod_{k=n+1}^{n+2l} q_k P_{q_k} \right) \Bigg\} \\
& \text{for } p > 0
\end{aligned} \tag{9}$$

where $\delta_{i,\,j} = \begin{Bmatrix} 0, & \text{for } j \neq k \\ 1, & \text{for } j = k \end{Bmatrix}$

and where $l!! = \begin{Bmatrix} 1 \cdot 3 \cdot 5 \cdot 7 \cdot \cdots \cdot l, & \text{for } l \text{ odd} \\ 2 \cdot 4 \cdot 6 \cdot 8 \cdot \cdots \cdot l, & \text{for } l \text{ even} \end{Bmatrix}.$

V. THE SURFACE

For the comparisons to be made shortly, we shall use the following sinusoidal surface

$$\begin{aligned}
s(x) &= A[\sin(\alpha x) + D] \\
&= (-A/2i)\exp(-i\alpha x) + AD + (A/2i)\exp(i\alpha x) \\
\alpha &= 2\pi/L.
\end{aligned} \tag{10}$$

Hence, $P_0 = AD$, $P_{\pm 1} = \pm A/2i$, and $P_m = 0$ for $m \neq 0, \pm 1$.

VI. AVERAGE MODIFIED NORMALIZED SURFACE IMPEDANCE

Although it is instructive to examine the scattering amplitudes of each mode (spectral order m) individually, a more compact representation is the average (effective) modified normalized surface impedance Δ'. Having in mind a mean flat surface parallel to the xy plane ($z = z_0$) through one period of the rough surface, we let

$$\begin{aligned} E_x &= -\eta_0 H_0 \exp(ik_0 S_0 x) e(x) \\ H_y &= H_0 \exp(-ik_0 S_0 x) h(x). \end{aligned} \tag{11}$$

Then we consider

$$\begin{aligned} \Delta' &= \overline{e}/\overline{h}, \\ \overline{e} &= (1/L) \int_0^L e(x)\, dx \\ \overline{h} &= (1/L) \int_0^L h(x)\, dx. \end{aligned} \tag{12}$$

Using (1), (3), (11), and (12), it is not difficult to show [2] that

$$\Delta' = C_0 \frac{1 - a_0 \exp(-i2k_0 C_0 z_0)}{1 + a_0 \exp(-i2k_0 C_0 z_0)}. \tag{13}$$

Recall in (13) that a_0 is a complex quantity with both real and imaginary parts.

VII. Barrick–Rice and Kirchhoff Models

The most obvious method to validate a theoretical model is by way of comparison with experimental data. In the absence of such data, however, an alternative is comparison against other existing accepted models. This latter course is adopted here. Therefore, at grazing incidence ($\theta_0 \cong 90°$) we will compare against Barrick's [8] model, while near normal incidence ($\theta_0 \cong 0°$) we will compare against the Kirchhoff approximation (model). It must be borne in mind, however, the Kirchhoff approximation neglects multiple scattering and shadowing effects and has all of its modes uncoupled. As a result, one cannot expect exact agreement even near normal incidence.

It is shown in Rosich [2] that Barrick's [8] average modified normalized surface impedance is given by

$$\Delta' = \Delta + A_{00}^{(2)} \tag{14}$$

where

$$\begin{aligned} A_{00}^{(2)} = &\left\{ \frac{k_0 \alpha^2}{b(h-1,0) D(-1,0)} + \Delta \left[\frac{\alpha^2 + k_0 \alpha}{D(-1,0)} + \left(-k_0 \alpha + \frac{\alpha^2}{2} \right) \right] \right\} \frac{A^2}{4} \\ &+ \left\{ \frac{k_0 \alpha^2}{b(h+1,0) D(+1,0)} + \Delta \left[\frac{\alpha^2 - k_0 \alpha}{D(+1,0)} + \left(+k_0 \alpha + \frac{\alpha^2}{2} \right) \right] \right\} \frac{A^2}{4}, \end{aligned} \tag{15}$$

