Impedance-Type Boundary Conditions for a Periodic Interface Between a Dielectric and a Highly Conducting Medium

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***Abstract*—Using the homogenization method, we derive a generalized impedance-type equivalent boundary condition for the electromagnetic (EM) field at a two-dimensional (2-D) periodic highly conducting rough surface with small-scale roughness. The results obtained in this paper generalize ones obtained previously for the case of a perfectly conducting rough surface. We will show that the coefficients in this equivalent boundary condition can be interpreted in terms of electric and magnetic polarizability densities. We also show that when the roughness dimensions are small compared to a skin depth of the conducting region (a smooth interface), the generalized impedance boundary condition given here reduces to the standard Leontovich condition. Results for the reflection coefficient of a plane wave incident onto a 2-D conducting interface are presented. We show the importance of the boundary-layer fields (as used in this study) over that of classical methods when calculating the reflection coefficient from a highly conducting rough interface. This work will lead to an analysis of the effects of surface roughness on power loss in MIMIC circuits.**

***Index Terms*—Conducting materials, impedance boundary conditions, nonhomogeneous media, periodic boundary conditions.**

I. Introduction

THE problem of theoretically describing the interaction between electromagnetic (EM) waves with a rough surface is an old one, with the first attempts dating back at least as far as those of Lord Rayleigh. We refer the reader to [1] for a brief review of the literature. Scattering from ocean waves or from rough terrain are but a few of the applications to which the results of such a problem might be applied [2]–[4]. More recently, there has been interest in modeling accurately the effect of the surface roughness of finitely conducting metal on the losses produced in microwave structures (see, e.g., [5]–[9]). However, in microwave circles, only qualitative descriptions of roughness effects rather than quantitative modeling tools for use in precise designs have been available up until now.

The Rayleigh-Rice technique is a perturbation method that assumes the height of the surface roughness is small compared to a wavelength and also that the slope of the roughness is small (which means that the height scale of the roughness is small compared to its width scale). Sanderson [7] has used the Rayleigh–Rice method in his treatment of roughness effects on metallic loss.

When the slope of the roughness is not small, Biot [10] and Wait [11] proposed a method by which the roughness was characterized by a random distribution of protrusions or "bosses" from an otherwise plane perfectly conducting surface. Each boss has an effect on the scattered field expressed in terms of its polarizabilities, whose density is used to obtain a boundary condition relating the Hertz potential to the field at the plane surface. Unfortunately, in this theory only the bosses may have finite conductivity; losses in the plane on which they lie are absent.

There are various other results for different roughness profiles found throughout the literature for imperfectly conducting surfaces. Morgan [5] and Baryshnikov *et al.* [8] solve a quasistatic eddy-current problem for a two-dimensional (2-D) periodic rough surface, with a view to computing the additional losses due to surface geometry. The problem of rough interfaces with superconductors has also been discussed in the literature using this method [12]–[15]. Attempts to apply fractal theory to the analysis of roughness effects have created a new research area known as fractal electrodynamics. A good account of this work to the present date can be found in [16].

Until the present, only equivalent boundary conditions for perfectly conducting rough surfaces have been developed; no one as yet has developed an accurate equivalent impedance type boundary condition for a general rough interface of small period. In this paper, the technique of homogenization is used to analyze periodic rough surfaces in order to determine an equivalent impedance type boundary condition for conducting interfaces. The results presented here are an extension of recent work [1], where homogenization was used to derive boundary conditions for perfectly conducting periodic surfaces.

The problem of interest is a 2-D periodic interface between a dielectric and a highly conducting medium [as shown in Fig. 1(a)]. Due to the geometry of the rough surface, one expects that the field should exhibit variations related to the periodicity of the roughness. Close to the rough interface, the total field should be composed of both a localized or boundary-layer field and an effective field. The effective field is what remains after we move a few periods away from the surface. By separating the boundary-layer field from the effective field (through homogenization), it is possible to derive an equivalent boundary condition for the effective field. Hence, the EM scattering from a rough periodic interface can be

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approximated by applying the equivalent boundary condition to an effective smooth surface [as shown in Fig. 1(b)]. This equivalent boundary condition along with Maxwell's equations are all that is needed to determine scattering and reflection from a rough interface. If desired, the boundary-layer fields can later be reconstructed from the effective fields and associated boundary conditions.

In previous applications of the homogenization method to rough surfaces with finite conductivity [17]–[19], the Leontovich impedance boundary condition [20] was imposed on the rough surface before homogenization was carried out. When the skin depth in the conductor is comparable to or greater than the length scale of the surface roughness, however, the Leontovich boundary condition can no longer be regarded as valid and so these results cannot be expected to be accurate. In this paper, effective or equivalent generalized impedance boundary conditions (EGIBCs) for the effective fields at a 2-D rough periodic interface with a highly conducting medium are determined using the method of homogenization. Section two presents the derivation of the equivalent boundary condition for the effective fields. In section three, it is shown that for a smooth interface, the generalized impedance boundary condition presented here reduces to the standard Leontovich surface impedance [20]. The final section shows results for a TM-polarized plane wave incident onto such a surface. In the process, we show the importance of using the boundary-layer fields compared to classical methods [21] when calculating the reflection coefficient from rough interfaces.

II. Derivation of the EGIBC

The EGIBC for the field at a rough interface to a highly conducting medium is determined here by the method of homogenization. This work is an extension of recent work, in which a perfectly conducting periodic rough boundary was treated [1]. The development of the EGIBC given here is in many ways similar to that used in [1] and we will omit some details when they can be found in the earlier work. This section is divided into several subsections, each covering different aspects of the derivation. The first subsections involve expanding the fields in powers of k_op (where p is the period of the structure and k_o is the free-space wavenumber) and determining boundary conditions for the different field components. This leads finally to an impedance type boundary condition for the effective fields.

A. Asymptotic Expansion of Maxwell's Equations

Assume that an EM field is incident onto a 2-D rough interface with a highly conducting medium as shown in Fig. 1(a). Due to this roughness, there are two spatial length scales, one (the free space wavelength λ_o) corresponding to the source or incident wave and the other corresponding to the microstructure of the roughness. The fields will exhibit a multiple-scale type variation that is associated with the microscopic (localized behavior) and macroscopic (global behavior) structures of the problem. In contrast with the problem treated in [1], here there are fields in the conducting region. Maxwell's equations are written as

$$\nabla \times \mathbf{E}^T = -j\omega\mathbf{B}^T$$

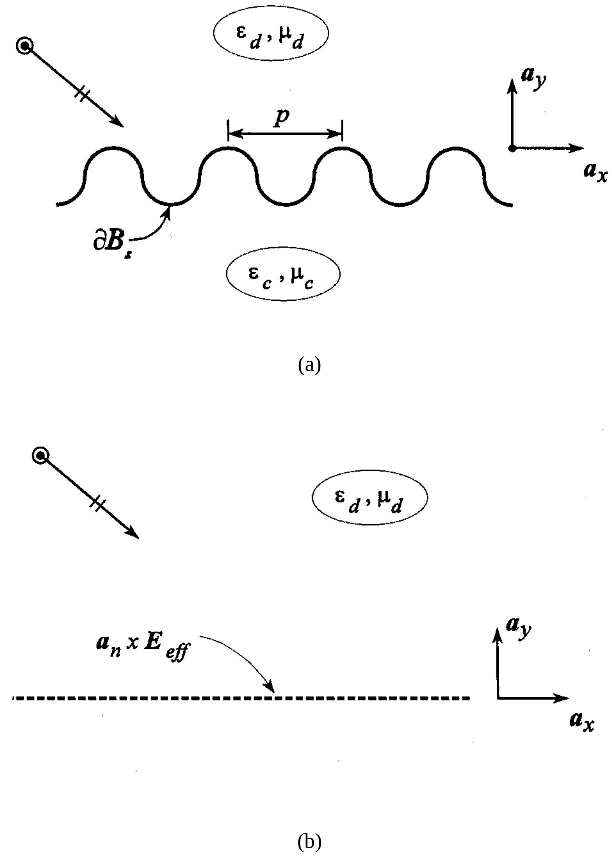

Fig. 1. (a) Geometry of a rough conducting interface. (b) Flat interface where the equivalent boundary condition is applied to the effective field.

$$\nabla \times \mathbf{H}^T = j\omega\mathbf{D}^T \tag{1}$$

where the total fields $\mathbf{E}^T, \mathbf{H}^T, \mathbf{D}^T$ and $\mathbf{B}^T$ (which contain both the localized and global behaviors) obey the constitutive equations

$$\begin{aligned}\mathbf{D}^T &= \epsilon\mathbf{E}^T\\ \mathbf{B}^T &= \mu\mathbf{H}^T.\end{aligned} \tag{2}$$

The material parameters are equal to ϵ_d and μ_d in the dielectric region above the interface and to ϵ_c and μ_c in the conductor region below the interface. When needed, we will consistently use a subscript d or c on a quantity to indicate its value in the dielectric region or the conductor region respectively. These material properties may be complex in lossy regions.

Similar to [1], a multiple-scale representation for these fields is used

$$\mathbf{E}^T(\mathbf{r}, \xi) = \mathbf{E}^T\left(\frac{\hat{\mathbf{r}}}{k_o}, \xi\right) \tag{3}$$

and so on. Here

$$\mathbf{r} = x\mathbf{a}_x + y\mathbf{a}_y + z\mathbf{a}_z \tag{4}$$

is the *slow* spatial variable, $\hat{\mathbf{r}}$ is the dimensionless *slow* variable given by [1]

$$\hat{\mathbf{r}} = k_o\mathbf{r} \tag{5}$$

and ξ is a scaled dimensionless variable referred to as the *fast* variable and defined as

$$\xi = \frac{\mathbf{r}}{p} \tag{6}$$

where p is the period of the roughness, which is small compared to all other lengths in the problem. The slow variable $\hat{\mathbf{r}}$ changes significantly over distances on the order of a wavelength, while the fast variable shows changes over much smaller distances comparable to p.

Microscopic variations with ξ should be expected close to the boundary, but once away from the boundary this behavior should die out. This suggests a boundary-layer field representation for the localized terms. The total fields can be expressed in a form making this boundary-layer effect explicit as follows:

$$\mathbf{E}^T = \mathbf{E}(\hat{\mathbf{r}}) + \mathbf{e}(\hat{\mathbf{r}}, \xi) \tag{7}$$

etc. Here $\mathbf{e}, \mathbf{h}, \mathbf{d}$ and $\mathbf{b}$ are the boundary-layer terms and due to the periodic nature of the interface, these fields are periodic in ξ_x (or x/p). $\mathbf{E}, \mathbf{H}, \mathbf{D}$ and $\mathbf{B}$ are the "nonboundary-layer" fields (to be referred to henceforth as the effective fields) in the dielectric region. In the conductor, the fields will exhibit a rapid decay away from the interface due to strong skin effect. We will assume that the skin depth in the conductor is of comparable size to the period p, so that the fields in the conductor are expected to exhibit only boundary-layer behavior (meaning that the effective fields vanish there). Further,

$$\mathbf{e}, \mathbf{h}, \mathbf{d} \quad \text{and} \quad \mathbf{b} = O\left(e^{-(\text{const})|\xi_y|}\right) \quad \text{as } |\xi_y| \to \infty \tag{8}$$

where $\xi_y = y/p$. Note that the boundary-layer terms are functions of both the *fast* and *slow* variables, while the effective fields are functions of the *slow* variables only.

