Dipole Radiation over an Inhomogeneous Thin Sheet

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***Abstract*—A general Sommerfeld integral formulation is given for the electromagnetic (EM) fields of an oscillating vertical magnetic or electric dipole over an electrically inhomogeneous thin sheet. The electrical properties of the sheet are characterized by a conductance function that is an arbitrary function of spatial coordinates. When the conductance function has axial symmetry relative to the source dipole, the general solution form simplifies to a Fredholm integral equation of the third kind. The general solution is shown to reduce to the special case of an infinite sheet having uniform conductance. When the sheet conductance is either uniform or varies linearly, the field expressions show an algebraic dependence on the conductance. For a general inhomogeneous conductance distribution, the field dependence is not algebraic.**

***Index Terms*—Dipole antenna, nonhomogeneous materials, thin films.**

I. Introduction

THE presence of electric conductors affects the performance of electromagnetic (EM) devices. For many antennas a flat metal sheet is used as a reflector, or ground plane, that redirects half the radiation into the opposite direction, improving the antenna gain by 3 dB and partially shielding objects on the other side. Although conductive surfaces are useful as reflectors, they also reverse the phase of reflected waves and support propagating surface waves. Without quarter-wavelength separation of the conductive ground plane from the antenna, destructive interference occurs resulting in poor antenna efficiency [1]. The shape and size of the ground plane is controlled by the application environment of various antenna systems and influences radiation characteristics [2]–[5]. One approach for examining ground plane effects with a view to tailoring radiation characteristics is to consider a vertical magnetic or electric dipole above an electrically inhomogeneous thin sheet. Here we consider the limiting case where the EM response of the conductive sheet depends only on the conductance S and not on the conductivity or sheet thickness d considered separately; that is, $\sigma \to \infty$ and $d \to 0$ so that σd is finite. We allow S to be inhomogeneous, that is

$$S = \sigma d = f(\rho, \phi). \tag{1}$$

The problem geometry for a magnetic dipole or an electric dipole over such an inhomogeneous sheet is shown in Fig. 1.

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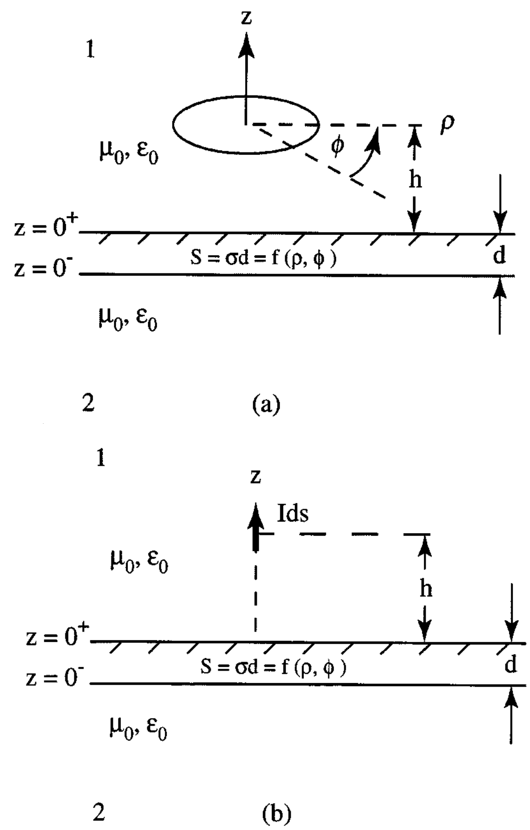

Fig. 1. (a) Vertical magnetic dipole or (b) vertical electric dipole over an electrically inhomogeneous conducting thin sheet.

II. General Hertzian Potential Formulation

Because the thin sheet is not spatially uniform, both induced electric and magnetic sources must be considered. In general, for a region containing both electric and magnetic sources the field for $e^{j\omega t}$ time dependence may be expressed as

$$\mathbf{E} = -\gamma^2 \mathbf{\Pi}^e + \nabla(\nabla \cdot \mathbf{\Pi}^e) - j\omega\mu_0 \nabla \times \mathbf{\Pi}^m \tag{2}$$

and

$$\mathbf{H} = \gamma^2 \mathbf{\Pi}^m + \nabla(\nabla \cdot \mathbf{\Pi}^m) + (\sigma + j\omega\epsilon_0) \nabla \times \mathbf{\Pi}^e \tag{3}$$

where

$\mathbf{\Pi}^e$ and $\mathbf{\Pi}^m$	electric and magnetic Hertz vectors, respectively;
$\gamma = jk$	propagation constant characterizing the medium of interest;
$k = (\mu_0\epsilon_0)^{1/2}\omega$	free-space;
ϵ_0, μ_0	free-space permittivity (8.854×10^{-12} F/m) and permeability ($4\pi \times 10^{-7}$ H/m).