$$D(\pm 1, 0) \cong 1 + \frac{k_0}{b(h \pm 1, 0)} \Delta \left[\frac{b^2(h \mp 1, 0)}{k_0^2} + 1 \right] \tag{16}$$

$$b(h \pm 1, 0) = [k_0^2 - \alpha^2 (h \pm 1)^2]^{1/2} \tag{17}$$

$$h = k_0/\alpha = L/\lambda_0 \tag{18}$$

and where A is the amplitude of the surface $s(x)$ [see (10)].

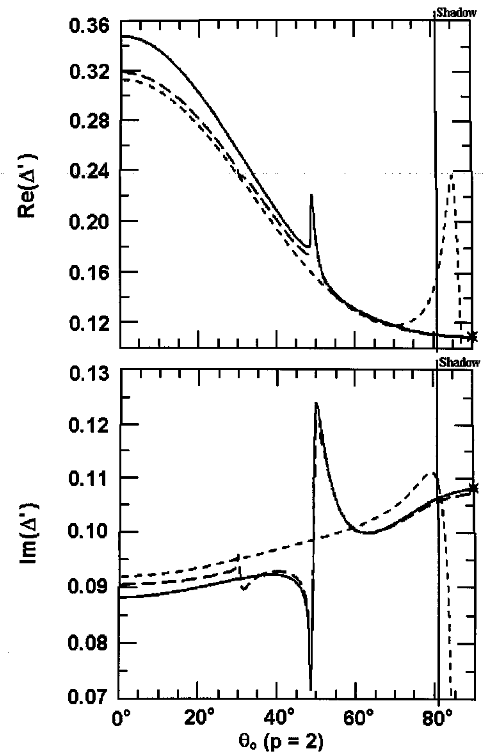

Fig. 4. Δ' versus θ_0 for $\Delta = 0.01 + i0.10$ at $p = 2$.

Similarly, Rosich [2] contains the details of the derivation of the average modified normalized surface impedance for the Kirchhoff approximation (model), which is given by

$$\Delta' = C_0 \frac{2C_0 - (1 + C_0^2 - S_0^2) J_0(-4\pi A C_0/\lambda_0)}{2C_0 + (1 + C_0^2 - S_0^2) J_0(-4\pi A C_0/\lambda_0)} \tag{19}$$

where $J_0(x)$ is the zeroth-order Bessel function of the first kind with argument x.

VIII. Numerical Results—Convergence Tests

In order to gain some insight into the convergence of the perturbation solution given by (8) and (9), we will next examine numerical results for two of the examples from [2]. These were considered likely to converge somewhat slowly due to the value of $\max[\gamma(x)]$ (0.157) being only 35% below Millar's [11]–[13] limit of 0.448. Examples that violated Millar's limit by over 40% were also presented in [2]. Since no hint of convergence could be observed in these latter examples through $m = \pm 10$ and $p = 10$, however, these examples are not presented here. In the two examples below, $A = 1m.$, $L = 40m.$, $\lambda_0 = 10m.$, $D = 0$, and $z_0 = 0m.$, while in the first $\Delta = 0.10 + i0.10$ and in the second $\Delta = 0 + i0$.

In the spirit of "the nth term test" and "the comparison test," Figs. 1–3 show the real (top) and the imaginary (bottom) parts of $a_0^{(p)}$ versus p for $p = 0$ through $p = 6$. Figs. 1 and 2 are for $\Delta = 0.10 + i0.10$ at $\theta_0 = 0°$ and $\theta_0 = 90°$, respectively, while Fig. 3 is for $\Delta = 0 + i0$ at $\theta_0 = 0°$. The results at $\theta_0 = 90°$ for the latter case are not presented due to numerical accuracy problems which occur for infinitely conducting surfaces ($\Delta = 0 + i0$) in the limit as $\theta_0 \to 90°$. This is discussed further in [2].

