Scattering of Electromagnetic Waves by a Perfectly Conducting Cylinder with a Thin Lossy Magnetic Coating

Hans C. Strifors, *Member, IEEE,* and Guillermo C. Gaunaurd, *Fellow, IEEE*

***Abstract*—We study the scattering interaction of electromagnetic (EM) waves with an infinite cylinder coated with a lossy dielectric material with frequency-dependent material properties. These properties are hypothetical, yet representative of a wide class of available materials. The monostatic and bistatic scattered widths (SW) are evaluated for the TM or TE polarization cases. These calculations require the use of algorithms to evaluate Bessel–Hankel functions of complex arguments. These algorithms are based on a continued fraction approach, which ensures stability of the recursion relations. The bistatic plots of the TM and TE scattering widths for the coated body are displayed in a convenient color-graded scale. The *reductions* in the scattering widths produced by this type of coating are determined in selected frequency bands and angular sectors, in both polarization cases. It is quantitatively shown how curvature and polarization shift the effectiveness band of the coating. The determined regions in which the SW are minimally affected are the most suitable for target identification purposes.**

***Index Terms*—Electromagnetic (EM) scattering, radar scattering widths.**

I. INTRODUCTION

SCATTERING of plane electromagnetic (EM) waves by spherical or cylindrical objects is a well-studied area [1], [2]. The analytical treatment of these problems has considered materials that produce little amount of absorption of the incident signal power, if any at all. For targets covered with coatings of radar absorbing material (RAM), complex-valued arguments enter the Bessel–Hankel functions present in the partial-wave solution for the radar cross section (RCS). Conventional evaluation algorithms [3] then break down, because the needed recurrence relations become unstable.

We will study an infinite perfectly conducting cylinder with a thin *lossy* magnetic layer, which has frequency-dependent material properties. The dielectric permittivity and magnetic permeability of the coating are hypothetical, yet representative of a wide class of available materials. The coating thickness is chosen to approximately center its effectiveness band within the interval of $0 \leq f \leq 20$ GHz. We quantitatively determine the reduction in RCS [or backscattering width (BSW)] resulting when plane waves of two polarizations are scattered by the above described coated cylinder. We also determine and display (in color plots) the bistatic plots of the SW at all angles throughout the above wide band.

A previous work has shown calculations [4] for the BSW for some discrete values of the frequency, and for some (constant) values of $\varepsilon_r \equiv \varepsilon_1/\varepsilon$, and $\mu_r = 1$. A later example [5] exhibits BSW calculations in the nondimensional frequency band $0 \leq ka \leq 50$, for complex but *constant* values of ε_r (*viz.*, $\varepsilon_r = 2.56$ or $\varepsilon_r = 2.56 + i0.1024$) and for $\mu_r = 1$, which we have used to verify our formulation. Cylinders of several cross-sectional shapes coated with a thin film with $\mu_r = 0.01 - j0.03$ have also received some attention [6]. Additional studies of dielectrically coated cylinders [7]–[9] have emphasized the high-frequency regime, the creeping waves, and have considered coatings with frequency-constant material properties, with no attention placed on the quantitatively determination of the reductions in BSW produced by such layers.

II. OUTLINE OF THE SCATTERING SOLUTION FOR CYLINDERS WITH LOSSY DIELECTRICS

An infinitely long perfectly conducting cylinder of radius a is covered with a magnetic lossy coating of thickness d. A plane wave $\mathbf{E}^{\text{in}}$ of time-dependence $\exp(-i\omega t)$ is normally incident on the cylinder. The incident E-field components parallel to and normal to the cylinder axis z are $E_{\text{TM}}^{\text{in}}$ and $E_{\text{TE}}^{\text{in}}$. The outer medium is free- space, and $\omega = kc$. For the two mentioned polarizations, the normalized radar scattering widths σ_{TM} and σ_{TE} take the forms

$$\frac{1}{\pi a}\begin{Bmatrix}\sigma_{\text{TM}}(\phi, x)\\ \sigma_{\text{TE}}(\phi, x)\end{Bmatrix} = \frac{4}{\pi x}\left|\sum_{n=0}^{\infty}(-1)^n(2-\delta_{n0})\begin{Bmatrix}A_n(x)\\ B_n(x)\end{Bmatrix}\cos n\phi\right|^2 \quad (1)$$

where $\phi = 0$ denotes the backscattering direction or monostatic case. We introduce the definitions $x \equiv k(a+d)$, $x_1 \equiv k_1(a+d) = m_1 x$, and $x_2 \equiv k_1 a = m_1 x a/(a+d)$. The index of refraction m_1 is: $m_1 = k_1/k \equiv \sqrt{\varepsilon_r \mu_r}$. This last form is expressed in terms of the (possibly) complex relative dielectric permittivity $\varepsilon_1/\varepsilon$ and magnetic permeability μ_1/μ of the coating. Then, the scattering coefficients assume the forms