A. Vertical Magnetic Dipole

The primary Hertz vector for the vertical magnetic dipole above the sheet is only of the magnetic type and has only a nonvanishing z-component in (2) and (3), which may be written as

$$\Pi_{1,p}^m(\rho,\phi,z) = M\frac{e^{-\gamma(\rho^2+(z-h)^2)^{1/2}}}{[\rho^2+(z-h)^2]^{1/2}} = M\int_0^\infty e^{-u|z-h|}\frac{J_0(\lambda\rho)}{u}\lambda\,d\lambda \tag{4}$$

where $u=(\lambda^2-k^2)^{1/2}$ and $M=IdA/4\pi$ and A is the area of the loop. The well-known expressions for the field components of a magnetic dipole in free-space may be obtained from (4) by means of (2) and (3) without the need of an electric Hertz potential. The dipole, however, interacts with the conductive sheet to produce secondary fields that require both electric and magnetic Hertz potentials.

The boundary conditions at the thin sheet [6], [7] are simply expressed as

$$E_{\phi,1}=E_{\phi,2}, \tag{5}$$
$$E_{\rho,1}=E_{\rho,2}, \tag{6}$$
$$H_{\phi,1}-H_{\phi,2}=-J_\rho d=-\sigma dE_\rho=-f(\rho,\phi)E_\rho \tag{7}$$
$$H_{\rho,1}-H_{\rho,2}=J_\phi d=\sigma dE_\phi=f(\rho,\phi)E_\phi \tag{8}$$

where J_ρ and J_ϕ are the components of the current density within the sheet.

The secondary Hertz potentials may be written in general form as

$$\Pi_{1,s}^m = M\sum_{n=-\infty}^{\infty}\int_0^\infty A_n(\lambda)e^{-uz}\frac{\lambda}{u}J_n(\lambda\rho)\,d\lambda\,e^{jn\phi} \tag{9}$$
$$\Pi_{1,s}^e = M\sum_{n=-\infty}^{\infty}\int_0^\infty B_n(\lambda)e^{-uz}\frac{\lambda}{u}J_n(\lambda\rho)\,d\lambda\,e^{jn\phi} \tag{10}$$
$$\Pi_{2,s}^m = M\sum_{n=-\infty}^{\infty}\int_0^\infty C_n(\lambda)e^{uz}\frac{\lambda}{u}J_n(\lambda\rho)\,d\lambda\,e^{jn\phi} \tag{11}$$
$$\Pi_{2,s}^e = M\sum_{n=-\infty}^{\infty}\int_0^\infty D_n(\lambda)e^{uz}\frac{\lambda}{u}J_n(\lambda\rho)\,d\lambda\,e^{jn\phi} \tag{12}$$

where $A_n(\lambda), B_n(\lambda), C_n(\lambda)$, and $D_n(\lambda)$ are functions to be determined based on the conductance function of the thin sheet and the boundary conditions.

From (2) and (3)

$$E_\rho = \frac{\partial^2\Pi^e}{\partial\rho\partial z}-\frac{j\omega\mu_0}{\rho}\frac{\partial\Pi^m}{\partial\phi} \tag{13}$$
$$E_\phi = \frac{1}{\rho}\frac{\partial^2\Pi^e}{\partial\phi\partial z}+j\omega\mu_0\frac{\partial\Pi^m}{\partial\rho} \tag{14}$$
$$H_\rho = \frac{\partial^2\Pi^m}{\partial\rho\partial z}+\frac{(\sigma+j\omega\epsilon_0)}{\rho}\frac{\partial\Pi^e}{\partial\phi} \tag{15}$$
$$H_\phi = \frac{1}{\rho}\frac{\partial^2\Pi^m}{\partial\phi\partial z}-(\sigma+j\omega\epsilon_0)\frac{\partial\Pi^e}{\partial\rho}. \tag{16}$$