In this analysis it is assumed that the roughness profile has no variation in z, nor hence in ξ_z. Since the sources or the incident fields are also assumed to be independent of ξ_z, the boundary-layer fields will also be independent of ξ_z. Furthermore, since the boundary-layer fields decay rapidly as the vertical coordinate moves away from the surface (as expressed in terms of the ξ_y variable), it can be shown ([22, pp. 49–53]) and [1] that the boundary-layer fields must be independent of the $\hat{y}$ variable (all variations in the vertical direction are incorporated into the ξ_y variable). Thus, the boundary-layer fields are functions of only four variables: the slow variables $(\hat{x}, \hat{z})$ represented succinctly by the position vector $\hat{\mathbf{r}}_o = \mathbf{a}_x\hat{x} + \mathbf{a}_z\hat{z} \equiv k_o\mathbf{r}_o$ and the components (ξ_x, ξ_y) of ξ transverse to z, i.e.,

$$\mathbf{e}(\hat{\mathbf{r}}_o, \xi_x, \xi_y). \tag{9}$$

Before we proceed with the homogenization technique, we need to decide how to handle the large magnitude of ϵ_c. For a good conductor, we have

$$\left|\frac{\epsilon_c}{\epsilon_d}\right| \gg 1.$$

In this situation, standard homogenization will not be valid (see [23] and [24]) and a so-called dense or stiff method of homogenization must be used. The largeness of ϵ_c must be quantified relative to a small dimensionless parameter ν defined as

$$\nu = k_o p.$$

If the skin depth is to be of the same order as p (the period of the roughness) [24], we must have

$$\epsilon_c = \frac{\epsilon_o G}{\nu^2} \tag{10}$$

where G is a complex constant of order one

$$|G| \sim O(1).$$

For a "good" conductor, $\epsilon_c \approx -j\sigma_c/\omega$; G can be approximated in this case by

$$G = k_o^2 p^2 \frac{\epsilon_c}{\epsilon_o} \approx -j\frac{2\mu_o}{\mu_c}\left(\frac{p}{\delta_c}\right)^2 \tag{11}$$

where δ_c is the skin depth in the conducting region

$$\delta_c = \sqrt{\frac{2}{\omega\mu_c\sigma_c}}. \tag{12}$$

The del operator must be expressed in terms of the fast and slow scaled variables, as presented in [1]. In this way, Maxwell's equations become

$$\begin{aligned} \nabla_{\hat{r}} \times \mathbf{E} + \nabla_{\hat{r}} \times \mathbf{e} + \frac{1}{\nu}\nabla_\xi \times \mathbf{e} &= -jc(\mathbf{B} + \mathbf{b}) \\ \nabla_{\hat{r}} \times \mathbf{H} + \nabla_{\hat{r}} \times \mathbf{h} + \frac{1}{\nu}\nabla_\xi \times \mathbf{h} &= jc(\mathbf{D} + \mathbf{d}) \end{aligned} \tag{13}$$

where

$$c = \frac{1}{\sqrt{\mu_o\epsilon_o}}$$

is the speed of light *in vacuo*. The constitutive equations (2) become

$$\begin{aligned} \mathbf{D} + \mathbf{d} &= \epsilon_o \begin{Bmatrix} \epsilon_{rd} \\ \frac{G}{\nu^2} \end{Bmatrix} (\mathbf{E} + \mathbf{e}) \\ \mathbf{B} + \mathbf{b} &= \mu_o \begin{Bmatrix} \mu_{rd} \\ \mu_{rc} \end{Bmatrix} (\mathbf{H} + \mathbf{h}) \end{aligned} \tag{14}$$

where the upper quantities in the curly brackets apply to the dielectric region and the lower ones to the conductor region (keep in mind that the effective fields vanish in the conductor). Here ϵ_{rd} is the relative permittivity in the dielectric and μ_{rd}, μ_{rc} are the relative permeabilities of the dielectric and conductor, respectively.

Now, the boundary-layer terms vanish as $|\xi_y| \to \infty$ by (8). Thus, from (13) the following is obtained for the fields in the dielectric, away from the boundary:

$$\begin{aligned} \nabla_{\hat{r}} \times \mathbf{E} &= -jc\mathbf{B} \\ \nabla_{\hat{r}} \times \mathbf{H} &= jc\mathbf{D} \end{aligned} \tag{15}$$

where

$$\mathbf{D} = \epsilon_o\epsilon_{rd}\mathbf{E}; \quad \mathbf{B} = \mu_o\mu_{rd}\mathbf{H}. \tag{16}$$

But since $\mathbf{E}$ and $\mathbf{H}$ are independent of ξ, (15) and (16) are true for all $\hat{\mathbf{r}}$ in the dielectric. Removing the effective fields from (13) and (14) by means of (15) and (16), we obtain for the boundary-layer fields

$$\nabla_{\hat{r}} \times \mathbf{e} + \frac{1}{\nu}\nabla_{\xi} \times \mathbf{e} = -jc\mathbf{b}$$
$$\nabla_{\hat{r}} \times \mathbf{e} + \frac{1}{\nu}\nabla_{\xi} \times \mathbf{h} = jc\mathbf{d} \tag{17}$$

along with

$$\mathbf{d} = \epsilon_o \left\{ \begin{matrix} \epsilon_{rd} \\ \frac{G}{\nu^2} \end{matrix} \right\} \mathbf{e}$$
$$\mathbf{b} = \mu_o \left\{ \begin{matrix} \mu_{rd} \\ \mu_{rc} \end{matrix} \right\} \mathbf{h}. \tag{18}$$

For this problem, we are interested in the case when the period is small compared to a wavelength, which corresponds to $\nu \ll 1$. Thus, it is useful to expand the fields in powers of ν, i.e.,

$$\mathbf{E} \sim \mathbf{E}^o(\mathbf{r}) + \nu\mathbf{E}^1(\mathbf{r}) + O(\nu^2)$$
$$\mathbf{e} \sim \mathbf{e}^o(\mathbf{r}_o, \xi_x, \xi_y) + \nu\mathbf{e}^1(\mathbf{r}_o, \xi_x, \xi_y) + O(\nu^2) \tag{19}$$

and similarly for $\mathbf{H}(\mathbf{r})$, $\mathbf{D}(\mathbf{r})$, $\mathbf{B}(\mathbf{r})$, and $\mathbf{h}(\mathbf{r}_o, \xi_x, \xi_y), \mathbf{d}(\mathbf{r}_o, \xi_x, \xi_y), \mathbf{b}(\mathbf{r}_o, \xi_x, \xi_y)$. With some hindsight, it can be seen that the expansion for $\mathbf{d}$ must start at order ν^{-1} (at least in the conductor region), with coefficient $\mathbf{d}^{-1}$. The zeroth-order effective field will contain the incident field as well as any zeroth-order scattered field. By substituting these expansions into (15), (17), (16) and (18) and grouping like powers of ν as in [1], it can be shown that each order of effective fields ($\mathbf{E}^i, \mathbf{H}^i, \mathbf{D}^i$ and $\mathbf{H}^i$) satisfies Maxwell's equations (15) and the constitutive equations (16) individually. For the boundary layer fields

$$\nu^i : \nabla_{\xi} \times \mathbf{e}^{i+1} = -jc\mathbf{b}^i - \nabla_{\hat{r}} \times \mathbf{e}^i$$
$$\nabla_{\xi} \times \mathbf{h}^{i+1} = jc\mathbf{d}^i - \nabla_{\hat{r}} \times \mathbf{h}^i \tag{20}$$

and furthermore, by taking the fast divergence of these curl equations and employing some vector identities:

$$\nabla_{\xi} \cdot \mathbf{b}^{i+1} = -\nabla_{\hat{r}} \cdot \mathbf{b}^i$$
$$\nabla_{\xi} \cdot \mathbf{d}^{i+1} = -\nabla_{\hat{r}} \cdot \mathbf{d}^i \tag{21}$$

for $i = -1, 0, 1, \ldots$. We adopt the convention that orders of the boundary-layer fields not appearing in the expansion (19) are identically zero, e.g.: $\mathbf{e}^{-1} \equiv 0, \mathbf{d}^{-2} \equiv 0, \mathbf{b}^{-1} \equiv 0$ and $\mathbf{h}^{-1} \equiv 0$.

Since the constitutive equations (18) for the boundary-layer fields intermix orders of ν differently in the conductor than in the dielectric, it is convenient to write them out separately for each region. In the dielectric, we have

$$\mathbf{d}_d^i = \epsilon_o\epsilon_{rd}\mathbf{e}_d^i$$
$$\mathbf{b}_d^i = \mu_o\mu_{rd}\mathbf{h}_d^i. \tag{22}$$

In particular, we see that $\mathbf{d}_d^{-1} \equiv 0$. In the conductor region,

$$\mathbf{d}_c^i = \epsilon_o G\mathbf{e}_c^{i+2}$$
$$\mathbf{b}_c^i = \mu_o\mu_{rc}\mathbf{h}_c^i. \tag{23}$$

From this we observe that $\mathbf{e}_c^o \equiv 0$. We see that the lowest-order fields $\mathbf{e}_d^o, \mathbf{h}_d^o, \mathbf{d}_d^o$ and $\mathbf{b}_d^o$ in the dielectric are 2-D static fields which are periodic in ξ_x. The lowest-order fields in the conductor are $\mathbf{e}_c^1, \mathbf{h}_c^o, \mathbf{d}_c^{-1}$ and $\mathbf{b}_c^o$. They obey the classic eddy current problem

$$\nabla_{\xi} \times \mathbf{e}_c^1 = -jc\mathbf{b}_c^o$$
$$\nabla_{\xi} \times \mathbf{h}_c^o = jc\mathbf{d}_c^{-1}$$
$$\nabla_{\xi} \cdot \mathbf{b}_c^o = 0$$
$$\nabla_{\xi} \cdot \mathbf{d}_c^{-1} = 0 \tag{24}$$

with the constitutive equations

$$\mathbf{d}_c^{-1} = \epsilon_o G\mathbf{e}_c^1$$
$$\mathbf{b}_c^o = \mu_o\mu_{rc}\mathbf{h}_c^0 \tag{25}$$

as discussed by Landau and Lifshitz [25]. The electric fields can be eliminated from (24) and (25) and from the field equations in the dielectric to give the equations for the magnetic field at lowest order. In the conductor, the $\mathbf{h}^o$ field is governed by Helmholtz' equation, while it is a magnetostatic field in the dielectric:

$$\nabla_{\xi} \times \mathbf{h}_d^o = 0$$
$$\nabla_{\xi} \cdot \mathbf{b}_d^o = 0$$

in the dielectric, and

$$\left(\nabla_{\xi}^2 + G\right)\mathbf{h}_c^o = 0$$

in the conductor

. (26)

B. Boundary Conditions Applied to the Conducting Interface

The boundary conditions for the fields must now be applied. The tangential $\mathbf{E}$ field is examined first: on the rough interface ∂B_s (see Fig. 2) we have

$$\mathbf{a}_n \times \left[\mathbf{E}_d^T - \mathbf{E}_c^T\right]\Big|_{\partial B_s} = 0. \tag{27}$$

As in [1], we assume that the skin depth, the period and the height of the roughness are comparable and both small compared to other length scales in the problem. Hence, we can define the surface by

$$F(\xi_x, \xi_y) = 0 \tag{28}$$

for some function F is a function which has period one in ξ_x. The normal unit vector $\mathbf{a}_n$ to the rough surface which appears in (27) can then be expressed as

$$\mathbf{a}_n = \frac{\nabla_{\xi}F}{|\nabla_{\xi}F|}. \tag{29}$$

Note that the unit normal is a function of the fast variables only.

