Scattering from Dielectric-Coated Impedance Elliptic Cylinder

Abdel-Razik Sebak, *Senior Member, IEEE*

***Abstract*—The scattering properties of an impedance elliptic cylinder coated with a homogeneous material are investigated analytically. The method of separation of variables together with the impedance boundary condition (IBC) are used to determine the field distributions in each region for both the TM and TE excitations. The technique can be easily extended to handle any number of layers. The behavior of the scattered field in the far zone is illustrated with numerical results for different core and coating material types, axial ratio, and electrical sizes.**

***Index Terms*—Cylinders, electromagnetic (EM) scattering, impedance boundary conditions (IBCs), nonhomogeneous media.**

I. Introduction

EXACT analytic solutions for the problem of electromagnetic (EM) wave scattering by homogeneous objects are known for a limited number of object shapes. These solutions play an important role in the investigation of scattering and radiation problems. The exact solution for scattering by dielectric and coated circular cylinder have been obtained by Wait [1] and Wang [2]. The general series solution for scattering of EM waves by a dielectric elliptic cylinder has been investigated by Yeh [3]. Ragheb and Shafai [4] have studied the exact series solution of scattering by a coated, perfectly conducting elliptic cylinder. Aside from providing exact solutions for their respective geometries, they are invaluable in the evaluation of approximate and numerical solutions. For imperfectly conducting and coated objects with electrically dissimilar materials, the concept of surface impedance [5] may be used to model their characteristics. The surface impedance is commonly used to model imperfectly conducting scatterers, perfectly conducting objects coated with a penetrable or absorbing layer, or scatterers with corrugated or rough surfaces. A general treatment of high-frequency scattering by impedance circular cylinder is given by Uslenghi [6]. The Galerkin's method combined with elliptic cylindrical harmonics functions and the impedance boundary condition (IBC) are used in [7] to investigate the scattering from an elliptic cylinder with discontinuous surface impedances. The scattering properties of impedance spheres [8], spheroids [9] and the usefulness of the IBC [10] have been studied previously. The scattering of a plane EM wave by a perfectly conducting cylinder coated with a material layer of large complex index of refraction has been treated by Syed and Volakis [11] who used a more accurate higher order IBC on the outer surface of the coating layer to simplify their analysis. The formulation for scattering from an imperfectly conducting cylinder coated with a lossy dielectric has been given by Parrikar *et al.* [12]. They applied the exact boundary conditions at the coating surfaces and an IBC on the cylindrical core. Also, the IBC has been successfully used by Wait and Conda [13] to compute the radiation pattern of an antenna on a curved lossy surface. A general treatment of uniformly coated antennas using IBCs is discussed by Huddleston and Wang [14]. In addition, an efficient method for the computation of ground wave attenuation for a curved earth with arbitrary surface impedance has been developed by Hill and Wait [15].

This paper is concerned with the problem of EM scattering from an elliptic cylinder with an IBC coated with a confocal dielectric or magnetic material. This problem has not yet been investigated analytically and it has wide-ranging applications in areas such as reduction of radar cross section using different coating material, controlling the radiation characteristics of coated antennas, and in verification of new developed numerical and approximate techniques. For imperfectly conducting objects or objects with relatively thin coatings, the IBC is a reasonable one and eliminates the need for the inclusion of the interior fields. This simplifies the formulation and reduces the computation time considerably. The analysis described in this paper is based on the eigenfunction technique, where the total fields inside the dielectric (magnetic) coating and in the exterior regions are expanded in terms of elliptical harmonics, i.e., Mathieu and modified Mathieu functions with unknown expansion coefficients. These unknown coefficients are determined by an infinite system of simultaneous equations, which are determined by imposing appropriate boundary conditions. In order to generate numerical results, the series solution is truncated in a suitable fashion to obtain a finite matrix. The order of such a matrix depends on the electrical size and property of each region and the degree of required accuracy from the solution.

In the following sections, the exact series solution to the scattering problem for the TM polarization, is presented. The solution is general and valid for both lossless and lossy dielectric materials by allowing a complex value for the permittivity in the coating region. Numerical results related to the amplitude of the scattered far field for various geometrical and material parameters as well as for different angles of incidence are included and discussed.

Manuscript received August 30, 1999; revised May 8, 2000. This work was supported by the National Sciences and Engineering Research Council of Canada.

The author is with the Department of Electrical and Computer Engineering, University of Manitoba, Winnipeg, MB, R3T 5V6, Canada (e-mail: sebak@ee.umanitoba.ca).

Publisher Item Identifier S 0018-926X(00)09357-1.