At a field point $0<z\le h$,

$$\Pi_1^m = \Pi_{1,p}^m+\Pi_{1,s}^m. \tag{17}$$

or

$$\Pi_1^m = M\left[\int_0^\infty e^{u(z-h)}\frac{J_0(\lambda\rho)}{u}\lambda\,d\lambda + \sum_n\int_0^\infty A_n(\lambda)e^{-uz}\frac{\lambda}{u}J_n(\lambda\rho)\,d\lambda\,e^{jn\phi}\right] \tag{18}$$

and

$$\Pi_1^e = M\sum_n\int_0^\infty B_n(\lambda)e^{-uz}\frac{\lambda}{u}J_n(\lambda\rho)\,d\lambda\,e^{jn\phi}. \tag{19}$$

For $z<0$

$$\Pi_2^m = M\sum_n\int_0^\infty C_n(\lambda)e^{uz}\frac{\lambda}{u}J_n(\lambda\rho)\,d\lambda\,e^{jn\phi} \tag{20}$$

and

$$\Pi_2^e = M\sum_n\int_0^\infty D_n(\lambda)e^{uz}\frac{\lambda}{u}J_n(\lambda\rho)\,d\lambda\,e^{jn\phi}. \tag{21}$$

Continuity of the radial and azimuthal electric field at $z=0$ yields

$$\begin{aligned}&-\sum_{n=-\infty}^{\infty}\int_0^\infty B_n(\lambda)\lambda^2J_n'(\lambda\rho)\,d\lambda\,e^{jn\phi} + \sum_{n=-\infty}^{\infty}\omega\mu_0 n\int_0^\infty\frac{A_n(\lambda)}{\rho}\frac{\lambda}{u}J_n(\lambda\rho)\,d\lambda\,e^{jn\phi}\\ &=\sum_{n=-\infty}^{\infty}\int_0^\infty D_n(\lambda)\lambda^2J_n'(\lambda\rho)\,d\lambda\,e^{jn\phi} + \omega\mu_0\sum_{n=-\infty}^{\infty}n\int_0^\infty\frac{C_n(\lambda)}{\rho}\frac{\lambda}{u}J_n(\lambda\rho)\,d\lambda\,e^{jn\phi}\end{aligned} \tag{22}$$

and

$$\begin{aligned}&-\frac{j}{\rho}\sum_{n=-\infty}^{\infty}n\int_0^\infty B_n(\lambda)\lambda J_n(\lambda\rho)\,d\lambda\,e^{jn\phi} + j\omega\mu_0\int_0^\infty\frac{\lambda^2}{u}J_0'(\lambda\rho)e^{-uh}d\lambda + j\omega\mu_0\sum_{n=-\infty}^{\infty}\int_0^\infty A_n(\lambda)\frac{\lambda^2}{u}J_n'(\lambda\rho)\,d\lambda\,e^{jn\phi}\\ &=\frac{j}{\rho}\sum_{n=-\infty}^{\infty}n\int_0^\infty D_n(\lambda)\lambda J_n(\lambda\rho)\,d\lambda\,e^{jn\phi} + j\omega\mu_0\sum_{n=-\infty}^{\infty}\int_0^\infty C_n(\lambda)\frac{\lambda^2}{u}J_n'(\lambda\rho)\,d\lambda\,e^{jn\phi}.\end{aligned} \tag{23}$$

The required discontinuities in the radial and azimuthal magnetic field components across $z=0$ are expressed by

$$\begin{aligned}&\int_0^\infty e^{-uh}\lambda J_0'(\lambda\rho)\,d\lambda - \sum_{n=-\infty}^{\infty}\int_0^\infty A_n(\lambda)\lambda^2J_n'(\lambda\rho)\,d\lambda\,e^{jn\phi} - \frac{\omega\epsilon_0}{\rho}\sum_{n=-\infty}^{\infty}n\int_0^\infty B_n(\lambda)\frac{\lambda}{u}J_n(\lambda\rho)\,d\lambda\,e^{j\omega\phi}\\ &=\sum_{n=-\infty}^{\infty}C_n(\lambda)\lambda^2J_n'(\lambda\rho)\,d\lambda\,e^{jn\phi}\end{aligned}$$