The eventual goal of this analysis is to determine an equivalent boundary condition for the effective fields at a plane located near the roughness profile (say, the $y = 0$ plane—see Fig. 2). If the height of the roughness profile is of the same order as p,

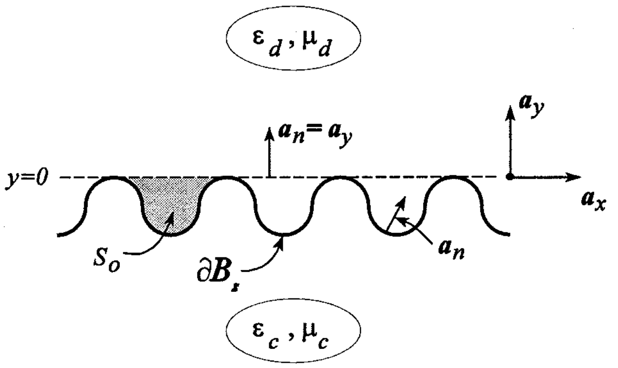

Fig. 2. Fictitious $y = 0$ plane along the top of the roughness profiles.

the effective fields can be evaluated in the boundary-layer by extrapolation relative to a reference level $y = 0$ (let us say, the top of the roughness profile for the moment) with the aid of a Taylor series in y [1]. Using this Taylor series and the asymptotic expansions for the fields, (27) is written as

$$\begin{aligned}\mathbf{a}_n \times \left[\mathbf{E}_d^T - \mathbf{E}_c^T\right]\big|_{\partial B_s} &= \mathbf{a}_n \times \mathbf{E}^o(\mathbf{r}_o) + \mathbf{a}_n \times \mathbf{e}_d^o + \nu\xi_y \mathbf{a}_n \times \left[\frac{\partial \mathbf{E}^o}{\partial \hat{y}}\right]_{y=0} \\ &\quad + \nu \mathbf{a}_n \times \mathbf{E}^1(\mathbf{r}_o) + \nu \mathbf{a}_n \times \mathbf{e}_d^1 - \nu \mathbf{a}_n \times \mathbf{e}_c^1 + O(\nu^2) = 0 \end{aligned} \tag{30}$$

where $\mathbf{r}_o = \mathbf{a}_x x + \mathbf{a}_z z$. By grouping powers of ν, we obtain

$$\nu^o : \mathbf{a}_n \times \mathbf{e}_d^o\big|_{\xi\in\partial B_s} = -\mathbf{a}_n \times \mathbf{E}^o(\mathbf{r}_o) \quad \text{for } \xi \in \partial B_s \tag{31}$$

and

$$\begin{aligned}\nu^1 : \mathbf{a}_n \times \mathbf{e}_d^1\big|_{\xi\in\partial B_s} &= \mathbf{a}_n \times \mathbf{e}_c^1\big|_{\xi\in\partial B_s} - \mathbf{a}_n \times \mathbf{E}^1(\mathbf{r}_0) \\ &\quad - \xi_y \mathbf{a}_n \times \left[\frac{\partial}{\partial \hat{y}} \mathbf{E}^o\right]_{y=0} \\ &\quad \text{for } \xi \in \partial B_s. \end{aligned} \tag{32}$$

These conditions contain additional boundary-layer field terms from the conductor region as compared to those in [1].

For the normal $\mathbf{D}$ field, we have

$$\mathbf{a}_n \cdot \left[\mathbf{D}_d^T - \mathbf{D}_c^T\right]\big|_{\partial B_s} = 0. \tag{33}$$

Extrapolating the effective fields to the $y = 0$ reference level, we have

$$\begin{aligned}\mathbf{D}_d^T = [\mathbf{D}^o(\mathbf{r}_o) + \mathbf{d}_d^o] + \nu\left\{\mathbf{D}^1(\mathbf{r}_o) + \mathbf{d}_d^1 + \xi_y\left[\frac{\partial \mathbf{D}^o}{\partial \hat{y}}\right]_{y=0}\right\} \\ + O(\nu^2) \end{aligned} \tag{34}$$

and

$$\mathbf{D}_c^T = \frac{1}{\nu}\mathbf{d}_c^{-1} + \mathbf{d}_c^o + \nu \mathbf{d}_c^1 + O(\nu^2). \tag{35}$$

Applying the boundary condition and grouping powers of ν, we get

$$\nu^{-1} : \mathbf{a}_n \cdot \mathbf{d}_c^{-1}\big|_{\xi\in\partial B_s} = 0 \quad \text{for } \xi \in \partial B_s \tag{36}$$

and

$$\begin{aligned}\nu^o : \mathbf{a}_n \cdot \mathbf{d}_d^o\big|_{y\xi\in\partial B_s} = \mathbf{a}_n \cdot \mathbf{d}_c^o\big|_{\xi\in\partial B_s} - \mathbf{a}_n \cdot \mathbf{D}^o(\mathbf{r}_o) \\ \text{for } \xi \in \partial B_s. \end{aligned} \tag{37}$$

For the tangential $\mathbf{H}$ fields, we have

$$\mathbf{a}_n \times \left[\mathbf{H}_d^T - \mathbf{H}_c^T\right]\big|_{\partial B_s} = 0. \tag{38}$$

Proceeding as above, we obtain

$$\begin{aligned}\nu^o : \mathbf{a}_n \times \mathbf{h}_d^o\big|_{\xi\in\partial B_s} = \mathbf{a}_n \times \mathbf{h}_c^o\big|_{\xi\in\partial B_s} - \mathbf{a}_n \times \mathbf{H}^o(\mathbf{r}_o) \\ \text{for } \xi \in \partial B_s, \end{aligned} \tag{39}$$

and

$$\begin{aligned}\nu^1 : \mathbf{a}_n \times \mathbf{h}_d^1\big|_{\xi\in\partial B_s} &= \mathbf{a}_n \times \mathbf{h}_c^1\big|_{\xi\in\partial B_s} - \mathbf{a}_n \times \mathbf{H}^1(\mathbf{r}_o) \\ &\quad - \xi_y \mathbf{a}_n \times \left[\frac{\partial \mathbf{H}^o}{\partial \hat{y}}\right]_{y=0} \quad \text{for } \xi \in \partial B_s. \end{aligned} \tag{40}$$

Likewise, the boundary conditions for the normal $\mathbf{B}$ lead to

$$\begin{aligned}\nu^o : \mathbf{a}_n \cdot \mathbf{b}_d^o\big|_{\xi\in\partial B_s} = \mathbf{a}_n \cdot \mathbf{b}_c^o\big|_{\xi\in\partial B_s} - \mathbf{a}_n \cdot \mathbf{B}^o(\mathbf{r}_o) \\ \text{for } \xi \in \partial B_s \end{aligned} \tag{41}$$

and

$$\begin{aligned}\nu^1 : \mathbf{a}_n \cdot \mathbf{b}_d^1\big|_{\xi\in\partial B_s} &= \mathbf{a}_n \cdot \mathbf{b}_c^1\big|_{\xi\in\partial B_s} - \mathbf{a}_n \cdot \mathbf{B}^1(\mathbf{r}_o) \\ &\quad - \xi_y \mathbf{a}_n \cdot \left[\frac{\partial \mathbf{B}^o}{\partial \hat{y}}\right]_{y=0} \quad \text{for } \xi \in \partial B_s. \end{aligned} \tag{42}$$

C. Investigation of the Lowest Order Boundary-Layer Terms

We next investigate each of the lowest order boundary-layer terms. In the dielectric, we have the electrostatic problem

$$\begin{aligned}\nabla_\xi \cdot \mathbf{d}_d^o = 0 \Rightarrow \xi \in B_d \\ \nabla_\xi \times \mathbf{e}_d^o = 0 \Rightarrow \xi \in B_d \end{aligned} \tag{43}$$

with constitutive equation

$$\mathbf{d}_d^o = \epsilon_o \epsilon_{rd} \mathbf{e}_d^o \Rightarrow \xi \in B_d \tag{44}$$

and boundary condition

$$\mathbf{a}_n \times \mathbf{e}_d^o\big|_{\xi\in\partial B_s} = -\mathbf{a}_n \times \mathbf{E}^o(\mathbf{r}_o) \Rightarrow \xi \in \partial B_s \tag{45}$$

where B_d is defined in Fig. 3. The magnetic field, on the other hand, is determined from the eddy current problem:

$$\begin{aligned}\nabla_\xi \cdot \mathbf{b}_d^o = 0 \Rightarrow \xi \in B_d \\ \nabla_\xi \times \mathbf{h}_d^o = 0 \Rightarrow \xi \in B_d \\ \nabla_\xi \times \mathbf{e}_c^1 = -jc\mathbf{b}_c^o \Rightarrow \xi \in B_c \\ \nabla_\xi \times \mathbf{h}_c^o = jc\mathbf{d}_c^{-1} \Rightarrow \xi \in B_c \end{aligned} \tag{46}$$

with the constitutive equations

$$\begin{aligned}\mathbf{b}_d^o = \mu_o \mu_{rd} \mathbf{h}_d^o \Rightarrow \xi \in B_d \\ \mathbf{d}_c^{-1} = \epsilon_o G \mathbf{e}_c^1 \Rightarrow \xi \in B_c \\ \mathbf{b}_c^o = \mu_o \mu_{rc} \mathbf{h}_c^o \Rightarrow \xi \in B_c \end{aligned} \tag{47}$$

and boundary conditions

$$\begin{aligned}\mathbf{a}_n \times \mathbf{h}_d^o|_{\xi\in\partial B_s} &= \mathbf{a}_n \times \mathbf{h}_c^o|_{\xi\in\partial B_s} - \mathbf{a}_n \\ &\quad \times \mathbf{H}^o(\mathbf{r}_o) \Rightarrow \xi \in \partial B_s \\ \mathbf{a}_n \cdot \mathbf{b}_d^o|_{\xi\in\partial B_s} &= \mathbf{a}_n \cdot \mathbf{b}_c^o|_{\xi\in\partial B_s} - \mathbf{a}_n \\ &\quad \cdot \mathbf{B}^o(\mathbf{r}_o)|_{y=0} \Rightarrow \xi \in \partial B_s \\ \mathbf{a}_n \cdot \mathbf{d}_c^{-1}|_{\xi\in\partial B_s} &= 0 \Rightarrow \xi \in \partial B_s \end{aligned} \tag{48}$$

where B_c is defined in Fig. 3.