II. Formulation

The geometry of the scattering problem and parameters employed in the formulation are shown in Fig. 1, where the core of the scatterer is an elliptic cylinder with semimajor and semiminor axes a and b and a semifocal length F. The core may be considered as a perfectly conducting elliptic cylinder totally coated with a homogeneous confocal absorbing or imperfectly conducting layer (dark area in Fig. 1). The core is characterized by an arbitrary surface impedance Z_s, which depends on its surface and EM properties. The impedance cylinder is coated with a confocal homogeneous layer of permittivity ϵ_1, permeability μ_1, and a characteristic impedance $Z_1 = (\mu_1/\epsilon_1)^{1/2}$, as shown in Fig. 1. The semimajor and semiminor axes are denoted by a_1 and b_1 for the outer surface of the impedance loading. The exterior environment is taken to be free space with permittivity ϵ_o, permeability μ_o and a characteristic impedance Z_o. An incident plane wave, with $\exp(j\omega t)$ time dependence, is normally incident, on the cylindrical structure. The incident wave is, in general, polarized arbitrarily which can be resolved into two components TM and TE waves. Each can be treated separately. Detailed analysis leading to the solution for the TM case is given below.

A. Incident Wave

A linearly polarized plane EM wave is incident at an angle ϕ^i, with respect to the x-axis, on the cylindrical structure of Fig. 1. For the TM case, the electric field, in each region, has only an axial component E_z. The electric field of the incident wave may be written as

$$E_z^i = E_o e^{jk(x\cos\phi^i + y\sin\phi^i)} \tag{1}$$

where E_o is the amplitude of the incident electric field and k is the wave number in free-space.

It is usually convenient to express the field quantities in the elliptic cylindrical coordinates (ξ, η, z), where $x = F\xi\eta$, $y = F\sqrt{\xi^2-1}\sqrt{1-\eta^2}$. The expansion of the incident electric field, in terms of Mathieu and modified Mathieu functions, may be written in the form

$$E_o e^{jk(x\cos\phi^i + y\sin\phi^i)} = \sum_{m=0}^{\infty} Ae_m \mathrm{Re_m}^{(1)}(c,\xi) Se_m(c,\eta) + \sum_{m=1}^{\infty} Ao_m Ro_m^{(1)}(c,\xi) So_m(c,\eta) \tag{2}$$

where

$$A_{\substack{e\\o}m} = \sqrt{8\pi} j^m E_o S_{\substack{e\\o}m}(c, \cos\phi^i)/N_{\substack{e\\o}m}(c) \tag{3}$$

and $c = kF$. Se_m and So_m are, respectively, the *even* and *odd* angular Mathieu functions of order m, $\mathrm{Re}_m^{(1)}$, $Ro_m^{(1)}$ are the even and odd modified Mathieu functions of first kind and Ne_m and No_m are normalized constants [16] and given by

$$N_{\substack{e\\o}n}(c) = \int_o^{2\pi} \left[S_{\substack{e\\o}n}(c,\eta)\right]^2 dv, \quad \eta = \cos v. \tag{4}$$

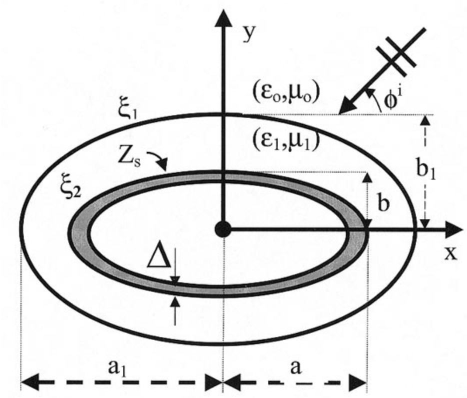

Fig. 1. Geometry of the scattering problem.

The subscripts e and o denote even and odd types, respectively.

B. Scattered and Transmitted Waves

Expressions for the scattered and transmitted fields inside the coating material can be derived by solving Helmholtz equation in the elliptical coordinate system using the separation of variable technique. The z-component of the scattered electric field, for $\xi > \xi_1$, may be expressed in the form

$$E_z^s = \sum_{m=0}^{\infty} Be_m \mathrm{Re}_m^{(4)}(c,\xi) Se_m(c,\eta) + \sum_{m=1}^{\infty} Bo_m Ro_m^{(4)}(c,\xi) So_m(c,\eta) \tag{5}$$

where $B_{\substack{e\\o}m}$ are unknown expansion coefficients and $\mathrm{Re}_m^{(4)}$ and $Ro_m^{(4)}$ are the *even* and *odd* modified Mathieu functions of fourth kind. The values of the expansion coefficients have to be determined through the use of the boundary conditions.