$$\begin{Bmatrix}A_n(x)\\ B_n(x)\end{Bmatrix} = -\frac{J_n(x) - i\begin{Bmatrix}Z_n(x)\\ Y_n(x)\end{Bmatrix}J_n'(x)}{H_n^{(1)}(x) - i\begin{Bmatrix}Z_n(x)\\ Y_n(x)\end{Bmatrix}H'^{(1)}_n(x)} \quad (2)$$

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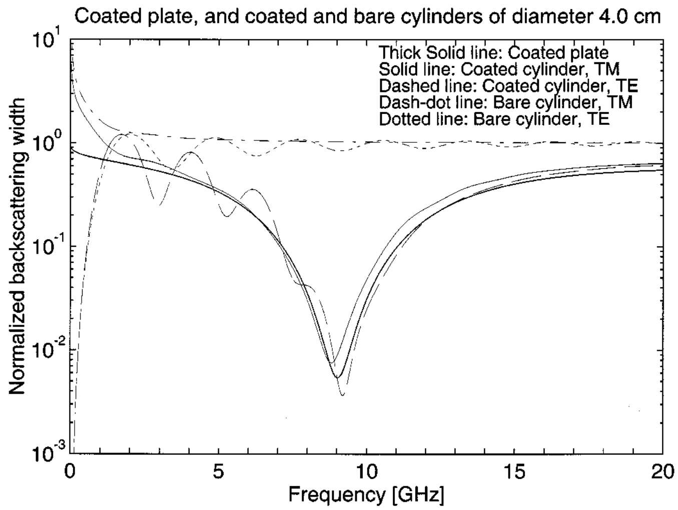

Fig. 1. Normalized backscattering width when a plane EM wave is incident in TM or TE polarization mode on a coated or bare perfectly conducting cylinder of diameter 4 cm and on a flat plate coated with the same material.

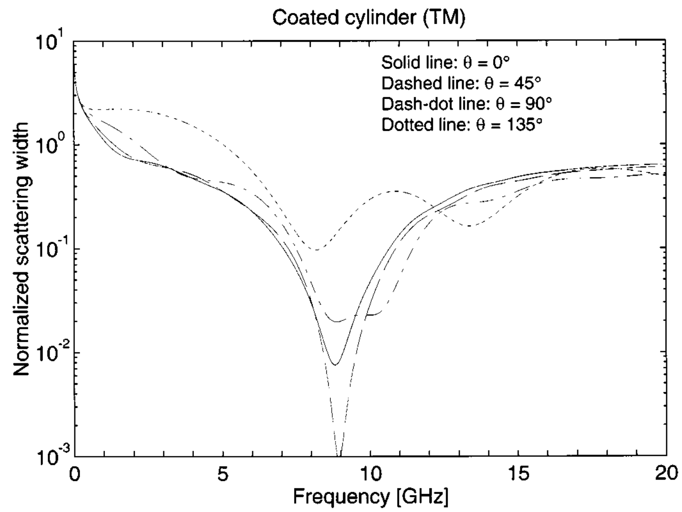

Fig. 2. Normalized scattering width in four bistatic directions (0°, 45°, 90°, 135°) when a plane wave is incident in TM mode on a *coated* perfectly conducting cylinder of diameter 4 cm.

where the modal impedances and admittances of the coated cylinder are

$$\begin{Bmatrix} iZ_n(x) \\ iY_n(x) \end{Bmatrix} = \begin{Bmatrix} \dfrac{\mu_r}{m_1} \\ \dfrac{m_1}{\mu_r} \end{Bmatrix} \cdot \left[\frac{J_n(x_1)\begin{Bmatrix} H_n^{(1)}(x_2) \\ H'^{(1)}_n(x_2) \end{Bmatrix} - H_n^{(1)}(x_1)\begin{Bmatrix} J_n(x_2) \\ J'_n(x_2) \end{Bmatrix}}{J'_n(x_1)\begin{Bmatrix} H_n^{(1)}(x_2) \\ H'^{(1)}_n(x_2) \end{Bmatrix} - H'^{(1)}_n(x_1)\begin{Bmatrix} J_n(x_2) \\ J'_n(x_2) \end{Bmatrix}} \right]. \tag{3}$$