The electrostatic problem (43), (44) and (45) is identical to the one encountered in [1]; from the results of that paper we have

$$\mathbf{a}_y \times \mathbf{E}^o(\mathbf{r}_o) = 0 \tag{49}$$

and thus, $\mathbf{E}^o$ has only a y-component. Likewise, it can be shown that

$$\mathbf{a}_y \cdot \mathbf{B}^o(\mathbf{r}_o) \equiv 0. \tag{50}$$

To zeroth order, then, the tangential effective $\mathbf{E}$ and the normal effective $\mathbf{B}$ fields see the rough interface as a smooth perfect conductor.

The *slow* $\mathbf{r}$ dependence of the zeroth-order boundary-layer fields can be factored out such that only a canonical *fast* ξ dependence of these fields remains; for details see [1]. As argued there, the effective electric field $E_y^o(\mathbf{r}_o)$ at the reference plane acts as a constant amplitude as far as ξ is concerned and as such can be factored out of the equations. Thus, by linearity $\mathbf{e}^o$ can be expressed as

$$\mathbf{e}^o = \mathbf{e}_d^o = E_y^o(\mathbf{r}_o)\mathcal{E}(\xi_x, \xi_y) \tag{51}$$

where $\mathcal{E}$ is a function of the fast variables only.

The magnetic field problem is more involved since $\mathbf{h}^o$ appears both in the dielectric and in the conductor, resulting in the coupled set of equations given in (46), (47), and (48). Although $\mathbf{H}^o(\mathbf{r}_o)$ has no y-component, in general both $H_x^o(\mathbf{r}_o) \neq 0$ and $H_z^o(\mathbf{r}_o) \neq 0$. Thus, by linearity $\mathbf{h}^o$ can be expressed as

$$\mathbf{h}^o = H_x^o(\mathbf{r}_o)\mathcal{H}^{(x)}(\xi_x, \xi_y) + H_z^o(\mathbf{r}_o)\mathcal{H}^{(z)}(\xi_x, \xi_y) \tag{52}$$

where $\mathcal{H}_c^{(x,z)}(\xi_x, \xi_y)$ is the $\mathbf{h}^o$-field produced by the unit effective field $H_{x,z}^o(\mathbf{r}_o) = 1$ alone (so that the superscript x or z denotes the polarization of the corresponding effective field). As in [1], it will prove convenient to introduce related normalized magnetic fields by

$$\mathcal{B}^{(x,z)} = \mu_r \mathcal{H}^{(x,z)} \tag{53}$$

with $\mu_r = \mu_{rd}$ or μ_{rc} as appropriate to the region and a normalized $\mathbf{d}$-field by

$$\mathcal{D} = \epsilon_{rd}\mathcal{E} \tag{54}$$

in the dielectric region.

By inspection of (46), it can be seen that $\mathcal{H}^{(z)} = \mathbf{a}_z \mathcal{H}^{(z)}$ has only a z-component and that $\mathcal{H}^{(x)}$ has only components transverse to z. Further, it can be shown using a similar argument as in [1] that $\mathbf{a}_z \cdot \mathbf{h}_d^o \equiv 0$ and thus that $\mathcal{H}^{(z)} \equiv 0$ in B_d. Since $\mathcal{E}, \mathcal{H}^{(x)}$ and $\mathcal{H}^{(z)}$ are functions of the fast variables only and $E_y^o(\mathbf{r}_o)$ and $\mathbf{H}^o(\mathbf{r}_o)$ are independent of y, (51) and (52) confirm that $\mathbf{e}^o$ and $\mathbf{h}^o$ are independent of y.

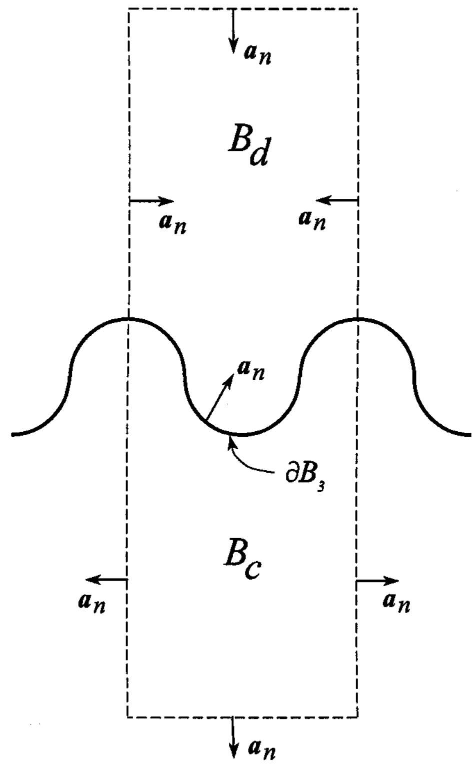

Fig. 3. Closed surface over which to apply Stokes' theorem.

With these definitions, (43) produces the following boundary problem for $\mathcal{E}$:

$$\begin{aligned}\nabla_\xi \cdot \mathcal{D} = 0 \Rightarrow \xi \in B_d \\ \nabla_\xi \times \mathcal{E} = 0 \Rightarrow \xi \in B_d\end{aligned} \tag{55}$$

with constitutive equation

$$\mathcal{D} = \epsilon_{rd}\mathcal{E} \Rightarrow \xi \in B_d \tag{56}$$

and boundary condition

$$\mathbf{a}_n \times \mathcal{E}|_{\xi\in\partial B_s} = -\mathbf{a}_n \times \mathbf{a}_y \tag{57}$$

which is the same static field problem for $\mathcal{E}$ as obtained for a perfectly conducting rough surface in [1]. All results pertaining to $\mathcal{E}$ from [1] can thus be used here as well.

By (52), there are two possible polarizations for the normalized magnetic field. We obtain the equations for each from (46), (47), and (48). For the x-polarization

$$\begin{aligned}\nabla_\xi \cdot \mathcal{B}_d^{(x)} = 0 \Rightarrow \xi \in B_d \\ \nabla_\xi \times \mathcal{H}_d = 0 \Rightarrow \xi \in B_d \\ \left(\nabla_\xi^2 + G\right)\mathcal{H}_c^{(x)} = 0 \Rightarrow \xi \in B_c\end{aligned} \tag{58}$$

with constitutive equations

$$\begin{aligned}\mathcal{B}_d^{(x)} = \mu_{rd}\mathcal{H}_d^{(x)} \Rightarrow \xi \in B_d \\ \mathcal{B}_c^{(x)} = \mu_{rc}\mathcal{H}_d^{(x)} \Rightarrow \xi \in B_c\end{aligned} \tag{59}$$

and boundary conditions

$$\mathbf{a}_n \times \left(\mathcal{H}_d^{(x)}\Big|_{\xi\in\partial B_s} - \mathcal{H}_c^{(x)}\Big|_{\xi\in\partial B_s} \right) = -\mathbf{a}_n \times \mathbf{a}_x$$
$$\mathbf{a}_n \cdot \left(\mathcal{B}_d^{(x)}\Big|_{\xi\in\partial B_s} - \mathcal{B}_c^{(x)}\Big|_{\xi\in\partial B_s} \right) = -\mu_{rd}\mathbf{a}_n \cdot \mathbf{a}_x. \tag{60}$$

For the z-polarization we have

$$\left(\nabla_\xi^2 + G\right)\mathcal{H}_c^{(z)} = 0 \Rightarrow \xi \in B_c \tag{61}$$

with constitutive equation

$$\mathcal{B}_c^{(z)} = \mu_{rc}\mathcal{H}_c^{(z)} \Rightarrow \xi \in B_c \tag{62}$$

and boundary condition

$$\mathcal{H}_c^{(z)}\Big|_{\partial B_s} = 1. \tag{63}$$

In the curl equations (20) for the first-order boundary-layer fields in the dielectric, we encounter the "slow curls" of the zeroth-order boundary-layer fields. With $\mathbf{e}_d^o$ and $\mathbf{h}_d^o$ each factored as the product of a function of $\hat{\mathbf{r}}$ and a function of ξ, it is now possible to evaluate $\nabla_{\hat{r}} \times \mathbf{e}_d^o$ and $\nabla_{\hat{r}} \times \mathbf{h}_d^o$. From (51), (52), (53) and (54)

$$\nabla_{\hat{r}} \times \mathbf{e}_d^o = -\mathcal{E} \times \nabla_{\hat{r}} E_y^o(\mathbf{r}_o)$$
$$\nabla_{\hat{r}} \times \mathbf{h}_d^o = -\mathcal{H}_d \times \nabla_{\hat{r}} H_x^o(\mathbf{r}_o). \tag{64}$$

With these expressions, the ν^o terms of (20) for the fields in the dielectric can be written as

$$\nabla_\xi \times \mathbf{e}_d^1 = \mathcal{E} \times \nabla_{\hat{r}} E_y^o(\mathbf{r}_o) - j\eta_o H_x^o(\mathbf{r}_o)\mathcal{B}_d^{(x)}$$
$$\nabla_\xi \times \mathbf{h}_d^1 = \mathcal{H}_d \times \nabla_{\hat{r}} H_x^o(\mathbf{r}_o) + \frac{j}{\eta_o} E_y^o(\mathbf{r}_o)\mathcal{D} \tag{65}$$

where

$$\eta_o = \sqrt{\frac{\mu_o}{\epsilon_o}}$$

is the wave impedance of free space.

D. Equivalent Boundary Condition for the Effective Fields

At this point, we have obtained boundary conditions for the zeroth-order boundary-layer and zeroth-order effective fields. The boundary values of the first-order effective fields are obtained next, from which an equivalent boundary condition for the total effective field will be determined. Just as the conditions for the zeroth-order effective fields did not require information about the zeroth-order boundary-layer fields, so the conditions on the first-order effective fields will require only knowledge of the zeroth-order boundary-layer fields.