Similarly, the transmitted electric field inside the coating region can be expressed in terms of appropriate modified Mathieu functions as

$$E_z^t = \sum_m \Big(\Big[Ce_m \mathrm{Re}_m^{(1)}(c_1,\xi) + De_m \mathrm{Re}_m^{(2)}(c_1,\xi) \Big] Se_m(c_1,\eta) + \Big[Co_m Ro_m^{(1)}(c_1,\xi) + Do_m Ro_m^{(2)}(c_1,\xi) \Big] So_m(c_1,\eta) \Big) \tag{6}$$

where $c_1 = k_1 F$, $\mathrm{Re}_m^{(2)}$ and $Ro_m^{(2)}$ are the *even* and *odd* modified Mathieu functions of second kind and $C_{\substack{e\\o}m}$ and $D_{\substack{e\\o}m}$ are unknown expansion coefficients. k_1 is the wave number in the coating region.

The unknown expansion coefficients given above in (5) and (6) can be determined by applying the boundary conditions, i.e., the IBC on the core surface $\xi = \xi_2$ and continuity of the tangential components of the electric and magnetic fields across the boundary $\xi = \xi_1$. The tangential magnetic field component may be expressed in terms of the corresponding axial electric field component and is given by

$$H_\eta = \frac{-j}{2\omega\mu F}\sqrt{\frac{\xi^2-1}{\xi^2-\eta^2}}\frac{\partial E_z}{\partial \xi}. \quad (7)$$

C. Formulation of the Boundary Conditions

1) Core Boundary: We now introduce the concept of the surface impedance Z_s and define a relative surface impedance η_s, such that $Z_s = \eta_s Z_1$. On a scattering surface with a surface impedance Z_s that depends on the surface coordinates and material properties of the scatterer, the IBC takes the form [5]

$$\overline{E} - (\overline{E}\cdot\hat{n}) = Z_s(\hat{n}\times\overline{H}) \quad (8)$$

where $\overline{E}$ and $\overline{H}$ are the total fields and $\hat{n}$ is the unit outward normal vector ($\hat{n} = \hat{\xi}$). In this study, the surface impedance is chosen to be independent of the axial direction z; that is, $Z_s = Z_s(\eta)$.

The equivalent form of (8) on the surface of the impedance cylinder $\xi = \xi_2$ may be written as

$$E_z^t(c1, \xi_2, \eta) = Z_s(\eta)H_\eta^t(c1, \xi_2, \eta). \quad (9)$$

Equation (9) must hold for all allowed values of $-1 \le \eta \le 1$. A substitution of the field expressions (6) and (7) in (9) provides an equation for the unknown expansion coefficients of the transmitted fields.

The essential complication of the elliptical geometry now becomes apparent. The individual terms in the series (6) and (7) cannot be matched term by term. We, therefore, use a method similar to the perfectly conducting elliptic case [4] and obtain a system of simultaneous equations for the unknown expansion coefficients. The above procedure provides an analytic solution for the general scattering problem involving an arbitrary surface impedance. For a perfectly conducting cylinder coated with an imperfectly conducting or absorbing layer of thickness Δ and relative permittivity and permeability ϵ_c and μ_c, the surface impedance on the outer surface of a thin coating is given by Uslenghi [6]

$$Z_s(\eta) \approx j\sqrt{\frac{\mu_c}{\epsilon_c}}\tan(k_1\Delta\sqrt{\epsilon_c\mu_c}) \approx jk_1\Delta(\eta)\mu_c Z_1 \quad (10)$$

where Z_1 is the characteristic impedance of the dielectric region. For a thin coating $\Delta(\eta)$ can be represented in terms of the metric coefficient h_ξ and $\delta\xi$, a small variation of ξ about ξ_2. It is given by

$$\Delta(\eta) = h_\xi\delta\xi = F\left[(\xi_2^2-\eta^2)/(\xi_2^2-1)\right]^{1/2}\delta\xi \quad (11)$$

which upon substitution in (10) yields

$$Z_s(\eta) = \eta_s(\xi_2^2-\eta^2)^{1/2}Z_1 \quad (12)$$

where $\eta_s = jc_1\mu_c\delta\xi/(\xi_2^2-1)$ is a complex quantity and μ_c may be real or complex.

Substituting for E_z's and H_η's from (6) and (7) into (9), multiplying the results by $S_{\substack{e\\o}n}(c_1, \eta)$ and applying the orthogonality of Mathieu functions [16], the terms involving *even* functions decouple completely from those of *odd* functions. Thus, one obtains two subsystems of linear equations for the even and odd unknown expansion coefficients. For the *even* functions case, the subsystem of equations is expressed as $n = 0, 1, 2, \ldots$ for

$$\left[Ce_n\mathrm{Re}_n^{(1)}(c_1, \xi_2) + De_n\mathrm{Re}_n^{(2)}(c_1, \xi_2)\right] = -j\frac{\eta_s}{c_1}\left[Ce_n\mathrm{Re}_n'^{(1)}(c_1, \xi_2) + De_n\mathrm{Re}_n'^{(2)}(c_1, \xi_2)\right]. \quad (13)$$