This is the cylindrical counterpart of the spherical formulation we gave in [10]. Although some arguments and factors look slightly different from that in [1], the overall formulation is completely equivalent to that in [1]. If there is no coating (i.e., $d \to 0$), this further reduces to the results in [11]. A distinguishing property of the dielectric coatings used for RCS reduction is that the index of refraction m_1 is complex-valued function of the angular frequency ω due to a complex permeability or permittivity or both. In the present case, we will examine a coating material with complex-values of μ_r and ε_r of the form

$$\begin{aligned} \mu_r &= \frac{0.85}{(f/f_0)^{0.4}} + i\frac{0.25}{(f/f_0)^{0.7}} \\ \varepsilon_r &= \frac{7.0}{(f/f_0)^{0.1}} + i\frac{0.80}{(f/f_0)^{0.2}} \end{aligned} \tag{4}$$

where the frequency $f_0 = 10$ GHz. This is a hypothetical material with complex and frequency-dependent values of μ_r, ε_r, that has a thickness of $d = 0.325$ cm, placed over a cylinder of outer diameter $2a = 4.0$ cm.

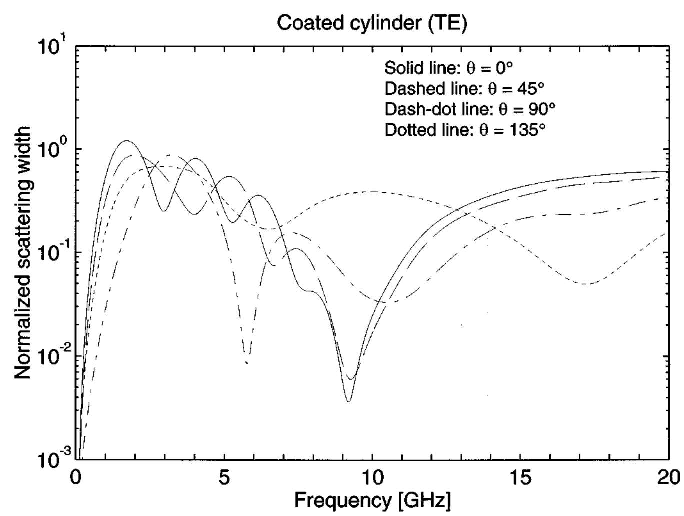

Fig. 3. Same as in Fig. 2, but for the TE mode.

III. Bessel and Hankel Functions of Complex-Valued Arguments

Scattering coefficients containing Bessel functions of complex arguments, often with a large imaginary part, severely restrict their successful numerical evaluation using traditional algorithms. In such cases, the ordinary recurrence relations for the Bessel functions become unstable. We use an algorithm for the Bessel functions of the first kind developed by Lentz [12], [13] that rests on an intrinsically stable continued fraction of the ratios

$$\frac{J_{n-1}(z)}{J_n(z)} = a_1(z, n) + \cfrac{1}{a_2(z, n) + \cfrac{1}{a_3(z, n) + \ddots}} \tag{5}$$