We first integrate (32) over the boundary contour ∂B_3, proceeding as in [1] to compute $\mathbf{a}_y \times \mathbf{E}^1$

$$\mathbf{a}_y \times \mathbf{E}^1(\mathbf{r}_o) = S_o \mathbf{a}_y \times \left[\frac{\partial}{\partial \hat{y}}\mathbf{E}^o\right]_{y=0} + \int_{\partial B_3} \mathbf{a}_n \times \mathbf{e}_c^1\, dl_\xi - \int_{\partial B_3} \mathbf{a}_n \times \mathbf{e}_d^1\, dl_\xi \tag{66}$$

where S_o is the area under the period cell (i.e., the shaded area between the conductor surface and the $y = 0$ reference plane as shown in Fig. 2). The last two integrals in (66) can be extended to B_c and B_d respectively by application of a generalized Stokes' theorem [1]. Then (46) and the ν^o terms of (20) are used to evaluate $\nabla_\xi \times \mathbf{e}^1$, which leaves us with

$$\mathbf{a}_y \times \mathbf{E}^1(\mathbf{r}_o) = S_o \mathbf{a}_y \times \left[\frac{\partial}{\partial \hat{y}}\mathbf{E}^o\right]_{y=0} - \nabla_{\hat{r}} \times \int_{B_d} \mathbf{e}_d^o\, dS_\xi - jc\int_{B_d+B_c} \mathbf{b}^o\, dS_\xi. \tag{67}$$

From (51), (52) and (53), then, we have finally

$$\begin{aligned}\mathbf{a}_y \times \mathbf{E}^1|_{y=0} = {} & S_o \mathbf{a}_y \times \left[\frac{\partial}{\partial \hat{y}}\mathbf{E}^o\right]_{y=0} \\ & + \int_{B_d} \mathcal{E}\, dS_\xi \times \nabla_{\hat{r}} \mathbf{E}_y^o(\mathbf{r}_o) \\ & - j\eta_o \left[H_x^o(\mathbf{r}_o)\int_{B_d+B_c} \mathcal{B}^{(x)}\, dS_\xi \right. \\ & \left. + H_z^o(\mathbf{r}_o)\int_{B_c} \mathcal{B}^{(z)}\, dS_\xi\right].\end{aligned} \tag{68}$$

From Maxwell's equations for the effective field in the dielectric, it can be shown that

$$\left[\frac{\partial \mathbf{E}^o}{\partial \hat{y}}\right]_{y=0} = j\eta_o\mu_{rd}\left[\mathbf{a}_x H_z^o(\mathbf{r}_o) - \mathbf{a}_z H_x^o(\mathbf{r}_o)\right] + \nabla_{\hat{r}} E_y^o(\mathbf{r}_o). \tag{69}$$

With this result, the boundary condition (68) can be expressed as

$$\begin{aligned}\mathbf{a}_y \times \mathbf{E}^1|_{y=0} & \\ = {} & \left[\int_{B_d} \mathcal{E}\, dS_\xi + S_o\mathbf{a}_y\right] \times \nabla_{\hat{r}} \mathbf{E}_y^o(\mathbf{r}_o) \\ & - j\eta_o H_x^o(\mathbf{r}_o)\left[\mu_{rd} S_o \mathbf{a}_x + \int_{B_d+B_c} \mathcal{B}^{(x)}\, dS_\xi\right] \\ & - j\eta_o H_z^o(\mathbf{r}_o)\left[\mu_{rd} S_o \mathbf{a}_z + \int_{B_c} \mathcal{B}^{(z)}\, dS_\xi\right].\end{aligned} \tag{70}$$

The total effective field on the $y = 0$ plane is expressed to first order in ν as

$$\mathbf{a}_y \times \mathbf{E}(\mathbf{r}_o) = \mathbf{a}_y \times [\mathbf{E}^o(\mathbf{r}_o) + \nu\mathbf{E}^1(\mathbf{r}_o) + O(\nu^2)]. \tag{71}$$

Recalling (49), we have that the total effective E-field on the $y = 0$ plane is

$$\mathbf{a}_y \times \mathbf{E}(\mathbf{r}_o) = \nu\mathbf{a}_y \times \mathbf{E}^1(\mathbf{r}_o). \tag{72}$$

Then from (72) and from the fact that $\nu = pk_o$ and $(\partial/\partial\hat{x}) = (1/k_o)(\partial/\partial x)$, the boundary condition for the effective field can be written in terms of the original unscaled variables as

$$\begin{aligned}\mathbf{a}_y \times \mathbf{E}|_{y=0} & \\ = p\Bigg\{ & \left[\int_{B_d} \mathcal{E}\, dS_\xi + S_o\mathbf{a}_y\right] \times \nabla_r E_y(\mathbf{r}_o) \\ & - j\omega\mu_o H_x(\mathbf{r}_o)\left[\mu_{rd} S_o \mathbf{a}_x + \int_{B_d+B_c} \mathcal{B}^{(x)}\, dS_\xi\right] \\ & - j\omega\mu_o H_z(\mathbf{r}_o)\left[\mu_{rd} S_o \mathbf{a}_z + \int_{B_c} \mathcal{B}^{(z)}\, dS_\xi\right]\Bigg\}.\end{aligned} \tag{73}$$

In [1] it is shown that the integral of $\mathcal{E}$ over B_d has only a y component; as stated above $\mathcal{B}_c^{(z)}$ only has a z component and in Appendix A it is shown that the quantity

$$\int_{B_d+B_c} \mathcal{B}^{(x)}\, dS_\xi$$

has only an x component. Thus, (73) can be written

$$\begin{aligned}\mathbf{a}_y \times \mathbf{E}|_{y=0} &= p\Big\{\Big[\int_{B_d} \mathcal{E}_y\, dS_\xi + S_o\Big]\mathbf{a}_y \times \nabla_r E_y(\mathbf{r}_o) \\ &\quad - j\omega\mu_o \mathbf{a}_x H_x(\mathbf{r}_o)\Big[\mu_{rd} S_o + \int_{B_d+B_c} \mathcal{B}_x^{(x)}\, dS_\xi\Big] \\ &\quad - j\omega\mu_o \mathbf{a}_z H_z(\mathbf{r}_o)\Big[\mu_{rd} S_o + \int_{B_c} \mathcal{B}_z^{(z)}\, dS_\xi\Big]\Big\} \end{aligned} \tag{74}$$

where the subscripts x and z on $\mathcal{B}$ represent the x- and z-components.

The coefficients in this boundary condition can be interpreted as dyadic surface electric and magnetic polarizability densities α_{eS} and α_{mS} (see Appendix B). The relevant components of these dyadics can be expressed in terms of dimensionless parameters through $\alpha_{eS,yy} = p\alpha_{ey}, \alpha_{mS,xx} = p\alpha_{mx}$ and $\alpha_{mS,zz} = p\alpha_{mz}$, where

$$\alpha_{ey} = -\Big[\int_{B_d} \mathcal{E}_y\, dS_\xi + S_o\Big] \tag{75}$$

and

$$\begin{aligned}\alpha_{mx} &= -\Big[S_o + \frac{1}{\mu_{rd}}\int_{B_d+B_c} \mathcal{B}_x^{(x)}\, dS_\xi\Big] \\ \alpha_{mz} &= -\Big[S_o + \frac{1}{\mu_{rd}}\int_{B_c} \mathcal{B}_z^{(z)}\, dS_\xi\Big].\end{aligned} \tag{76}$$

With these polarizability densities, the boundary condition can be expressed simply as

$$\begin{aligned}\mathbf{a}_y \times \mathbf{E}(\mathbf{r}_o) &= j\omega p[\mathbf{a}_x \alpha_{mx} B_x(\mathbf{r}_o) + \mathbf{a}_z \alpha_{mz} B_z(\mathbf{r}_o)] \\ &\quad - p\alpha_{ey}\mathbf{a}_y \times \nabla_r E_y(\mathbf{r}_o).\end{aligned} \tag{77}$$

This effective generalized impedance boundary condition (EGIBC) has the same form as the boundary condition for a perfectly conducting rough surface presented in [1]. The difference between the two is that the magnetic polarizability densities for this work are complex, where in [1] they were purely real.

Written in this manner, this impedance boundary condition has the same form as the generalized impedance boundary conditions (GIBCs) for planar metal-backed dielectric layers presented in [26, ch. 5]. It can be shown, using Maxwell's equations as well as the expressions for the fields in terms of Hertz potentials, that the boundary condition obtained by Biot [10] and Wait [11] can be rewritten in a form like (77). Our version is to be preferred: 1) because it does not require the introduction of Hertz potentials and 2) the whole interface may be lossy, not just the "bumps" or bosses protruding from an otherwise plane perfectly conducting surface.

In summary, the EGIBC can be used to model the behavior of the effective EM field near the periodic interface between a dielectric and a highly conducting medium in the following manner. The actual rough interface is replaced with an equivalent smooth surface located at the $y = 0$ plane. The boundary condition given in (77) is then applied at this plane. All the effects of the roughness profile as well as the finite conductivity are incorporated in this boundary condition. As the period of the roughness get smaller, the boundary-layer fields become more localized; their effect on the total fields and on the boundary condition of the effective fields disappears and the case of a smooth impedance interface is obtained (i.e., the Leontovich surface impedance [20]), as shown in the sequel. The parameters α_{ey}, α_{mx} and α_{mz} in this boundary condition depend on the parameter S_o and the field quantities $\mathcal{E}, \mathcal{B}^{(x)}$, and $\mathcal{B}^{(z)}$, which must be obtained numerically and are governed by (55), (58), and (61).

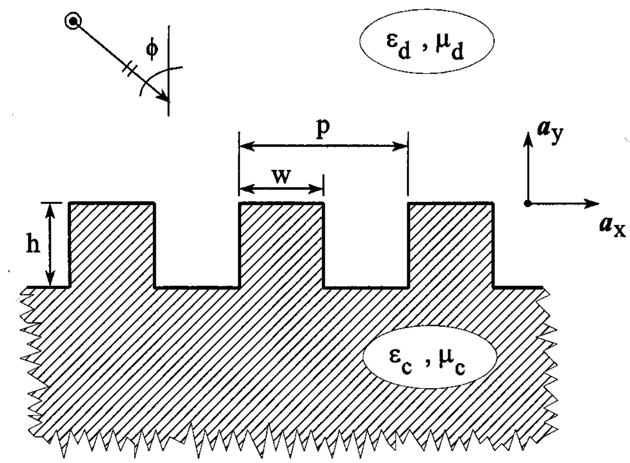

Fig. 4. Two-dimensional rectangular profile.

III. Plane Conducting Interface

In this section, we obtain the parameters $\alpha_{mx}, \alpha_{mz}, \alpha_{ey}$, and S_o of the EGIBC (77) for the case of a plane conducting interface. The area S_o under one period cell is clearly $S_o \equiv 0$. From [1], the electric polarizability density for a plane surface (take the case of no cover layer therein) is found to be:

$$\alpha_{ey} \equiv 0. \tag{78}$$

For the plane interface, then, the EGIBC reduces to

$$\mathbf{a}_y \times \mathbf{E}(\mathbf{r}_o) = j\omega\mu_o\mu_{rd} p\,[\mathbf{a}_x \alpha_{mx} H_x(\mathbf{r}_o) + \mathbf{a}_z \alpha_{mz} H_z(\mathbf{r}_o)] \tag{79}$$

which is a Leontovich (impedance) type boundary condition for a highly conducting surface.