For the *odd* functions case, the subsystem of equations is expressed as $n = 1, 2, \ldots$ for

$$\left[Co_nRo_n^{(1)}(c_1, \xi_2) + Do_nRo_n^{(2)}(c_1, \xi_2)\right] = -j\frac{\eta_s}{c_1}\left[Co_nRo_n'^{(1)}(c_1, \xi_2) + Do_nRo_n'^{(2)}(c_1, \xi_2)\right] \quad (14)$$

where the prime denotes derivative with respect to ξ. Equations (13) and (14) may be used to eliminate De_n and Do_n, i.e.,

$$D_{\substack{e\\o}n} = -P_{\substack{e\\o}n}\cdot C_{\substack{e\\o}n} \quad (15)$$

with

$$P_{\substack{e\\o}n} = \frac{R_{\substack{e\\o}n}^{(1)}(c_1, \xi_2) + j\eta_s/c_1R_{\substack{e\\o}n}'^{(1)}(c_1, \xi_2)}{R_{\substack{e\\o}n}^{(2)}(c_1, \xi_2) + j\eta_s/c_1R_{\substack{e\\o}n}'^{(2)}(c_1, \xi_2)}. \quad (16)$$

2) Coating-Free Space Boundary: The second set of the boundary conditions on the surface $\xi = \xi_1$ is given by

$$E_z^i + E_z^s = E_z^t \quad \text{and} \quad H_\eta^i + H_\eta^s = H_\eta^t, \quad \xi = \xi_1. \quad (17)$$

Substituting for E_z's and H_η's from (2), through (7) into (17), multiplying the results by $S_{\substack{e\\o}n}(c_1, \eta)$, integrating over the contour of the cylindrical boundary and applying the orthogonality of Mathieu functions [16], the terms involving *even* functions decouple completely from those of *odd* functions. Thus, one obtains two subsystems of linear equations for the even and odd unknown expansion coefficients. For the *even* functions case, the subsystem of equations is expressed as $n = 0, 1, 2, \ldots$ for

$$\sum_{m=0,1,2}^{\infty} Me_{nm}(c_1, c)\left[Ae_m\mathrm{Re}_m^{(1)}(c, \xi_1) + Be_m\mathrm{Re}_m^{(4)}(c, \xi_1)\right] = Ne_n(c_1)Ce_n\left[\mathrm{Re}_n^{(1)}(c_1, \xi_1) - Pe_n\mathrm{Re}_n^{(2)}(c_1, \xi_1)\right] \quad (18)$$

$$\mu_1/\mu_o\sum_{m=0,1,2}^{\infty} Me_{nm}(c_1, c)\cdot\left[Ae_m\mathrm{Re}_m'^{(1)}(c, \xi_1) + Be_m\mathrm{Re}_m'^{(4)}(c, \xi_1)\right] = Ne_n(c_1)Ce_n\left[\mathrm{Re}_n'^{(1)}(c_1, \xi_1) - Pe_n\mathrm{Re}_n'^{(2)}(c_1, \xi_1)\right]. \quad (19)$$

For the odd functions case $n = 1, 2, 3, \ldots$ the corresponding subsystem of equations is given by

$$\sum_{m=0,1,2}^{\infty} Mo_{nm}(c_1, c)\left[Ao_mRo_m^{(1)}(c, \xi_1) + Bo_mRo_m^{(4)}(c, \xi_1)\right] = No_n(c_1)Co_n\left[Ro_n^{(1)}(c_1, \xi_1) - Po_nRo_n^{(2)}(c_1, \xi_1)\right] \quad (20)$$

$$\mu_1/\mu_o \sum_{m=0,1,2}^{\infty} Mo_{nm}(c_1, c) \cdot \left[Ao_m Ro'^{(1)}_m(c, \xi_1) + Bo_m Ro'^{(4)}_m(c, \xi_1)\right] = No_n(c_1) Co_n \left[Ro'^{(1)}_n(c_1, \xi_1) - Po_n Ro'^{(2)}_n(c_1, \xi_1)\right] \quad (21)$$

where $N_{\substack{e\\o}n}$ is given by (4) and

$$M_{\substack{e\\o}mn}(c, c_1) = \int_o^{2\pi} S_{\substack{e\\o}m}(c, \eta) S_{\substack{e\\o}n}(c_1, \eta)\, dv, \quad \eta = \cos v. \quad (22)$$

Equations (18)–(21) may be used to eliminate $C_{\substack{e\\o}n}$. The final result is a system of linear equations for the unknown expansion coefficients of the scattered fields which may be solved for Be_m, $m = 0, 1, 2, \ldots$ and Bo_m, $m = 1, 2, 3, \ldots$.

The case of a TE polarized wave scattered by an impedance elliptic cylinder coated with a homogeneous material can be treated in the same way. The resuls for the matrix elements are identical to (18)–(21) and may be obtained by substituting $(\mu_1, \epsilon_1, \eta_s)$ by $(\epsilon_1, \mu_1, 1/\eta_s)$ with $Z_s(\eta) = \eta_s(\xi_2^2 - \eta^2)^{-1/2} Z_1$ for the TE case.