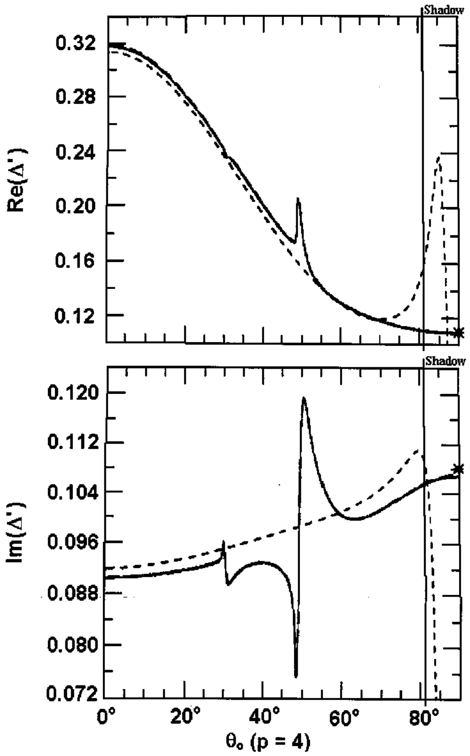

Fig. 5. Δ' versus θ_0 for $\Delta = 0.01 + i0.10$ at $p = 4$.

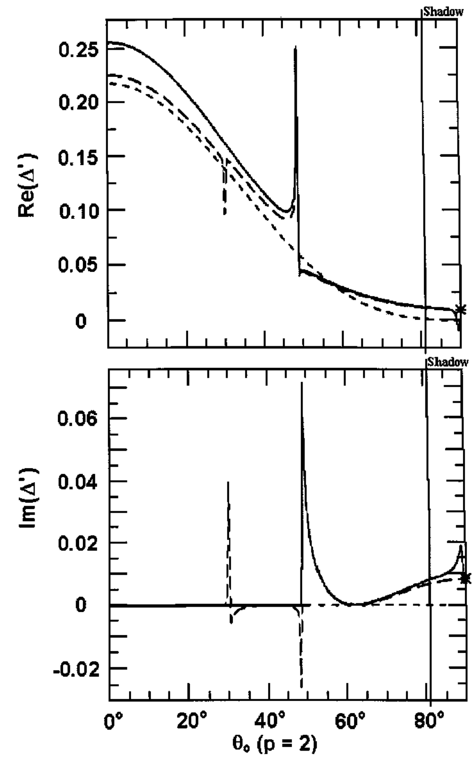

Fig. 6. Δ' versus θ_0 for $\Delta = 0 + i0$ at $p = 2$.

The *dashed curves* in Figs. 1–3 are proportional to $(p+1)^{-1}$ while the *dotted curves* are proportional to $(p+1)^{-2}$.

The proportionality constant, K, in each case is equal to $\max |a_0^{(p)}|$ over the range of p shown in the figure. As can be seen, Figs. 1–3 "seem to" satisfy both "the nth term test" and "the convergence test," although they are not at all completely convincing!