For a plane interface, the two eddy current problems given in (58) and (61) are simple and can be solved analytically. Leaving out the details, it can be shown that for this case

$$\alpha_{mz} = \alpha_{mx} = \frac{j}{\mu_{rd}}\sqrt{\frac{\mu_{rc}}{G}} = \frac{(j-1)}{2}\frac{\delta_c}{p}\frac{\mu_{rc}}{\mu_{rd}} \tag{80}$$

where G is related to the skin depth δ by (11). The EGIBC for the effective fields corresponding to a plane interface thus reduces to

$$\mathbf{a}_y \times \mathbf{E}(\mathbf{r}_o) = -Z_s\,[\mathbf{a}_x H_x(\mathbf{r}_o) + \mathbf{a}_z H_z(\mathbf{r}_o)] \tag{81}$$

which is the standard Leontovich condition [20] with surface impedance

$$Z_s = \frac{1+j}{\sigma_c \delta_c}. \tag{82}$$

It is remarkable that this condition can be viewed in terms of magnetic polarizability densities as we do here, a fact that does not seem to have been reported previously.

In [27], the impedance boundary condition presented in this paper is used to calculate the power loss associated with rough conducting interfaces. Results for conducting and superconducting surfaces are shown and compared to other results presented in the literature. In the following section, the impedance boundary condition is applied to determine the reflection coefficient of an H polarized plane wave incident onto a rectangular profile.

IV. Reflection Coefficient of an H Polarized Plane Wave from a Rough Conducting Interface

Assume an H polarized plane wave is incident onto the 2-D profile shown in Fig. 1, such that the total H field is given by

$$\mathbf{H} = \mathbf{a}_z H_o [e^{Uy} + R_H e^{-Uy}] e^{-j\lambda x}. \tag{83}$$

Following a similar procedure as [1], the reflection coefficient R_H is expressed as

$$R_H \sim \frac{1 + jkp\left[\frac{\alpha_{mz}}{\cos\phi} - \alpha_{ey}\frac{\sin^2\phi}{\cos\phi}\right]}{1 - jkp\left[\frac{\alpha_{mz}}{\cos\phi} - \alpha_{ey}\frac{\sin^2\phi}{\cos\phi}\right]}. \tag{84}$$

This expression is true for a general 2-D rough conducting interface. It illustrates that the reflection coefficient depends on the surface itself only through the electric and magnetic polarizability densities of the rough interface.

The reflection coefficient predicted from this expression has been calculated for the rectangular roughness profile shown in Fig. 4 with $h/p = 0.5$ and $w/p = 0.5$ (all media are assumed to be nonmagnetic for the results presented in this paper: $\mu_c = \mu_d = \mu_o$). The value of S_o is simply the area under one of the roughness periods as defined earlier; here, $S_o = 0.25$. On the other hand, α_{ey} and α_{mz} must be determined numerically; α_{ey} is calculated as indicated in [1] and found to be $\alpha_{ey} \simeq -0.043$ for this geometry.

The magnetic polarizability densities α_{mz} and α_{mx} are defined in (76), which involves integrals of the fields $\mathcal{B}^{(x,z)}$, which are governed by (58) and (61). These eddy current problems present some unusual features, which are addressed in a separate publication [28]. A finite-element program was written to compute the magnetic fields of the eddy current problem [24] and from those, the magnetic polarizabilities. In Fig. 5, the real part of α_{mz} is plotted against p/δ_c. We see that, rather than approaching zero for large p/δ_c, it approaches $-S_o$ instead, the value for a perfectly conducting surface. The behavior of the imaginary part of α_{mz}, normalized to its value for a plane conducting interface, is shown in Fig. 6. As $p/\delta_c \to \infty$, this normalized value approaches 2 (the factor by which the transverse path length over the rough surface exceeds that of the plane surface per period). This agrees with the results of [17]–[19] which started from the application of a Leontovich boundary condition directly on the rough surface, rather than consideration of the fields within the conductor as we have done here. We see that for moderate or small values of p/δ_c, those results are no longer valid. Moreover, the real part of α_{mz} is not accurately predicted by the method of [17]–[19] for any value of p/δ_c unless S_o comprises only a small part of its total value.

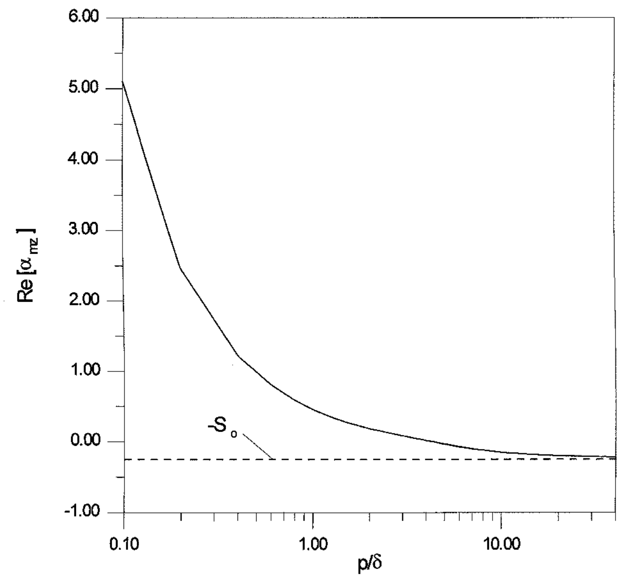

Fig. 5. The real part of α_{mz} versus p/δ_c, with $w/p = 0.5$ and $h/p = 0.5$.

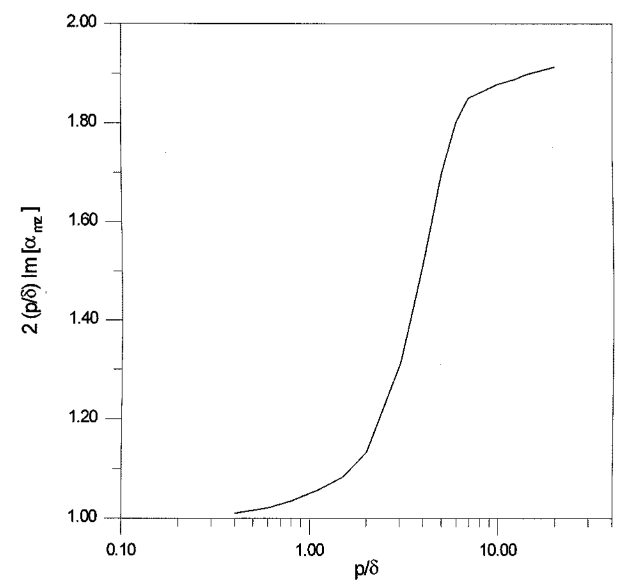

Fig. 6. The imaginary part of α_{mz} normalized to $\delta_c/2p$ (the imaginary part of α_{mz} for a plane surface), versus p/δ_c, with $w/p = 0.5$ and $h/p = 0.5$.

With α_{mz} and α_{ey} in hand, for the reflection coefficient R_H given in (84) can be computed. Fig. 7 shows the phase of R_H for various p/δ_c with an incidence angle equal to 0°. When p/δ_c becomes large, Fig. 7 indicates that the phase of R_H approaches 42.8°, the same value obtained in [1] for the case of a perfectly

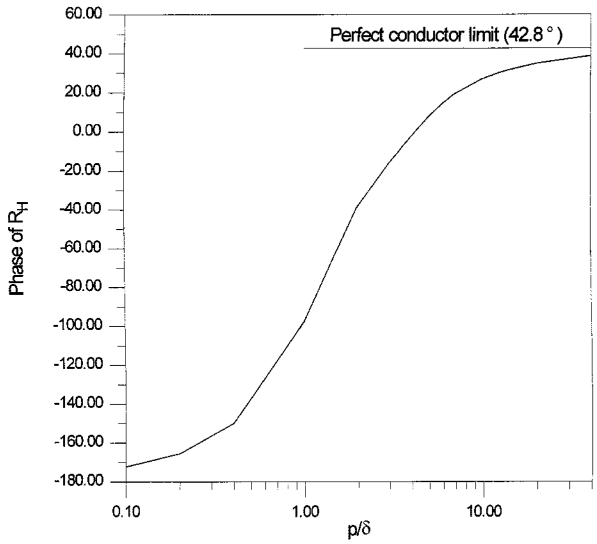

Fig. 7. The phase of R_H versus p/δ_c, with $w/p = 0.5, h/p = 0.5$ and $\phi = 0°$.

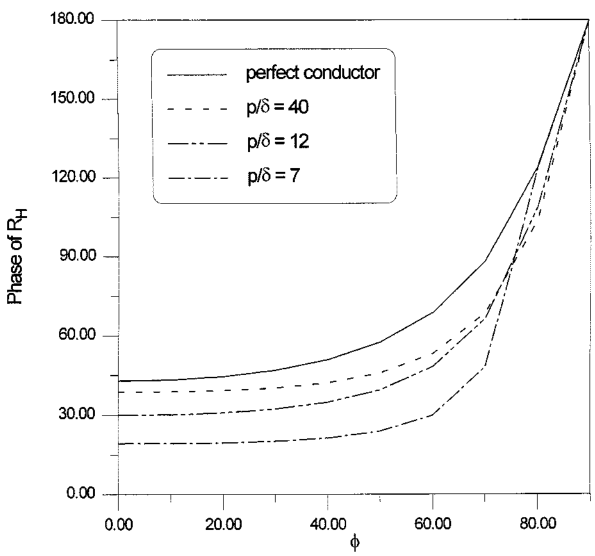

Fig. 8. The phase of R_H versus p/δ, with $w/p = 0.5, h/p = 0.5$, and ϕ ranging from 0° to 90°.

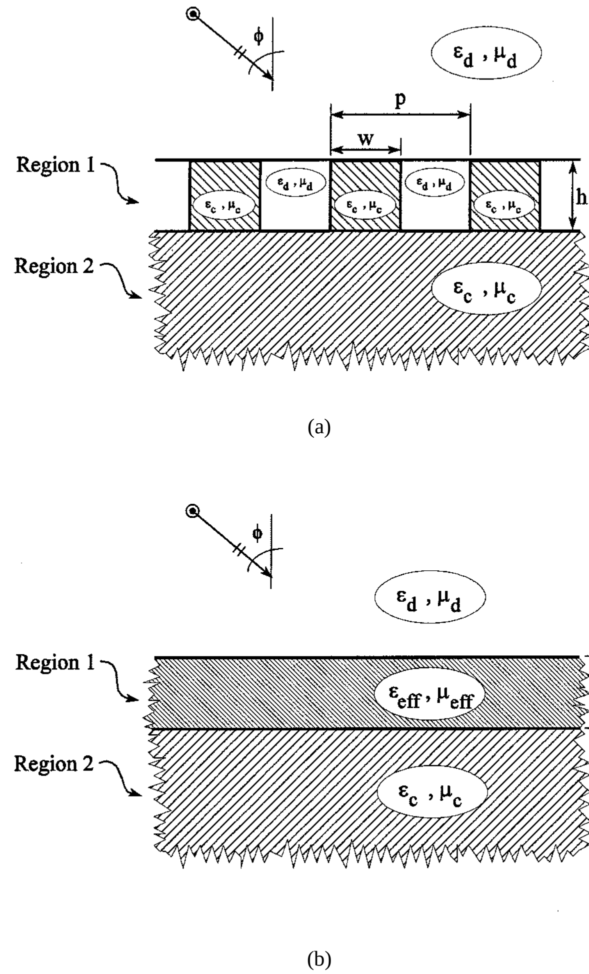

Fig. 9. (a) Two-region representation of the rectangular roughness profile. (b) Representation of region 1 as an effective medium with permittivity $\epsilon_{\rm eff}$ and permeability $\mu_{\rm eff}$.

conducting rough surface. Fig. 8 shows the phase of R_H for angles of incidence from $\phi = 0°$ to $\phi = 90°$. Also shown on this plot are the results obtained in [1] for a perfectly conducting surface. As expected, the results presented here for large p/δ_c approach those of a perfectly conducting surface. Large p/δ_c corresponds to the situation where there is very little field penetration into the conducting region and the profile behaves as a perfectly conducting surface in this limit.