III. Far-Zone Fields

The near and far scattered fields can be computed once the coefficients Be_m and Bo_m are known. The scattered far field at very large distances from the cylindrical structure can be deduced from (5) by using the asymptotic expansions of the modified Mathieu functions [16]. In the far zone, the following limits can also be made

$$\lim_{c\xi \to \infty} c\xi \longrightarrow k_o\rho, \quad \text{and} \quad \eta = \cos\phi. \quad (23)$$

Using the above limiting values in (5), the expressions for the z-component of TM polarized wave is given by

$$E_z^{(s)} = \sqrt{\frac{j}{k_o\rho}} e^{-jk\rho} \sum_m j^m \left[Be_m Se_m(c, \eta) + Bo_m So_m(c, \eta)\right]. \quad (24)$$

The scattering cross section per unit length, i.e., the echo width, is of particular interest in scattering problems. It is defined for the TM wave as

$$\sigma(\phi) = 2\pi\rho \lim_{rho \to \infty} \frac{|E_z^s|^2}{|E_z^i|^2} = \lambda \sum_m j^m \left[Be_m Se_m(c, \eta) + Bo_m So_m(c, \eta)\right]. \quad (25)$$

IV. Numerical Results

In this analysis, the exact solution for EM scattering by an impedance elliptical cylinder coated with a homogeneous material is given in terms of four uncoupled linear systems of equations for the even- and odd-ordered expansion coefficients of both *even* and *odd* function. In order to solve these linear systems of equations for Be_m and Bo_m, the infinite series (2), (5), and (6) are terminated to include only the first N terms, where N, in general, is a suitable truncation number proportional to the structure's electrical size and shape. In the following results, the value of N has been chosen to impose a convergence condition that provides solution accuracy better than four significant figures. It is found that N, for even- or odd-ordered coefficients, is of the same order as $k_1 a_1$. Hence, the total number of terms in the series solution is about $2k_1 a_1$. The axial ratio and surface impedance have a negligible effects on N. The following numerical data are generated for nonpermeable coating materials, i.e., $\mu_{r1} = 1$, although the analysis is carried out and similar results can be presented for any μ_r.

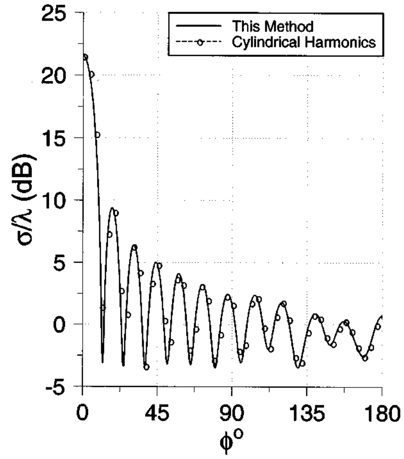

Fig. 2. Normalized TM echo width σ/λ for a circular dielectric-coated impedance cylinder ($a/\lambda = 1.5$, $a_1/\lambda = 2$, $\epsilon_r 1 = 2$, $\phi^i = 180°$, $\eta_s = 0.64 - j0.37$).

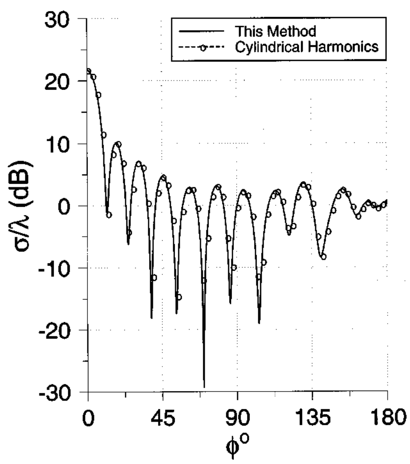

Fig. 3. Normalized TE echo width σ/λ for a circular dielectric-coated impedance cylinder ($a/\lambda = 1.5$, $a_1/\lambda = 2$, $\epsilon_r 1 = 2$, $\phi^i = 180°$, $\eta_s = 0.64 - j0.37$).