IX. Numerical Results—Model Comparisons

Figs. 4–7 offer far more convincing evidence of the convergence of the perturbation solution. These four figures show the real (top) and the imaginary (bottom) parts of the average modified normalized surface impedance Δ' versus θ_0 (from 0° to 90°) at $p = 2$ (Figs. 4 and 6) and at $p = 4$ (Figs. 5 and 7). Figs. 4 and 5 are for the first example ($\Delta = 0.10 + i0.10$), while Figs. 6 and 7 are for the second example ($\Delta = 0 + i0$; infinite conductivity). In each of these four figures, the *solid line* shows the results for the perturbation solution, while the *medium length dashed line* shows the numerical point-matching results, the *short dashed line* shows the Kirchhoff solution, and the asterisk (*) at $\theta_0 = 90°$ shows Barrick's solution. The vertical line at about $\theta_0 = 81°$ indicates the onset of shadowing; that is, the maximum slope angle of the surface. Normally, one would expect the Kirchhoff solution to begin to break down due to multiple scattering before the onset of shadowing and then to break down catastrophically after the onset of shadowing. Figs. 4 and 5 clearly show that multiple scattering appears to begin about 10° before the shadowing angle. The "graph scaling" makes this far less clear in Figs. 6 and 7. This is due to the fact that for infinitely conducting surfaces ($\Delta = 0 + i0$) the Rayleigh–Wood anomalies have very narrow large-amplitude "spikes" in the plots of the real and the imaginary parts of Δ' versus θ_0. This occurs here (when $S_m^2 = 1$) at $\theta_0 = 30°$ and at $\theta_0 \cong 48.59°$. This naturally impacts the visibility of the shadowing and multiple scattering effects in these figures.

However, as promised, Figs. 4–7 do provide far more convincing evidence of the convergence of the perturbation solution given by (8) and (9) than do Figs. 1–3. Recall that the numeric point-matching, the Kirchhoff, and the Barrick solutions are not subject to the same "convergence considerations" as the perturbation solution—although as was pointed out above, the Kirchhoff and the Barrick solutions each have their own limitations. Note in Figs. 4–7 that while the numeric point-matching, the Kirchhoff, and the Barrick solutions do not change at all between the results for $p = 2$ (Figs. 4 and 6) and for $p = 4$ (Figs. 5 and 7), the perturbation solution very clearly does. Whereas the perturbation solution at $p = 2$ in each case differs very noticeably from the numeric point-matching solution, at $p = 4$ the results from these two solutions seem to be almost identical. This is true for both $\Delta = 0.10 + i0.10$ in Figs. 4 and 5 and for $\Delta = 0 + i0$ in Figs. 6 and 7. This provides the clearest evidence found in [2] for the convergence of the perturbation solution in the cases presented here—both of which obey Millar's [11]–[13] slope limit.

X. Convergence—Analytical Considerations

Unfortunately, thus far, the complexity of (9) has defied all of our analytical attempts to extract expressions for the radius of

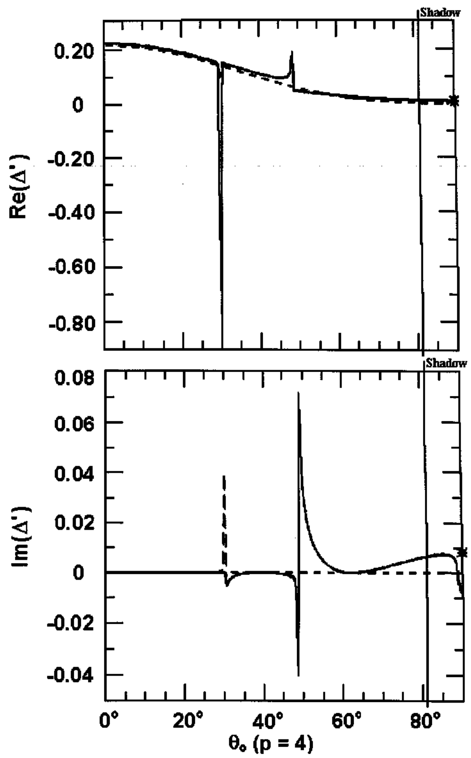

Fig. 7. Δ' versus θ_0 for $\Delta = 0 + i0$ at $p = 4$.

convergence and for the conditions of convergence of the perturbation solution. Such convergence conditions as are known will be discussed in the next section, however.

XI. Conclusion

As stated earlier, our goal is to shed some light on the convergence and the conditions for convergence of the general perturbation series originally presented in [1] and [2]. In order to do so, we first examined $a_0^{(p)}$ as a function of p in the spirit of "the nth term test" and "the comparison test." Although $a_0^{(p)}$ appeared to behave as it should for a convergent series, the evidence was not entirely convincing nor was it exhaustive.