$$- \frac{\omega\epsilon_0}{\rho} \sum_{n=-\infty}^{\infty} n \int_0^\infty D_n(\lambda) \frac{\lambda}{u} J_n(\lambda\rho)\, d\lambda\, e^{jn\phi} + f(\rho,\phi) \left[-\frac{j}{\rho} \sum_{-\infty}^{\infty} n \int_0^\infty B_n(\lambda) \lambda J_n(\lambda\rho)\, d\lambda\, e^{jn\phi} + j\omega\mu_0 \left(\int_0^\infty e^{-uh} \frac{\lambda^2}{u} J_0'(\lambda\rho)\, d\lambda + \sum_{n=-\infty}^{\infty} \int_0^\infty A_n(\lambda) \frac{\lambda^2}{u} J_n'(\lambda\rho)\, d\lambda\, e^{jn\phi} \right) \right] \quad (24)$$

and

$$-\frac{j}{\rho} \sum_{n=-\infty}^{\infty} n \int_0^\infty A_n(\lambda) \lambda J_n(\lambda\rho)\, d\lambda\, e^{jn\phi} - j\omega\epsilon_0 \sum_{n=-\infty}^{\infty} \int_0^\infty B_n(\lambda) \frac{\lambda^2}{u} J_n'(\lambda\rho)\, d\lambda\, e^{jn\phi}$$
$$= \frac{j}{\rho} \sum_{n=-\infty}^{\infty} n \int_0^\infty C_n(\lambda) \lambda J_n(\lambda\rho)\, d\lambda\, e^{jn\phi} - j\omega\epsilon_0 \sum_{n=-\infty}^{\infty} \int_0^\infty D_n(\lambda) \frac{\lambda^2}{u} J_n'(\lambda\rho)\, d\lambda\, e^{jn\phi} + f(\rho,\phi) \left[\sum_{n=-\infty}^{\infty} \int_0^\infty B_n(\lambda) \lambda^2 J_n'(\lambda\rho)\, d\lambda\, e^{jn\phi} - \frac{\omega\mu_0}{\rho} \sum_{n=-\infty}^{\infty} n \int_0^\infty A_n(\lambda) \frac{\lambda}{u} J_n(\lambda\rho)\, d\lambda\, e^{jn\phi} \right]. \quad (25)$$

For the general inhomogeneous case, an infinite set of integral equations must be solved for A_n, B_n, C_n, and D_n; practically, this set must be truncated, but it is difficult to make any mathematical generalizations with respect to truncation without á priori knowledge of the conductance variation $f(\rho,\phi)$.

1) Azimuthal symmetry of sheet conductance: One simplification of the sheet conductance is that it be axi-symmetric or that $S = f(\rho)$. Then

$$\Pi_{1,s}^m(\rho,z) = \int_0^\infty A(\lambda) e^{-uz} \frac{\lambda}{u} J_0(\lambda\rho)\, d\lambda; \quad z > 0 \quad (26)$$
$$\Pi_{2,s}^m(\rho,z) = \int_0^\infty C(\lambda) e^{uz} \frac{\lambda}{u} J_0(\lambda\rho)\, d\lambda; \quad z < 0 \quad (27)$$

and $B(\lambda) = D(\lambda) = 0$. The field components are then written

$$E_\rho = H_\phi = 0, \quad (28)$$
$$E_\phi = j\omega\mu_0 \frac{\partial \Pi^m}{\partial\rho} \quad (29)$$
$$H_\rho = \frac{\partial^2 \Pi^m}{\partial\rho\partial z} \quad (30)$$
$$H_z = \frac{\partial^2 \Pi^m}{\partial z^2} - \gamma^2 \Pi^m. \quad (31)$$

For an observer position at $0 < z \le h$

$$\Pi_1^m = \Pi_{1,p}^m + \Pi_{1,s}^m \quad (32)$$

or

$$\Pi_1^m(\rho,z) = M \left[\int_0^\infty e^{u(z-h)} \frac{\lambda}{u} J_0(\lambda\rho)\, d\lambda + \int_0^\infty A(\lambda) e^{-uz} \frac{\lambda}{u} J_0(\lambda\rho)\, d\lambda \right]; \quad 0 < z \le h. \quad (33)$$

Enforcement of the boundary conditions at the sheet surface yields

$$A(\lambda) = C(\lambda) - e^{-uh}. \quad (34)$$

For the other coefficient an integral equation in $C(\lambda)$ remains to be solved

$$C(\lambda) = e^{-uh} - \frac{j\omega\mu_0}{2\lambda} \int_0^\infty f(\rho) \times \int_0^\infty C(\xi) \frac{\xi}{\bar{u}} [\xi\rho J_1(\xi\rho) J_1(\lambda\rho)]\, d\xi\, d\rho \quad (35)$$

where $\bar{u} = (\xi^2 - k^2)^{1/2}$.

