An Effective Approach for the Optimal Focusing of Array Fields Subject to Arbitrary Upper Bounds

Tommaso Isernia, Paolo Di Iorio, and Francesco Soldovieri

***Abstract*—A new approach to the optimal focusing of array fields subject to arbitrary upper bounds is presented. The approach formulates the problem as the minimization of a linear function in a convex set. Unlike other approaches, this one guarantees the achievement of the global optimum by using local optimization techniques and can, moreover, deal with any convex constraint on the unknowns, such as near field constraints. Optimization is performed by two ad hoc developed solution algorithms, which exploit the geometrical characteristics of the problem at hand, thus leading to extremely effective and computationally efficient numerical codes. An extensive numerical analysis has been performed in all cases of linear, planar, and circular arc arrays. The enhanced performance of the proposed technique with respect to the solution algorithms available in the literature fully confirms the effectiveness of the approach.**

***Index Terms*—Antenna synthesis, arrays, near-field constraints, optimal focusing, optimal synthesis.**

I. Introduction

IT is well known that antenna power synthesis allows designers to deal with a greater number of "degrees of freedom" than is possible with more conventional field synthesis problems, as it is not necessary to fix in advance the far-field phase distribution [1]. Power synthesis has thus become a classic problem in electromagnetics.

In its wider sense, power pattern synthesis is a nonconvex optimization problem [2]. All solution techniques can therefore be regarded as the search for the global optimum of a nonquadratic function of the parameters defining antenna excitations and structure. Consequently, design procedures based on gradient or Newton-like minimization techniques can be "trapped" into yielding a local optimum (rather than converging to the global optimum), thus providing a suboptimal (i.e., nonoptimal) solution to the problem at hand. In an array synthesis problem, for instance, such a "suboptimal" design would either imply excitations that do not fulfill the constraints or arrays that fulfill the constraints by using a redundant number of antennas.

A great deal of effort has recently been channeled into solving this problem. In [2], the properties of squared amplitude distributions of the fields are exploited to obtain the criteria needed in order to *a priori* establish the feasibility of a given shaped beam power pattern synthesis problem for a fixed geometry array. These properties are then used to understand and tackle, by means of geometrical arguments and suitable deterministic algorithms, the local optima occurrence. This approach can synthesize, for given mask constrained power patterns, array antennas having a minimum number of elements, and application of the above feasibility criteria shows that it is not possible to further decrease this number.

A more widely used approach entailing the use of global optimization procedures has been proposed in order to guarantee the "global optimality" of the design. In particular, the use of genetic algorithms has become very popular [3], [4]. However, optimization procedures based on genetic or other global optimization algorithms are very cumbersome from the computational point of view, thus making them very difficult to use (or unreliable) for solving problems with more than a couple of dozen unknowns [4].

Rather than being in contrast, the two approaches can be considered complementary to each other as a better understanding of the causes and the number of local optima can also help to recognize the actual need, performance, and ultimate limitations of "global optimization" approaches.

In contrast to the shaped beam case, some important synthesis problems, such as "optimal synthesis" [5], can be solved in a globally optimal fashion, although only in a limited set of cases.

In the case of linear uniformly spaced arrays with constant sidelobe levels, for example, globally optimal solutions to the focusing problem are obtained by resorting to the properties of the Chebyshev polynomials [6]. A recent advance in this field is presented in [7], which introduces a nonexplicit exploitation of the Chebyshev polynomials. The approach also makes it possible to determine the minimum number of elements required to achieve the desired beamwidth and sidelobe level (SLL) without resorting to an iterative process.

The classical Dolph–Chebyshev approach has also been extended to uniformly spaced planar arrays, with the same limitations as in the linear case, thanks to Baklanov [8] or McClellan [9] transformations, which reduce the two-dimensional (2-D) problem to a one-dimensional one. However, such transformations introduce a symmetry in the synthesized excitation coefficients [10], which reduces the number of degrees of freedom available to the designer.

A new class of linear arrays having equal sidelobe levels in their radiation patterns has recently been introduced [11]. Once the number of elements, (the uniform) spacing, and the SLL have been fixed, these arrays exhibit a larger directivity than conventional Chebyshev arrays, provided the number of elements exceeds a minimum value depending on the SLL and element spacing. However, these "modified" Chebyshev polynomials also exhibit a larger beamwidth.

Manuscript received February 5, 1999; revised June 8, 2000.

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Publisher Item Identifier S 0018-926X(00)10855-5.

However, all the above approaches are unable to deal with situations where *nonuniform* behavior of the sidelobes is required, as happens when we want to reduce the array response over a range of angles where an undesired interference is present.

A second important limitation in all the above approaches is that they can only deal with arrays having a uniform spacing on a line or on a plane and whose identical elements radiate the same identical field. Therefore, nonuniform or conformal arrays, as well as arrays where the elements radiate different patterns, cannot be dealt with.