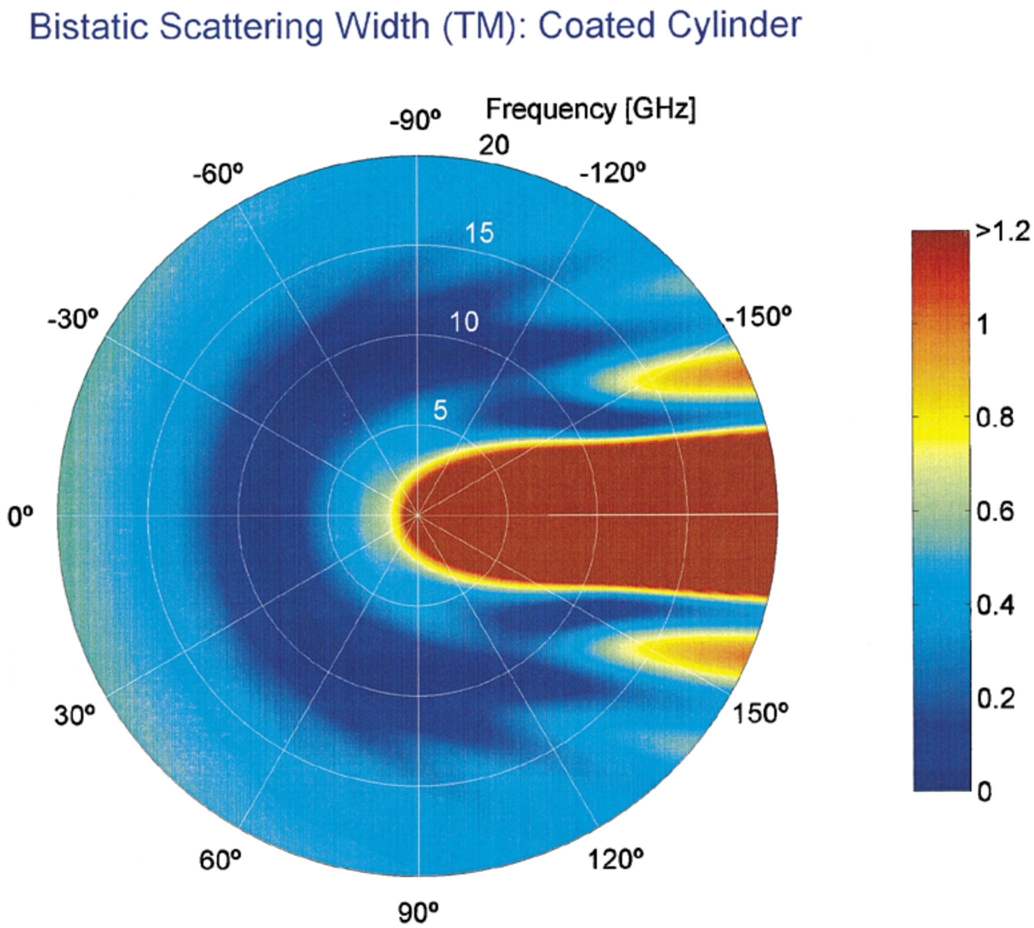

Fig. 4. Bistatic scattering width $\sigma_{\rm TM}$ for a *coated* perfectly conducting cylinder of diameter 4 cm.

where the coefficients a_m are given by

$$a_m(z, n) = (-1)^{m+1} 2(n+m-1) z^{-1}. \tag{6}$$

Using the Lentz algorithm, a vector of consecutive ratios $J_0(z)/J_1(z), J_1(z)/J_2(z) \cdots$ is computed and the Bessel functions of each desired order by

$$1/J_n(z) = 1/J_0(z) \frac{J_0}{J_1} \frac{J_1}{J_2} \frac{J_2}{J_3} \cdots \frac{J_{n-1}}{J_n} \tag{7}$$

once the initial function $J_0(z)$ is known. When $J_0(z)$ is unknown, as is the case, in general, any nonzero value can be first assigned to it, then the whole set of Bessel functions $J_0(z)$, $J_1(z), \cdots, J_n(z)$ can be computed and properly scaled using the infinite series in [14]. The number of terms in the series that is needed for convergence is a few tens larger than n and it can be estimated using asymptotic expansions for Bessel functions of large orders.

The Hankel functions of first kind are then calculated using the Wronskian determinant [15]:

$$H_n^{(1)}(z) = \left[J_n(z) H_{n-1}^{(1)}(z) - 2i/(\pi z) \right] \Big/ J_{n-1}(z) \tag{8}$$

and the initial function $H_0^{(1)}(z)$, which can be computed using [16] and the already computed values of the Bessel functions of first order. These evaluations of Bessel and Hankel functions of complex arguments can be conveniently performed in a PC and are carried out here in this fashion.

IV. Normal Incidence on a Coated Flat Plate

A continuous wave (CW) normally incident on a perfectly conducting plate (of infinite extent) coated by a dielectric layer of thickness d has reflection coefficient R given by [2], [10]

$$R = \frac{1 - Y_1 - (1+Y_1) e^{i2k_1 d}}{1 + Y_1 - (1-Y_1) e^{i2k_1 d}} e^{-i2kd} \tag{9}$$

where $Y_1 = \sqrt{\varepsilon_r/\mu_r}$ is the relative admittance of the layer. The "power reflection coefficient" is $|R|^2$. Computations based on (9) will illustrate, by contrast, the differences resulting when the object being coated is flat or curved. Coating performance is still often (erroneously) evaluated by placing them on flat plates, thus ignoring the curvature of the object they coat.

V. Numerical Results

Fig. 1 shows plots of the (normalized) BSW $\sigma_{\rm TM}$ and $\sigma_{\rm TE}$ for the bare cylinder ($d \to 0$) of diameter 4.0 cm and for the coated cylinder with a layer of material properties given by (4) in the frequency band $0 \leq f \leq 20$ GHz. Expressions similar to those in (4) have been used before to model experimentally