More convincing evidence of convergence was presented in the form of "model comparisons." The average modified normalized surface impedance Δ' as a function of θ_0 was compared for: 1) the general perturbation series solution; 2) the numerical point-matching (collocation) solution; 3) the Barrick–Rice solution; and 4) the Kirchhoff solution. The most convincing evidence came from the comparison of "1" and "2" with "3" and "4" providing an independent secondary test. This, however, naturally raises another issue. This concerns the convergence of the numerical point-matching (collocation) solution and its relationship to the Rayleigh hypothesis.

An early resolution of part of this controversy can be found in [11]–[15], while more recent results can be found in [16]–[19]. Briefly, the former references present results on the validity of the Rayleigh hypothesis. The latter references, on the other hand, show that collocation solutions such as those given here may converge when the Rayleigh criterion is not fulfilled and may diverge when the Rayleigh criterion is fulfilled—depending decisively on the positioning of the collocation points.

Because the collocation points used here are randomly distributed on the open interval $(0, L)$, they do not follow the prescription given in [16]–[19]. Furthermore, the numerical point-matching solution used here minimizes neither the L_2 nor the L_∞ norm. As a result, [11]–[19] provide no guidance on the convergence of our numerical point-matching results.

The complexity of the perturbation solution given in (8) and (9) has thus far defied our attempts to obtain expressions for the radius of convergence and for the conditions of convergence. Therefore, the numerical results presented here and in [2] must be considered entirely on their own merits. Although not mathematically rigorous, the numerical results for the model comparisons do appear rather convincing!

Acknowledgment

The first author would like to thank the reviewers for pointing out the need to clarify the issue of the convergence of the point-matching (collocation) solution and its relationship to the Rayleigh hypothesis. He would also like to thank J. A. DeSanto for several very fruitful discussions of this topic and for pointing out the references in [16]–[19].

In Memorium

During the time I worked on my Ph.D. dissertation under James R. Wait, he always seemed to come up with the right mixture of respect for my need for independence and my often-unrealized need for some gentle guidance, especially when I had managed to "paint myself into a corner." Without this, it is not at all clear to me how long it would have taken me to arrive at the general perturbation solution discussed herein or even if I would have ever arrived at it! For this I shall always owe Jim my deepest gratitude. Following the completion of my dissertation, although we no longer seemed to work on any problems of mutual interest, I typically would get a call from Jim once or twice a year. He always had some interesting new result to report on that was somehow at least loosely related to my earlier work. This always resulted in a very stimulating discussion that reactivated my "EM juices." Unfortunately, the press of my then current job generally served to fairly quickly dissipate my excitement. However, a few of these discussions did result in "Letters to the Editor" that produced much enjoyment for me while I was preparing them. As a result, I can never adequately thank Jim for the intellectual stimulation that he added to my life. I will certainly miss his periodic telephone calls!

References

[1] R. K. Rosich and J. R. Wait, "A general perturbation solution for reflection from two-dimensional periodic surfaces," *Radio Sci.*, vol. 12, no. 5, pp. 719–729, Sept./Oct. 1977.

[2] R. K. Rosich, "Electromagnetic scattering from periodic rough surfaces," Ph.D. dissertation, Phys. Dept., Univ. Colorado, Boulder, Dec. 1977.

[3] J. R. Wait, "Perturbation analysis for reflection from two-dimensional sea waves," *Radio Sci.*, vol. 6, no. 3, pp. 387–391, 1971.