To check the validity and accuracy of the proposed method and associated code, the angular distributions of the normalized radar cross section per unit length (echo width) are shown in Fig. 2 for the TM case and in Fig. 3 for the TE case. The scatterer is a coated impedance circular cylinder. For both TM and TE cases, the normalized echo width σ/λ for a core with $\eta_s = 0.64 - j0.37$, corresponding to a typical ferrite absorbing material coated with a dielectric layer $\epsilon_r 1 = 2$, is in perfect

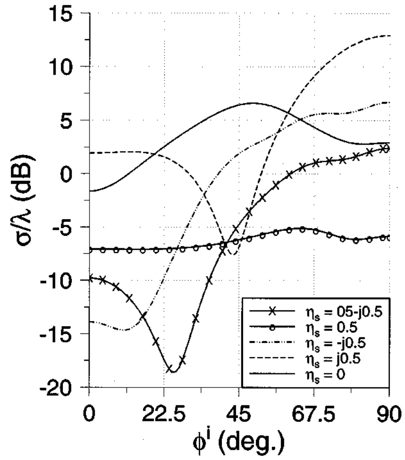

Fig. 4. Normalized TE backscattering echo widths σ/λ for an impedance elliptic cylinder coated with a dielectric versus incident angle ($a/\lambda = 0.7$, $a_1/\lambda = 1$, $a/b = 2$, $a_1/b_1 = 2.2$, $\epsilon_r 1 = 2$).

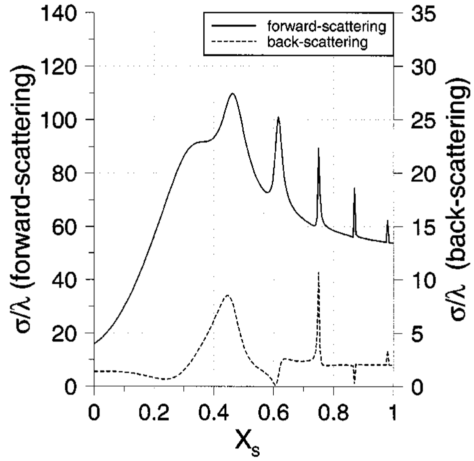

Fig. 6. Normalized TE forward and backscattering echo widths σ/λ for a reactive-loaded elliptic cylinder versus surface reactance ($a/\lambda = 2$, $a_1/\lambda = 2.1$, $a/b = 2$, $a_1/b_1 = 1.76$, $\epsilon_r 1 = 1$, $\phi^i = 0°$).

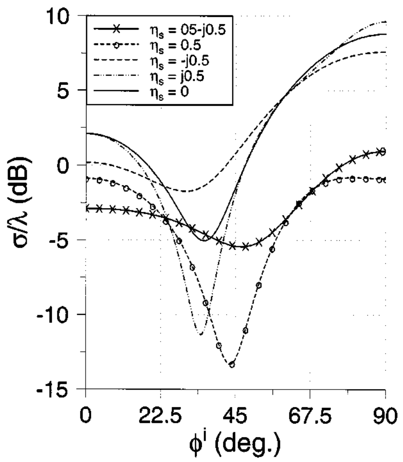

Fig. 5. Same as in Fig. 4, but for the TM polarization case.

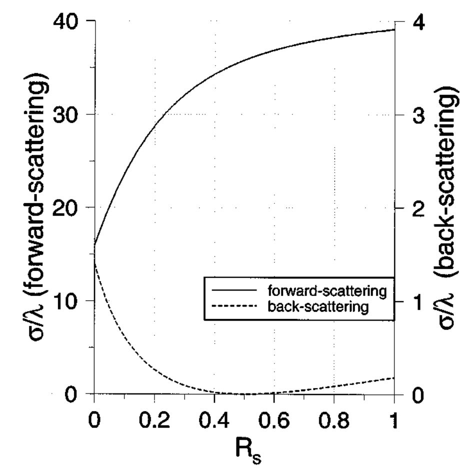

Fig. 7. Same as in Fig. 6, but for a resistive-loaded elliptic cylinder.

agreement with the result generated by cylindrical harmonics solution similar to the one given in [6].

The dependence of the backscattered field on the angle of incidencee for different surface impedances is shown in Fig. 4 for the TE case and in Fig. 5 for the TM case. For all directions of incidence, it may observed that both conducting ($\eta_s = 0.5 - j0.5$) and resistive ($\eta_s = 0.5$) coatings tend to decrease the backscattering echo width compared with the perfectly conducting case with no coating. An inductive coating ($\eta_s = j0.5$) tends, for a wide range of incident angles, to increase the backscattering echo width.

Numerical results for the TE backscattered and forward scattered echo widths for impedance elliptic cylinder are shown in Fig. 6 for reactive coating ($\eta_s = X_s$) and in Fig. 7 for resistive coating ($\eta_s = R_s$). As the reactance increases the contribution of the surface wave to the back and forward scattered fields increases due to the propagation of the coupled energy without attenuation around the cylinder. The resonance-like behavior in Fig. 6 is due to the interference of the surface wave with the wave reflected back at the front of the cylinder. Such kind of interaction depends on the circumference of the cylinder, ϵ_r, λ, and X_s. The resistive coating, however, reduces the backscattered, but enhance the forward scattered field.