2) Complete uniformity of sheet conductance: As a check on the results thus far, we note that for $S = f(\rho) = \sigma d =$ constant (uniform conductance of sheet) that

$$C(\lambda) = e^{-uh} - \frac{j\omega\mu_0\sigma d}{2\lambda} \int_0^\infty \frac{\xi^2 C(\xi)}{\bar{u}} \times \int_0^\infty \rho J_1(\xi\rho) J_1(\lambda\rho)\, d\rho\, d\xi \quad (36)$$

so that by use of the identity

$$\frac{1}{\xi}\delta(\xi - \lambda) = \int_0^\infty \rho J_1(\xi\rho) J_1(\lambda\rho)\, d\rho \quad (37)$$

we obtain

$$C(\lambda) = \frac{2ue^{-uh}}{2u + j\omega\mu_0\sigma d} \quad (38)$$

which is the expected result [7].

B. Vertical Electric Dipole

Sommerfeld [9] has derived formal expressions for the fields due to an electric dipole current element of strength Ids over a conducting half-space. For the uniformly conducting half-space, the fields can be represented everywhere in terms of a Hertz vector with only a z component. In the case of the inhomogeneous sheet, the primary Hertz potential above the sheet is only of the electric type and may be written as

$$\Pi_{1,p}^e(\rho,\phi,z) = \frac{Ids}{4\pi} \frac{e^{-\gamma R}}{R} = \frac{Ids}{4\pi} \int_0^\infty e^{-u|z-h|} \frac{J_0(\lambda\rho)}{u} \lambda\, d\lambda \quad (39)$$

where $R = [\rho^2 + (z-h)^2]^{1/2}$. Again the dipole interacts with the inhomogeneous conductive sheet to produce secondary fields that require both electric and magnetic Hertz vectors.

The general solution forms may be written in a form analogous to those of a vertical magnetic dipole over an inhomogeneous sheet. At $0 < z \leq h$, the z-directed electric and magnetic Hertz potentials are written

$$\Pi_1^e = M^e \left[\int_0^\infty e^{u(z-h)} \frac{J_0(\lambda\rho)}{u} \lambda \, d\lambda + \sum_{n=-\infty}^{\infty} \int_0^\infty Q_n(\lambda) e^{-uz} \frac{\lambda}{u} J_n(\lambda\rho) \, d\lambda \, e^{jn\phi} \right] \quad (40)$$

$$\Pi_1^m = M^e \sum_{n=-\infty}^{\infty} \int_0^\infty P_n(\lambda) e^{uz} \frac{\lambda}{u} J_n(\lambda\rho) \, d\lambda \, e^{jn\phi} \quad (41)$$

where $M^e = Ids/(4\pi)$. For $z < 0$,

$$\Pi_2^m = M^e \sum_{n=-\infty}^{\infty} \int_0^\infty U_n(\lambda) e^{uz} \frac{\lambda}{u} J_n(\lambda\rho) \, d\lambda \, e^{jn\phi} \quad (42)$$

and

$$\Pi_2^e = M^e \sum_{n=-\infty}^{\infty} \int_0^\infty V_n e^{uz} \frac{\lambda}{u} J_n(\lambda\rho) e^{jn\phi}. \quad (43)$$

Boundary conditions given by [5]–[8] may be applied for a general conductance function, and in the case of azimuthal symmetry, where $P(\lambda) = U(\lambda) = 0$

$$\Pi_1^e(\rho, z) = M^e \left[\int_0^\infty e^{u(z-h)} \frac{\lambda}{u} J_0(\lambda\rho) \, d\lambda + \int_0^\infty Q(\lambda) e^{uz} \frac{\lambda}{u} J_0(\lambda\rho) \, d\lambda \right] \quad (44)$$

and

$$\Pi_2^e(\rho, z) = M^e \int_0^\infty V(\lambda) e^{uz} \frac{\lambda}{u} J_0(\lambda\rho) \, d\lambda. \quad (45)$$