[4] R. K. Rosich, "Comment on 'Scattering from conduncting rough surfaces: A general perturbation solution'," *IEEE Trans. Antennas Propagat.*, vol. 42, pp. 1198–1199, Aug. 1994.
[5] J. W. Strutt, Lord Rayleigh, *Theory of Sound*, 2nd ed. New York: Macmillan, 1896, vol. II.
[6] ——, "On the dynamical theory of gratings," *Proc. Roy. Soc. London*, vol. A79(532), pp. 399–416, 1907.
[7] S. O. Rice, "Reflection of electromagnetic waves from a slightly rough surface" (in *Theory of Electromagnetic Waves*, M. Kline, Ed. New York: Dover, 1965, pp. 351–378), *Commun. Pure Appl. Math.*, vol. 4, pp. 351–378, 1951.
[8] D. E. Barrick, "Theory of HF/VHF propagation across the rough sea—Parts 1, 2," *Radio Sci.*, vol. 6, no. 5, pp. 517–533, 1971.
[9] J. W. Strutt, Lord Rayleigh, "Note on the remarkable case of diffraction spectra described by Prof. Wood," *Phil. Mag.*, vol. 14, no. 79, pp. 60–65, 1907.
[10] R. W. Wood, "On a remarkable case of uneven distribution of light in a diffraction grating spectrum," *Phil. Mag.*, vol. 4, no. 21, pp. 396–402, 1902.
[11] R. F. Millar, "Rayleigh hypothesis in scattering problems," *Electron. Lett.*, vol. 5, no. 17, pp. 416–418, 1969.
[12] ——, "On the Rayleigh assumption in scattering by a periodic surface," *Proc. Cambridge Phil. Soc.*, vol. 65, no. 3, pp. 773–791, 1969.
[13] ——, "On the Rayleigh assumption in scattering by periodic surfaces," *Proc. Cambridge Phil. Soc.*, vol. 69, no. 1, pp. 217–225, 1971.
[14] P. M. van den Berg and J. T. Fokkema, "The Rayleigh hypothesis in the theory of reflection by a grating," *J. Opt. Soc. Amer.*, vol. 69, no. 1, pp. 27–31, 1979.
[15] R. Petit and M. Cadilhac, "Sur la diffraction d'une onde plane par un réseau infiniment conducteur," *Comptes. Rendus Academie Sci. Paris*, ser. B, vol. 262, pp. 468–471, 1966.
[16] A. I. Kleev and A. B. Manenkov, "The convergence of point-matching techniques," *IEEE Trans. Antennas Propagat.*, vol. 37, pp. 50–54, Jan. 1989.
[17] S. Christiansen and R. E. Kleinman, "On a misconception involving point collocation and the Rayleigh hypothesis," *IEEE Trans. Antennas Propagat.*, vol. 44, pp. 1309–1316, Oct. 1996.
[18] A. B. Manenkov, "Comments on 'On a misconception involving point collocation and the Rayleigh hypothesis'," *IEEE Trans. Antennas Propagat.*, vol. 46, p. 1765, Nov. 1998.
[19] S. Christiansen and R. E. Kleinman, "Authors' reply ," *IEEE Trans. Antennas Propagat.*, vol. 46, pp. 1765–1766, Nov. 1998.

Rayner K. Rosich (S'60–M'63–SM'86) received the B.S. (*cum laude*) and M.S. degrees in physics from the University of Michigan, Ann Arbor, in 1962 and 1963, respectively, the Ph.D. degree from the University of Colorado, Boulder, in 1977, and the M.B.A. degree from the University of Denver, CO, in 1982.

In April 2000, he became a Senior Consulting Engineer with Time0, Inc., with responsibility for measuring and modeling the performance of business-to-business e-commerce systems. From October 1999 to April 2000, he was a Senior Network Architect with Redstone Networks. Prior to that, he was a Network Architect at Galileo International and was responsible for upgrading the architecture of United Airline's world-wide multiprotocol intranet. Before that, he held a number of positions involved with the architecture and design of communication networks, with software development, and with electromagnetic propagation and scattering.

Dr. Rosich first met J. R. Wait during his own tenure as a Physicist (physics of the atmosphere, 1967–1980) at the Institute for Telecommunication Sciences, U.S. Department of Commerce, Boulder, CO.

James R. Wait, deceased.