Fig. 8 shows the effect of changing the dielectric coating thickness on the TM backscattered field for a resistive coating, also shown as a reference the perfectly conducting case. For all coating thickness' considered, the resistive coating decreases the backscattered field. As the dielectric coating thickness increases, resonance-like peaks occur in the backscattered field due to the contribution of the surface wave that trapped inside the dielectric and propagated around the cylinder.

The effect of the coating dielectric constant ($\epsilon_r 1$) for different kind of impedance conditions on the TM backscattered field is shown in Fig. 9. The core is an impedance elliptic cylinder with two different values of η_s coated with a thick dielectric material. It is evident from the results shown in Fig. 9 that the backscattered field is considerably reduced for the resistive coating ($\eta_s = 1$).

Finally, the normalized backscattering echo width σ/ka_1 from a dielectric-coated impedance cylinder as a function of the frequency is shown in Fig. 10. The results show that both the magnitude and width of the resonance-like envelopes depend on coating conditions. For all considered values of

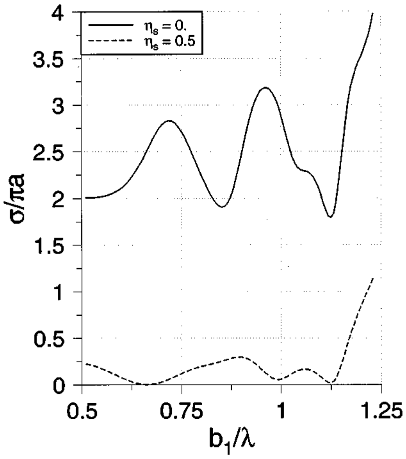

Fig. 8. Normalized TM backscattering echo widths σ/λ for an impedance elliptic cylinder coated with a dielectric versus coating thickness ($b/\lambda = 0.5$, $a/b = 2$, $\epsilon_r 1 = 2$, $\phi^i = 90°$).

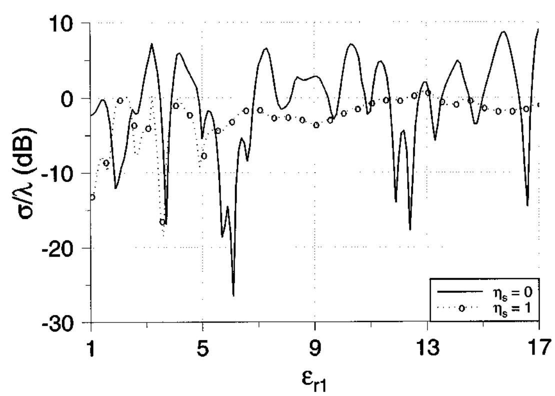

Fig. 9. Normalized TM backscattering echo widths σ/λ for an impedance elliptic cylinder coated with a dielectric versus coating dielectric constant ($a/\lambda = 0.5$, $a_1/\lambda = 1$, $a/b = 2$, $a_1/b_1 = 1.11$, $\phi^i = 0°$).

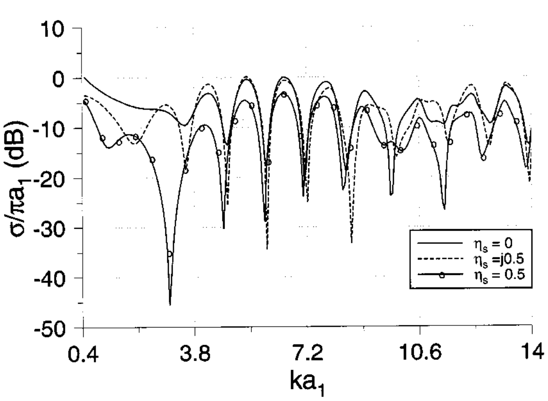

Fig. 10. Normalized TM backscattering echo widths σ/λ for an impedance elliptic cylinder coated with a dielectric versus electrical size ($a_1/a = 1.5$, $a/b = 2$, $a_1/b_1 = 1.225$, $\epsilon_r 1 = 2$, $\phi^i = 0°$).

the frequency, the resistive coating results are less than those corresponding to perfectly conducting case with no surface impedance loading.

V. Conclusion

In this paper, an analytical solution for EM scattering by a dielectric coated impedance elliptic cylinder was developed and numerical results in the resonance region were presented for both TM and TE polarizations. The scattered and transmitted fields were expressed in terms of Mathieu and proper modified Mathieu functions in each region in terms of unknown expansion coefficients. These coefficients were obtained by numerically solving four uncoupled systems of linear equations.

The validity and accuracy of the numerical results were examined by making use of limiting cases such as circular cylindrical structures and perfectly conducting elliptic cylinders coated with dielectrics. Numerical results were also given for different angles of incidence, coating materials, surface impedance, and operating frequency, which were of useful values for development of approximate and numerical techniques as well as antennas and radar applications. The computed results showed that the magnitude of the scattered fields may change considerably with small changes in these parameters.