The field components are given by

$$H_\rho = E_\phi = H_z = 0 \quad (46)$$

$$H_\phi = -(\sigma + j\omega\epsilon_0) \frac{\partial \Pi^e}{\partial \rho} \quad (47)$$

$$E_\rho = \frac{\partial^2 \Pi^e}{\partial \rho \partial z} \quad (48)$$

$$E_z = \frac{\partial^2 \Pi^e}{\partial z^2} - \gamma^2 \Pi^e \quad (49)$$

for $0 < z \leq h$. Enforcement of the continuity of the radial electric field at the sheet surface and the discontinuity in the azimuthal magnetic field yields

$$Q(\lambda) = e^{-uh} - V(\lambda) \quad (50)$$

where

$$V(\lambda) = e^{-uh} - \frac{u}{j\omega\epsilon_0 \lambda (1 + e^{-uh})} \int_0^\infty f(\rho) \times \int_0^\infty V(\xi) \xi [\xi\rho J_1(\xi\rho) J_1(\lambda\rho)] \, d\xi \, d\rho. \quad (51)$$

For complete uniformity of the sheet

$$V(\lambda) = \frac{j\omega\epsilon_0 (1 + e^{-uh}) e^{-uh}}{j\omega\epsilon_0 (1 + e^{-uh}) + u\sigma d}. \quad (52)$$

As long as the conductance of the thin sheet is uniform, the dependence of the Hertzian potential components varies algebraically with respect to σd in the case of magnetic or electric dipole excitation. Except for the case of linear variation of the conductance function, however, this is generally not true for an inhomogeneous sheet.

III. Numerical Example

Consider, for example, the case of a Gaussian variation in the conductance function for the axi-symmetric case of vertical magnetic dipole excitation; that is

$$f(\rho) = a e^{-b^2 \rho^2} \quad (53)$$

so that

$$C(\lambda) = e^{-uh} - \frac{j\omega\mu_0 a}{4\lambda b^2} \int_0^\infty C(\xi) \frac{\xi^2}{\bar{u}} \times e^{-(\xi^2+\lambda^2)/(4b^2)} I_1(\lambda\xi/(4b^2)) \, d\xi \quad (54)$$

where $\bar{u} = (\xi^2 + \gamma^2)^{1/2}$ and I_1 is the modified Bessel function of order one. Equation (54) is a Fredholm integral equation of the third kind with a nonseparable nonsymmetric kernel. For numerical convenience, we form a Fredholm integral equation of the second kind with a symmetric kernel by multiplying both sides of (54) by the factor $\lambda^{3/2}(\lambda^2 - k^2)^{-1/4}$ so as to obtain

$$G(\lambda) = F(\lambda) + \int_0^\infty K(\lambda; \xi) G(\xi) \, d\xi \quad (55)$$

where $G(\lambda) = \lambda^{3/2} C(\lambda)/(\lambda^2 - k^2)^{1/4}$, $F(\lambda) = \lambda^{3/2} e^{-(\lambda^2-k^2)^{1/2} h}/(\lambda^2 - k^2)^{1/4}$ and the modified kernel for a Gaussian conductance variation is

$$K(\lambda; \xi) = K(\xi; \lambda) = -\frac{j\omega\mu_0 a}{4b^2} \left[\frac{\lambda\xi}{(\lambda^2 - k^2)^{1/2} (\xi^2 - k^2)^{1/2}} \right]^{1/2} \times e^{-(\xi^2+\lambda^2)/(4b^2)} I_1\left(\frac{\lambda\xi}{4b^2} \right). \quad (56)$$

The modified integral equation may now be solved by

$$G(\lambda_i) = F(\lambda_i) + \sum_{j=1}^{N} D_j K(\lambda_i; \lambda_j) G(\lambda_j), \quad i = 1, 2, \ldots, N \quad (57)$$

where D_j are quadrature weighting coefficients. In matrix notation

$$[G] = [F] + [K][D][G] \quad (58)$$

so that

$$[G] = ([I] - [K][D])^{-1} [F]. \quad (59)$$

We now evaluate $[G]$ for N points and find $C(\lambda_i)$ and $A(\lambda_i)$ for $i = 1, 2, \ldots, N$. Equations (30), (31), and (33) are then used to determine the secondary magnetic field components arising from the prescribed conductance distribution. The required Hankel transforms are evaluated by using a quadratic Lagrange collocation polynomial for unequally spaced arguments

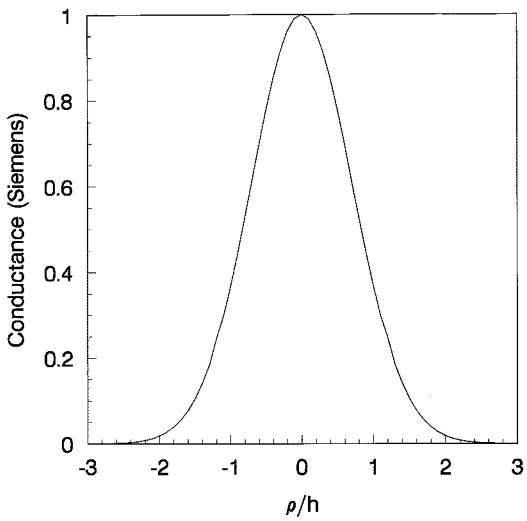

Fig. 2. Sheet conductance variation as function of radial distance from vertical magnetic dipole axis.