For the case when p/δ_c is small, we can compare the results presented here to those obtained using an effective medium approach. Small p/δ_c corresponds to the situation where there is deep field penetration into the conducting profile. To examine this limit, the rectangular profile in Fig. 4 is represented as a layered two-component composite medium [see Fig. 9(a)]. Region 1 consists of alternating slabs of dielectric (ϵ_d, μ_d) and conducting (ϵ_c, μ_c) media and has height h. Region 2 is a semi-infinite conducting medium. For this small p/δ_c limit, this rectangular profile can be modeled as an anisotropic homogeneous region [see Fig. 9(b)] with effective material properties [29]. The effective material properties of alternating slabs of material (where one slab is highly conducting) for $\delta \gg p$ are given by Rytov [30]; for a normally incident H-polarized plane wave, we need these elements of the effective permittivity and permeability tensors

$$\frac{1}{\epsilon_{xx,\rm eff}} = \frac{w/p}{\epsilon_c} + \frac{1 - w/p}{\epsilon_d} \tag{85}$$

and

$$\mu_{zz,\rm eff} = \mu_c \frac{w}{p} + \mu_d \left(1 - \frac{w}{p}\right). \tag{86}$$

Using this expression for the material properties of region 1 and assuming that a plane wave is incident onto the layered media, R_H is then calculated using classical layered media expressions [21]. Fig. 10 compares the effective medium model to the results obtained from (84). The effective medium model works well

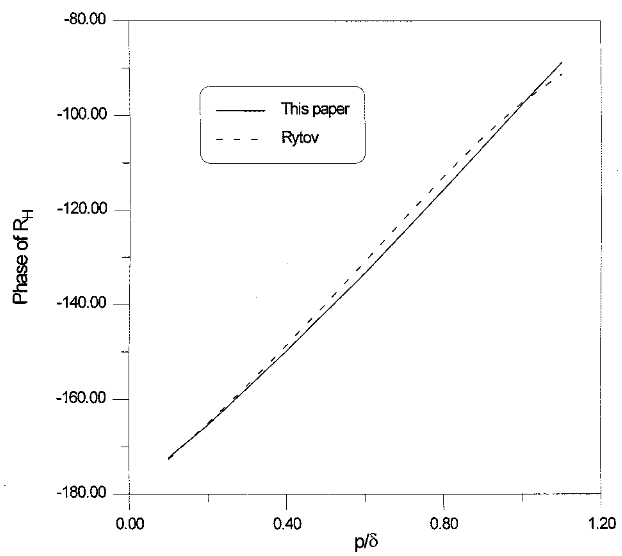

Fig. 10. Comparisons of the effective-medium model to the results obtained from (84) with $w/p = 0.5, h/p = 0.5$, and $\phi = 0.0°$.

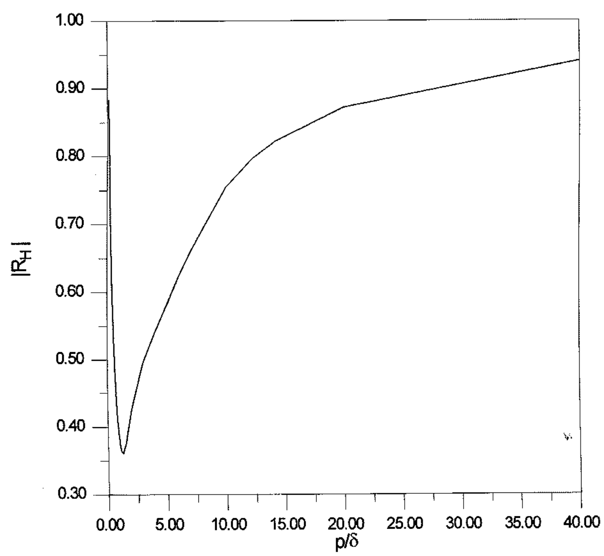

Fig. 11. The magnitude of R_H versus p/δ_c with $w/p = 0.5, h/p = 0.5$, and $\phi = 0.0°$.

for small values of p/δ (which is based on a zeroth-order approach which ignores the boundary-layer fields), as illustrated in Fig. 10. We conclude that the boundary-layer effects are less important in this case and field interactions with such an interface can be accurately determined from classical methods [21], with the interface handled as an effective medium. The same is not true, however, for large p/δ. This point was also illustrated in [31], where a finite-difference time-domain (FDTD) model was used to show that the reflection coefficient of a conducting wedge obtained from an effective material properties model correlates to the full-wave solution (FDTD) only when the conductivity of a wedge absorber is sufficiently small. It is shown in [31] that for high conductivities, the effective medium model fails. Thus, the boundary-layer effects presented in this paper are very important in analyzing EM problems with highly conducting rough interfaces.

Fig. 11 shows results for the magnitude of the reflection coefficient. We see that as p/δ_c gets very large, the magnitude of the reflection coefficient approaches 1. This is expected because as p/δ_c gets large, the skin depth becomes very small compared to the period and the problem approaches that of the perfectly conducting rough surface in which the magnitude of the reflection coefficient becomes one. This figure shows a strong dip in the reflection coefficient for $p/\delta \approx 1.25$, where the magnitude is about 0.36. This type of dip in the reflection coefficient has also been observed for similar problems in optics [32] and [33]. These results suggest that a metallic absorber, a frequency selective surface (FSS) or a photonic bandgap (PBG) material could be designed using appropriate choices for the roughness dimensions. Such structures could have wide applications and will be the topic of future research.

V. Conclusion

In this paper, we have presented the derivation of a generalized impedance type boundary condition for a 2-D conducting periodic interface using the technique of homogenization. This equivalent boundary condition along with Maxwell's equations are all that are needed to determine scattering and reflection from a rough interface. The advantage of homogenization above other techniques is that it allows one to obtain parameters for periodic structures in a systematic manner. In addition, one could in principle recover the field close to the interface by suitable inclusion of the boundary-layer fields.

The validity of the impedance boundary condition derived here is illustrated by showing results for the reflection coefficient for an H-polarized plane wave incident onto a 2-D rough interface. In [1] it was shown that the reactive part of this boundary condition (or essentially α_{ey} and the real part of α_{mz}) was handled properly. The calculations given here show that the results for the H polarization approach the correct value for both small and large p/δ_c. They are the first steps needed to analyze the effects of surface roughness on power loss in MIMIC circuits. Although in this paper we have not shown results for the other polarization, in [27] calculations for the power loss in MIMIC circuits based on the theory given here are presented, which agree very well with other results in the literature. We have shown in the present paper that for a plane conducting interface, the EGIBC reduces to the standard Leontovich surface impedance condition. We have also shown both that the boundary-layer effects are very important in analyzing highly conducting EM interfaces and that the use of classical methods cannot accurately predict these effects.

Appendix A
Integrals of the Zeroth-Order Magnetic Fields

A vector potential representation of $\mathcal{B}$ is possible, which will be used here to show that

$$\int_{B_d+B_c} \mathcal{B}^{(x)}\, dS_\xi$$

only has an x component. Since $\nabla_\xi \cdot \mathcal{B}^{(x)} = 0$ in B_d and in B_c, we can express $\mathcal{B}^{(x)}$ in either region in terms of a vector potential with a single component along z, i.e.,

$$\mathcal{B}^{(x)} = \nabla_\xi \times (\mathbf{a}_z A) = -\mathbf{a}_z \times \nabla_\xi A \tag{87}$$

where A is periodic in ξ_x and may be discontinuous across ∂B_3. A boundary condition on A may be deduced from the one in (60)

$$\mathbf{a}_n \cdot [(-\mathbf{a}_z \times \nabla_\xi A_d) + (\mathbf{a}_z \times \nabla_\xi A_d) + \mu_{rd}\mathbf{a}_x] = 0 \quad \text{on } \partial B_s. \tag{88}$$

Following a similar procedure as in Appendix A of [1], it can be shown from (88) that

$$(A_d - A_c)_{\partial B_3} = \text{constant} \tag{89}$$

Having found that $A_d - A_c$ is constant on the boundary, we can now evaluate the requisite integral. Using the potential representation given in (87), we have

$$\begin{aligned}\int_{B_d} \mathcal{B}_d^{(x)}\, dS_\xi &= -\mathbf{a}_z \times \int_{B_d} \nabla_\xi A_d \, dS_\xi \\ &= \mathbf{a}_z \times \int_{\partial B_3} \mathbf{a}_n A_d \, dl_\xi \end{aligned}\tag{90}$$

and

$$\begin{aligned}\int_{B_c} \mathcal{B}_c^{(x)}\, dS_\xi &= -\mathbf{a}_z \times \int_{B_c} \nabla_\xi A_c \, dS_\xi \\ &= -\mathbf{a}_z \times \int_{\partial B_3} \mathbf{a}_n A_c \, dl_\xi. \end{aligned}\tag{91}$$

Since $A_d - A_c$ is constant on the boundary and $\int_{\partial B_3} \mathbf{a}_n \, dl_\xi = \mathbf{a}_y$, we have

$$\begin{aligned}\int_{B_d+B_c} \mathcal{B}^{(x)}\, dS_\xi &= \mathbf{a}_z \times \int_{\partial B_3} \mathbf{a}_n (A_d - A_c)\, dl_\xi \\ &= -\mathbf{a}_x (A_d - A_c)|_{\partial B_3}. \end{aligned}\tag{92}$$

Therefore, this quantity has only an x component.