References

[1] J. R. Wait, "Scattering of a plane wave from a circular dielectric cylinder at oblique incidence," *Can. J. Phys.*, vol. 33, p. 189, 1955.

[2] N. Wang, "Electromagnetic scattering from a dielectric-coated circular cylinder," *IEEE Trans. Antennas Propagat.*, vol. AP-30, pp. 960–963, Sept. 1985.

[3] C. Yeh, "Backscattering cross section of a dielectric elliptical cylinder," *J. Opt. Soc. Amer.*, vol. 55, pp. 309–314, 1965.

[4] H. Ragheb and L. Shafai, "Electromagnetic scattering from a dielectric-coated elliptic cylinder," *Can. J. Phys.*, vol. 66, pp. 1115–1122, 1988.

[5] M. A. Leontovich, "On the approximate boundary conditions," in *Investigation of Propagation Radiowaves*, B. A. Vredensky, Ed. Moscow, USSR: USSR Acad. Sci., 1948, pp. 5–20.

[6] P. L. E. Uslenghi, "High frequency scattering from a coated cylinder," *Can. J. Phys.*, vol. 42, pp. 2121–2128, 1964.

[7] N. G. Alexopoules and G. A. Tadler, "Electromagnetic scattering from an elliptic cylinder loaded with continuous and discontinuous surface impedances," *J. Appl. Phys.*, vol. 46, pp. 1128–1134, 1975.

[8] J. R. Wait and C. M. Jackson, "Calculations of the bistatic scattering cross section of a sphere with an impedance boundary condition," *J. Res. NBS*, vol. 69D, pp. 299–315, 1965.

[9] A. Sebak and L. Shafai, "Electromagnetic scattering by spheroidal objects with impedance boundary conditions at axial incident," *Radio Sci.*, vol. 23, pp. 1048–1060, 1988.

[10] N. G. Alexopoules and G. A. Tadler, "Accuracy of the Leontovich boundary condition for continuous and discontinuous surface impedance," *J. Appl. Phys.*, vol. 46, pp. 3326–3332, 1975.

[11] H. H. Syed and J. L. Volakis, "High-frequency scattering by a smooth coated cylinder simulated with generalized impedance boundary conditions," *Radio Sci.*, vol. 26, pp. 1305–1314, 1991.

[12] R. P. Parrikar, A. A. Kishk, and A. Z. Elsherbeni, "Scattering from an impedance cylinder embedded in a nonconcentric dielectric cylinder," *Proc. Inst. Elect. Eng.*, pt. H, vol. 138, pp. 169–175, 1991.

[13] J. R. Wait and A. M. Conda, "Pattern of an antenna on a curved lossy surface," *IEEE Trans. Antennas Propagat.*, vol. AP-6, pp. 348–359, Oct. 1958.

[14] P. L. Huddleston and D.-S. Wang, "An impedance boundary condition approach to radiation by uniformly coated antennas," *Radio Sci.*, vol. 24, pp. 427–432, 1989.

[15] D. Hill and J. R. Wait, "Ground wave attenuation function for a spherical earth with arbitrary surface impedance," *Radio Sci.*, vol. 15, pp. 637–643, 1980.

[16] P. M. Morse and H. Feshbach, *Methods of Theoretical Physics*. New York: McGraw-Hill, 1953, vol. I and II.

Abdel-Razik Sebak (S'81–M'84–SM'91) received the B.Sc. degree (honors) in electrical engineering from Cairo University, Egypt, in 1976, the B.Sc. degree in applied mathematics from Ein Shams University, Egypt, in 1978, and the M.Eng. and Ph.D. degrees from the University of Manitoba, Winnipeg, MB, Canada, in 1982 and 1984, respectively, both in electrical engineering.

From 1984 to 1986, he was with the Canadian Marconi Company, Kanata, ON, working on the design of microstrip phased-array antennas. He is currently a Professor of electrical and computer engineering, University of Manitoba. His current research interests include phased-array antennas, computational electromagnetics, integrated antennas, electromagnetic theory, detection of subsurface conducting objects, and electromagnetic interference.

Dr. Sebak received the 1992 University of Manitoba Merit Award for Outstanding Teaching and Research and the 1994 Rh Award for Outstanding Contributions to Scholarship and Research in Applied Sciences. He has served as Chair (1991–1992) of the joint IEEE AP/MTT/VT Winnipeg Chapter. As Chapter Chair, he received the 1992 IEEE Antennas and Propagation Society Best Chapter Award. Currently, he is the IEEE Canada CONAC Chair. He has served as the Chair of the IEEE Winnipeg Section (1996–1997). He has also served as the Treasurer (1992, 1996, and 2000) and Publicity Cochair (1994) for the Symposium on Antenna Technology and Applied Electromagnetics (ANTEM), and is a member of the International Union of Radio Science Commission B.