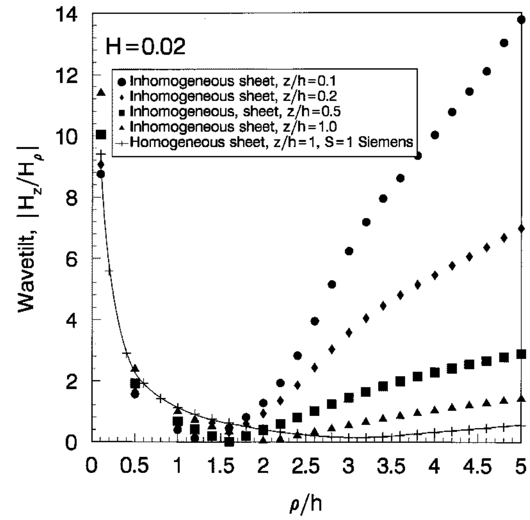

Fig. 3. Wavetilt behavior for vertical magnetic dipole excitation of inhomogeneous sheet having Gaussian conductance variation versus homogeneous sheet.

so that the Bessel function integrations are calculated analytically (see Appendix).

Fig. 2 depicts the normalized conductance variation of an inhomogeneous sheet from the axis of an elevated vertical magnetic dipole. Fig. 3 compares wavetilt, $|H_z/H_\rho|$, behavior over the inhomogeneous conducting sheet with conductance variation given in Fig. 2 with that of a sheet having a uniform conductance of 1 S. Here the normalized height $H = 2\pi(h/\lambda_0) = 0.02$, is much less than the free-space wavelength λ_0. Nulls in the wavetilt amplitude are shifted toward the direction where σd increases and deviations of the wavetilt from that of a homogeneous thin sheet become larger as the horizontal offset from the dipole axis becomes larger. Also shown is the variation of wavetilt as a function of normalized observation height, z/h. For $\rho/h \gg 1$, differences in wavetilt become larger as the observation point is closer to the sheet surface; wavetilt nulls, on the other hand, occur at greater normalized radial distances ρ/h as $z/h \to 1$.

IV. Summary Comments

Thin sheet models have successfully been used in subsurface communication studies at frequencies where the conducting sheet has a thickness small compared to a plane-wave skin depth [10] in the sheet. Such a model gives better agreement with transmission measurements than do the fields of a homogeneous half-space model. A general analysis of a dipole-excited inhomogenous thin sheet in terms of induced Hertzian electric and magnetic sources has been presented, both for magnetic and electric dipole excitation and where the sheet either has no conductance symmetry or is excited off any axis or plane of symmetry. This model is particularly simple in that it requires only the conductivity thickness product σd as a function of spatial coordinates. When the range of parameters is defined for which the model is applicable, it could potentially be used for tailoring the radiation characteristics of an antenna over an inhomogenous ground plane having a nonuniform surface impedance. It could also be used to define the limitations of remote sensing of buried conductive targets when an inhomogeneous conductive overburden is present [11].

Appendix

The expressions for the evaluation of the field components involve integrals of the type

$$g_m(\rho) = \int_a^b f(\lambda) J_m(\lambda\rho)\, d\lambda$$

where for axi-symmetric conductance distributions, $m = 0$ or 1. We approximate $f(\lambda)$ by second-order polynomials in short sections and perform the integrations analytically. If $\lambda, f(\lambda)$ is represented by the discrete pairs (λ_i, f_i), the quadratic polynomial is fit to three points $(\lambda_i, f_i), (\lambda_{i+1}, f_{i+1})$, and (λ_{i+2}, f_{i+2}). Thus, if $f(\lambda)$ is represented by the relation $f(\lambda) = l_0 + l_1\lambda + l_2\lambda^2$ where l_0, l_1, and l_2 are the Lagrange interpolation coefficients, we may write