APPENDIX B
MAGNETIC POLARIZABILITY DENSITIES

As in [1], the coefficients in the EGIBC can be identified with surface densities of electric and magnetic polarizability. The electric polarizability density is the same as in [1], so we do not repeat the derivation here. For the magnetic polarizability, the procedure is similar to that used in [1], so we will present only highlights of the derivation here. Let $\mathbf{J} = \sigma \mathbf{E} \simeq \nabla \times \mathbf{h}_c^o$ be the conduction current in the conducting region. The magnetic dipole moment associated with this current is

$$\frac{1}{2} \int \mathbf{r} \times (\nabla \times \mathbf{h}_c^o)\, dV$$

where the integral is carried out over the entire conducting region. Likewise there may a magnetization current equal to $\nabla \times \mathbf{M}$, where the magnetization density is given by $\mathbf{M} \simeq (\mu_{rc}/\mu_{rd} - 1)\mathbf{h}_c^o$. This current contributes a magnetic dipole moment

$$\frac{1}{2}\left[\frac{\mu_{rc}}{\mu_{rd}} - 1\right] \int \mathbf{r} \times (\nabla \times \mathbf{h}_c^o)\, dV.$$

Finally, there is a surface magnetization current equal to $\mathbf{J}_s \simeq -(\mu_{rc}/\mu_{rd} - 1)\mathbf{a}_n \times \mathbf{h}^o$ at the interface between conductor and dielectric, whose dipole moment is

$$-\frac{1}{2}\left[\frac{\mu_{rc}}{\mu_{rd}} - 1\right] \int \mathbf{r} \times (\mathbf{a}_n \times \mathbf{h}^o)\, dS.$$

The total magnetic dipole moment is thus

$$\begin{aligned}\mathbf{m} = {} & \frac{1}{2}\frac{\mu_{rc}}{\mu_{rd}} \int \mathbf{r} \times (\nabla \times \mathbf{h}_c^o)\, dV \\ & - \frac{1}{2}\left[\frac{\mu_{rc}}{\mu_{rd}} - 1\right] \int \mathbf{r} \times (\mathbf{a}_n \times \mathbf{h}^o)\, dS. \end{aligned}\tag{93}$$

Using some vector identities, the z-component of (93) can be shown to be

$$m_z = -\frac{1}{2} \int \mathbf{a}_n \cdot [(\mathbf{a}_z \times \mathbf{r}) \times \mathbf{h}_c^o]\, dS + \frac{\mu_{rc}}{\mu_{rd}} \int h_{c,z}^o \, dV. \tag{94}$$

A closed surface integral over the boundary of the conducting region (including surfaces at infinity) arises during the derivation of (94). It reduces to one over the finite part of the boundary ∂B_s only, due to decay of the field at infinity. We now convert the integrals to ones in the fast variables. The surface dipole density is then identified as

$$\begin{aligned} m_{Sz} = p \Bigg\{ & -\frac{1}{2} \int_{\partial B_3} \mathbf{a}_n \cdot [(\mathbf{a}_z \times \xi) \times \mathbf{h}_c^o]\, dl_\xi \\ & + \frac{\mu_{rc}}{\mu_{rd}} \int_{B_c} h_{c,z}^o \, dS_\xi \Bigg\} \end{aligned}\tag{95}$$

which further transforms to

$$m_{Sz} = p H_z^o(\mathbf{r}_o) \left[\frac{\mu_{rc}}{\mu_{rd}} \int_{B_c} \mathcal{H}_z^{(z)}\, dS_\xi + S_o\right]. \tag{96}$$

Thus, with a unit effective field $H_z^o(\mathbf{r}_o) = 1$, the zz-component of the magnetic polarizability per unit area is

$$\alpha_{mS,zz} = -p \left[\frac{1}{\mu_{rd}} \int_{B_c} \mathcal{B}_z^{(z)}\, dS_\xi + S_o\right] \tag{97}$$

in agreement with (76).

The x-component of (93) is handled similarly (keeping in mind the necessity to multiply the expression by two in order to account for the effect of currents at infinity which are not explicitly included in the expression for $\mathbf{m}$ [1]). We arrive at

$$m_{Sx} = p H_x^o(\mathbf{r}_o) \left[\frac{1}{\mu_{rd}} \int_{B_c+B_d} \mathcal{B}_x^{(x)}\, dS_\xi + S_o\right] \tag{98}$$

so that

$$\alpha_{mS,xx} = -p \left[\frac{1}{\mu_{rd}} \int_{B_c+B_d} \mathcal{B}_x^{(x)}\, dS_\xi + S_o\right] \tag{99}$$

again in agreement with (76).

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REFERENCES

[1] C. L. Holloway and E. F. Kuester, “Equivalent boundary conditions for a perfectly conducting periodic surface with a cover layer,” *Radio Sci.*, vol. 35, no. 3, pp. 661–681, May-June 2000.

[2] J. R. Wait, "Perturbation analysis for reflection from two-dimensional periodic sea waves," *Radio Sci.*, vol. 6, pp. 387–391, 1971.

[3] D. E. Barrick, "Theory of HF and VHF propagation across the rough sea, 1, the effective surface impedance for a slightly rough highly conducting medium at grazing incidence," *Radio Sci.*, vol. 6, pp. 517–526, 1971.

[4] R. K. Rosich and J. R. Wait, "A general perturbation solution for reflection from two-dimensional periodic surfaces," *Radio Sci.*, vol. 12, pp. 719–729, 1977.

[5] S. P. Morgan, "Effect of surface roughness on eddy current losses at microwave frequencies," *J. Appl. Phys.*, vol. 20, pp. 352–362, 1949.

[6] E. A. Marcatili, "Heat loss in grooved metallic surface," *Proc. IRE*, vol. 45, pp. 1134–1139, 1957.

[7] A. E. Sanderson, "Effect of surface roughness on propagation of the TEM mode," in *Advances in Microwaves*, L. Young, Ed. New York: Academic, 1971, vol. 7, pp. 1–57.

[8] I. V. Baryshnikov, V. A. Datskovskiy, and A. V. Upolovnev, "Effect of the surface roughness of a waveguiding system on the HF losses," *Radiotekh. Elektron.*, vol. 33, pp. 2029–2034, 1988. In Russian; Engl. transl. in *Sov. J. Commun. Technol. Electron.*, vol. 34, no. 5, pp. 51–56, 1989.

[9] A. S. Il'inskii (Ilyinsky), G. Ya. Slepyan, and A. Ya. Slepyan, *Propagation, Scattering and Dissipation of Electromagnetic Waves*. London, U.K.: Peter Peregrinus, 1993.

[10] M. A. Biot, "Some new aspects of the reflection of electromagnetic waves on a rough surface," *J. Appl. Phys.*, vol. 28, pp. 1455–1463, 1957. addendum, *ibid*., vol. 29, p. 998, 1958.

[11] J. R. Wait, "Guiding of electromagnetic waves by uniformly rough surfaces, part I," *IRE Trans. Antennas Propagat.*, vol. 7, pp. S154–S162, 1959.

[12] F. F. Mende, A. I. Spitsyn, and V. N. Podlesnyi, "Effect of roughness on surface impedance and effective penetration depth in superconductors," *Zh. Tekh. Fiz.*, vol. 51, pp. 2560–2564, 1981. in Russian; Engl. transl. in *Sov. Phys. Tech. Phys.*, vol. 26, pp. 1512–1515, 1982.

[13] F. F. Mende, A. I. Spitsyn, N. A. Kochkonyan, and A. V. Skugarevski, "Surface impedance of real superconducting surfaces," *Croygenics*, vol. 25, pp. 10–12, Jan. 1985.

[14] Z. Wu and L. E. Davis, "Surface roughness effect on surface impedance of superconductors," *J. Appl. Phys.*, vol. 76, pp. 3669–3672, 1994.

[15] F. F. Mende and A. I. Spitsyn, *Poverkhnostnyi Impedans Sverkhprovodnikov*, Kiev: Naukova Dumka, 1985, ch. 3.

[16] D. L. Jaggard, "On fractal electrodynamics," in *Recent Advances in Electromagnetic Theory*, H. N. Kritikos and D. L. Jaggard, Eds. Berlin, Germany: Springer-Verlag, 1990, ch. 6, pp. 183–224.

[17] E. Sanchez-Palencia, *Non-Homogeneous Media and Vibration Theory*. Berlin, Germany: Springer-Verlag, 1980, pp. 68–77. Lecture Notes Phys. no. 127.

[18] M. Codegone, "On the acoustic impedance condition for ondulated boundary," *Ann. Inst. Henri Poincaré Sect. A: Phys. Théor.*, vol. 36, pp. 1–18, 1982.

[19] A. G. Belyaev (Belyayev), A. G. Mikheev (Mikheyev), and A. S. Shamaev (Shamayev), "Diffraction of a plane wave by a rapidly oscillating surface," *Zh. Vychisl. Mat. Mat. Fiz.*, vol. 32, pp. 1258–1272, 1992. In Russian; Engl. transl. in *Comp. Math. Math. Phys.*, vol. 32, pp. 1121–1133, 1992.

[20] M. A. Leontovich, "Approximate boundary conditions for electromagnetic fields at the surface of a highly conducting body," in *Issledovaniya po Rasprostraneniyu Radiovoln*, Moscow, 1948, pt. 2, pp. 5–12. Russian; also M. A. Leontovich, *Izbrannye Trudy: Teoreticheskaya Fizika*, Nauka, Moscow, pp. 351–355, 1985.

[21] L. M. Brekhovskikh, *Waves in Layered Media*. New York: Academic, 1960.

[22] J.-L. Lions, *Some Methods in the Mathematical Analysis of Systems and Their Control*, N.Y.: Science Press, 1981.

[23] G. P. Panasenko, "Averaging of processes in highly heterogeneous structures," *Dokl. Akad. Nauk USSR*, vol. 298, pp. 76–79, 1988. In Russian; Engl. transl. in *Sov. Phys. Dokl.*, vol. 32, pp. 20–22, 1988.

[24] C. L. Holloway, "Edge and Surface Shape Effects on Conductor Loss Associated with Planar Circuits," Dept. Elec. Comp. Eng., Univ. Colorado, Boulder, MIMICAD Tech. Rep. 12, 1992.

[25] L. D. Landau and E. M. Lifshitz, *Electrodynamics of Continuous Media*. Oxford, U.K.: Pergamon, 1984.

[26] T. B. A. Senior and J. L. Volakis, *Approximate Boundary Conditions in Electromagnetics*. London, U.K.: Inst. Elect. Eng., 1995.

[27] C. L. Holloway and E. F. Kuester, "Power loss associated with conducting and superconducting rough interfaces," *IEEE Trans. Microwave Theory Tech*, vol. 48, pp. 1601–1610, Oct. 2000.

[28] ——, "A variational formulation for a general class of eddy current problem," *IEEE Trans. Antennas Propagat.*, to be published.

[29] E. F. Kuester and C. L. Holloway, "A low-frequency model for wedge or pyramid absorber array-I: Theory," *IEEE Trans. Electromagn. Compat.*, vol. 36, pp. 300–306, 1994.

[30] S. M. Rytov, "Electromagnetic properties of a finely stratified medium," *Sov. Phys. JETP*, vol. 2, no. 3, pp. 466–475, 1956.

[31] C. L. Holloway, P. McKenna, and R. DeLyser, "A numerical investigation on the accuracy of the use of homogenization for analyzing periodic absorbing arrays," in *Proc. Int. Symp. Electromagn. Theory—URSI*, St. Petersburg, Russia, May 1995, pp. 163–165.

[32] B. S. Thornton, "Limit of the Moth's eye principle and other impedance-matching corrugations for solar-absorber design," *J. Opt. Soc. Amer.*, vol. 65, pp. 267–270, 1975.

[33] T. K. Gaylord, E. N. Glytsis, and M. G. Moharam, "Zero-reflectivity homogeneous layers and high spatial frequency surface-relief gratings on lossy materials," *Appl. Opt.*, vol. 26, pp. 3123–3134, 1987.

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