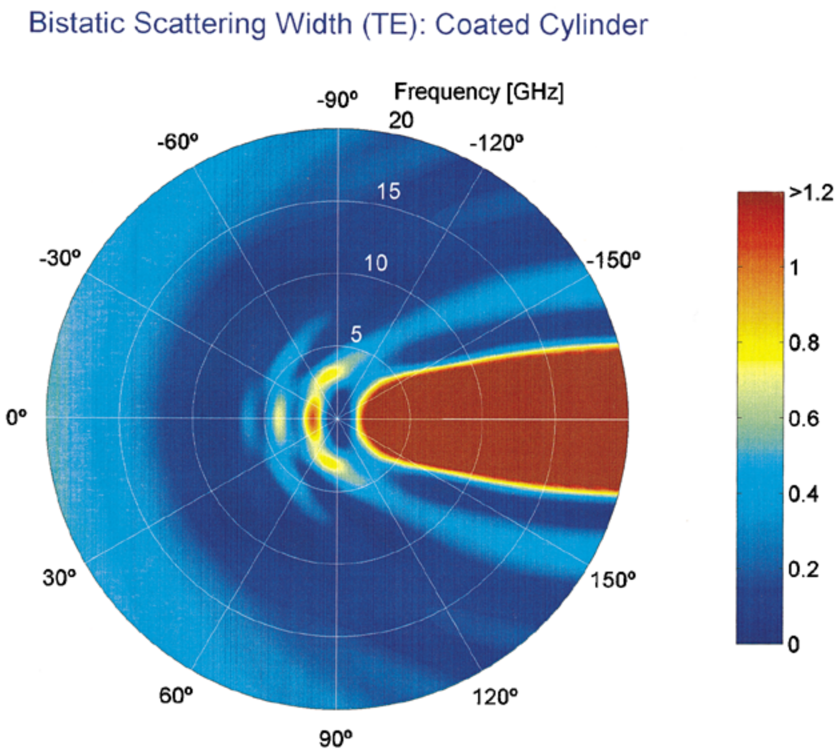

Fig. 5. Same as in Fig. 4, but for the TE polarization mode.

determined properties of real materials [17]. Also shown in this figure is the power reflection coefficient for the flat plate also covered with the same material and thickness (*viz.*, $d = 0.325$ cm), as found from (9). It is clear that the minimum value of the BSW occurs at different frequencies (*viz.*, 8.8 GHz, TM, and 9.2 GHz, TE) for the coated cylinder and for the coated flat plate (9.0 GHz) and that the minima have different values. Thus, target curvature and incidence polarization alter the results as was stated earlier [10].

Figs. 2 and 3 show plots of $\sigma_{\rm TM}$ and $\sigma_{\rm TE}$ for the coated cylinder in several bistatic directions (*viz.*, $\theta = 0^\circ, 45^\circ, 90^\circ, 135^\circ$). These figures show that the minimum scattering widths occur at nearly the same frequency for $\theta \leq 90^\circ$. For $\theta > 90^\circ$ the minimum value of the SW increases substantially in the TM case while various additional minima appear at various angles for the TE case.

Bistatic plots of $\sigma_{\rm TM}$, $\sigma_{\rm TE}$ are displayed for coated cylinders in Figs. 4 and 5 for all angles and for $0 \leq f \leq 20$ GHz (or $0 \leq ka \leq 8.4$). The SWs are represented by an arbitrary color scale (pseudocolors) graded from blue (0) to brown (≥ 1.2), going through magenta, green, yellow, and red. The incidence direction is from the left, i.e., $\theta = 0^\circ$. The forward scattered field is 180° out-of-phase with the incident and, when the two are added, the shadow region behind the cylinder is formed [2]. Away from $\theta = 180^\circ$ the SWs show different features. There are dark blue regions caused by the absorption in this coating, which often reduce the SW by 20 dB. For angles θ below 90°, the frequency extent of the regions of low SW seem to be independent of bistatic angle. Above 90°, it appears that the minima in the SW increase with θ. These results are in agreement with those in Figs. 2 and 3. The regions outside the efficiency band of the coating, particularly the ones at the lower frequencies and at the larger bistatic angles, are the best suited for target recognition purposes [18], [19].

VI. Conclusion

We have quantitatively evaluated the *reduction* in SW when incident TM and TE plane EM waves are scattered by a dielectrically coated cylinder. The material parameters in this analysis are complex and frequency-dependent. Algorithms to evaluate Bessel–Hankel functions of complex arguments were used. The results are displayed as BSWs (i.e., $\theta = 0^\circ$) for both polarizations and bistatically versus θ for a wide frequency range, using a color scheme. Results are also shown for flat plates covered with the same lossy coating to show the differences present when the target curvature is accounted for. The color plots permit a quantitative visualization of the angular (sectors) and frequency (annular) regions in which cross-sectional features of interest appear. Although reduced in some regions, often as much as 20 dB, these features are enhanced in others. This is useful for the identification of objects coated with lossy magnetic layers.

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