$$\begin{aligned} g_0(\rho) &= \int_a^b f(\lambda) J_0(\lambda\rho)\, d\lambda \\ &\approx \sum_{i=1}^{q} \int_{\lambda_i}^{\lambda_{i+2}} [l_0 + l_1\lambda + l_2\lambda^2] J_0(\lambda\rho)\, d\lambda \\ &= \sum_{i=1}^{q} \Bigg(\frac{1}{\rho}[J_1(\lambda_{i+2}\rho) f_{i+2} - J_1(\lambda_i\rho) f_i] \\ &\quad + \frac{l_0}{\rho}[J_{i1}(\lambda_i\rho) - J_{i1}(\lambda_{i+2}\rho)] \\ &\quad + \frac{l_2}{\rho^3}[J_{i1}(\lambda_{i+2}\rho) - J_{i1}(\lambda_i\rho)] \\ &\quad + \frac{l_2}{\rho^2}[\lambda_{i+2} J_0(\lambda_{i+2}\rho) - \lambda_i J_0(\lambda_i\rho)] \\ &\quad - \frac{l_2}{\rho^3}[J_1(\lambda_{i+2}\rho) - J_1(\lambda_i\rho)] \Bigg), \end{aligned}$$

and

$$\begin{aligned} g_1(\rho) &= \int_a^b f(\lambda) J_1(\lambda\rho)\, d\lambda \\ &\approx \sum_{i=1}^{q} \int_{\lambda_i}^{\lambda_{i+2}} [l_0 + l_1\lambda + \lambda_2\lambda^2] J_1(\lambda\rho)\, d\lambda \\ &= \sum_{i=1}^{q} \Bigg(\frac{1}{\rho}[J_0(\lambda_i\rho) f_i - J_0(\lambda_{i+2}\rho) f_{i+2}] \\ &\quad - \frac{1}{\rho^2}[l_1(J_1(\lambda_i\rho) - J_1(\lambda_{i+2}\rho)) \\ &\quad + 2l_2(\lambda_i J_1(\lambda_i\rho) - \lambda_{i+2} J_1(\lambda_{i+2}\rho))] \\ &\quad + \frac{l_1}{\rho^2}[J_{i1}(\lambda_i\rho) - J_{i1}(\lambda_{i+2}\rho)] \Bigg) \end{aligned}$$

where $q = (m-1)/2$, m is odd, and $f(\lambda)$ is represented by m points, and where

$$J_{i1}(y) = \int_y^\infty \frac{J_1(t)}{t}\, dt.$$

The Lagrange interpolation coefficients are determined to be

$$\begin{aligned} l_0 &= \frac{\lambda_{i+1}\lambda_{i+2}}{(\lambda_i - \lambda_{i+1})(\lambda_i - \lambda_{i+2})} f_i \\ &\quad + \frac{\lambda_i\lambda_{i+2}}{(\lambda_{i+1} - \lambda_i)(\lambda_{i+1} - \lambda_{i+2})} f_{i+1} \\ &\quad + \frac{\lambda_i\lambda_{i+1}}{(\lambda_{i+2} - \lambda_i)(\lambda_{i+2} - \lambda_{i+1})} f_{i+2}, \\ l_2 &= \frac{\lambda_{i+1} - \lambda_{i+2}}{(\lambda_{i+1} - \lambda_i)(\lambda_i - \lambda_{i+2})} f_i \\ &\quad + \frac{\lambda_i + \lambda_{i+2}}{(\lambda_i - \lambda_{i+1})(\lambda_{i+1} - \lambda_{i+2})} f_{i+1} \\ &\quad + \frac{\lambda_i + \lambda_{i+1}}{(\lambda_i - \lambda_{i+2})(\lambda_{i+2} - \lambda_{i+1})} f_{i+2}, \end{aligned}$$

and

$$\begin{aligned} l_2 &= \frac{1}{(\lambda_i - \lambda_{i+1})(\lambda_i - \lambda_{i+2})} f_i \\ &\quad + \frac{1}{(\lambda_{i+1} - \lambda_i)(\lambda_{i+1} - \lambda_{i+2})} f_{i+1} \\ &\quad + \frac{1}{(\lambda_{i+2} - \lambda_i)(\lambda_{i+2} - \lambda_{i+1})} f_{i+2}. \end{aligned}$$

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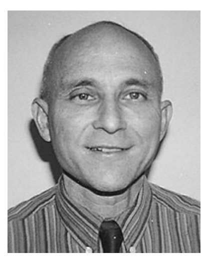

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