In this paper, we tackle the problem of determining the excitations of a given set of arbitrarily positioned sources so as to produce a far-field intensity that is maximum in a prescribed direction and subject to completely arbitrary upper bounds elsewhere. Note that the formulation includes any kind of fixed geometry arrays and can be naturally applied to considering mutual coupling (see Section V). This problem is of primary interest in the antenna and propagation community [12] and has more recently attracted attention in biomedical engineering applications [13], [14]. After quoting some of the existing solution procedures, the convex nature of this problem is demonstrated. Consequently, provided that the proper formulation is adopted, it does not admit local optima. Effective *ad hoc* solution schemes are then presented. Finally, numerical evidence of the effectiveness, power, and flexibility of the approach is furnished in the cases of linear, planar, and conformal arrays. By virtue of the chosen formulation and the developed codes, the performance of the achieved designs is equal to or better than the relative published results in all the examples we have examined.

II. The Proposed Approach and Its Properties

Although many efforts have been performed in order to remove the above limitations [13], [15]–[23], all existing approaches share the common properties that global optimality, in the sense defined in Section I, is not guaranteed. Therefore, a step in the right direction is certainly constituted by the recognition of the fact that the optimal focusing of scalar fields subject to arbitrary upper bounds can be formulated as a convex optimization problem, i.e., the minimization of a convex function (in particular a linear function) over a convex set [24]. In fact, such a formulation implies that in this case any local minimum of the objective function is also a global minimum [25]. Obviously, effective and computationally efficient solution codes are also needed.

The adopted formulation is briefly recalled in the following. Let us consider a set of N sources, each radiating a known scalar[1] field $\Psi_i(\underline{r})$, so that the overall field is given by

$$E(\underline{r}) = \sum_{i=1}^{N} a_i \psi_i(\underline{r}) \quad (1)$$

wherein $a_i = x_i + jy_i$ is the excitation coefficient of the ith element. The problem is to determine the set of excitations $\{a_i\}$ $i = 1, \ldots, N$ such that

$$|E(\underline{r}_0)|^2 = \left| \sum_{i=1}^{N} a_i \psi_i(\underline{r}_0) \right|^2 \quad (2a)$$

[1]For the sake of simplicity, we refer herein to scalar fields. This hypothesis will be removed in the following.

is maximum subject to

$$|E(\underline{r})|^2 = \left| \sum_{i=1}^{N} a_i \psi_i(\underline{r}) \right|^2 \leq \text{SLL } (\underline{r}) \quad (2b)$$

where $\text{SLL}(\underline{r})$ is a nonnegative function of the coordinate $\underline{r}$ spanning the observation space. Note that $\text{SLL}(\underline{r})$ will be fixed arbitrarily large in the pencil beam region surrounding r_0.

This formulation makes it possible to consider pencil beam synthesis problems relative to arrays with arbitrary but fixed structures and is therefore also recommended for the conformal array cases. Furthermore, the formulation also makes it possible to solve synthesis problems with constraints on the near field, which is of interest when the array is located in complex electromagnetic scenarios such as satellites, airplanes, or ships.

The bandlimitedness properties of radiated fields [26], considering a sufficiently dense grid of sampling points over the observation domain S and without any loss of degrees of freedom, mean that the problem can be formulated as: to determine the real and the imaginary parts of the excitations $(x_1, \ldots, x_N, y_1, \ldots, y_N)$ such that

$$\text{Re } (E(\underline{r}_0)) \text{ is minimum} \quad (3a)$$

with the constraints

$$\text{Im } (E(\underline{r}_0)) = 0 \quad (3b)$$

$$\begin{cases} |E(\underline{r}_1)|^2 & \leq \quad \text{SLL}(\underline{r}_1) \\ & \vdots \\ |E(\underline{r}_M)|^2 & \leq \quad \text{SLL}(\underline{r}_M) \end{cases} \quad (3c)$$

where M is the total number of points in the grid. Note that (3a) and (3b) correspond to maximizing the original objective function $|E(\underline{r}_0)|^2$ by fixing the (arbitrary) reference phase in such a way that the field has a π phase in the direction r_0. Also note that (1)–(3c) naturally extend to the optimal focusing of a single component of the field subject to arbitrary upper bounds on the total field. In fact, it suffices to consider the given field component in (3a) and (3b) while enforcing constraints (3c) on the total field so that the proposed approach may be conveniently exploited in applications where polarization and/or space reuse are required.

Since $|E(\underline{r}_i)|^2$ is a positive semidefinite quadratic form as a function of $x_1, \ldots, x_N, y_1, \ldots, y_N$, each constraint (3c) defines a hypercylinder, i.e., a convex set, in the space of the unknowns. Moreover, the constraint of (3b) is linear in terms of $x_1, \ldots, x_N, y_1, \ldots, y_N$, so that it also defines a convex set (a hyperplane) in the space of the unknowns. As the intersection of convex sets is still convex, (3b) and (3c) define a convex set. Finally, the objective function (3a) is a linear function of the unknowns, so that the whole problem is equivalent to the minimization of a linear function in a convex set, say, C.

Such a problem has been extensively analyzed in operations research, and it can be shown that it admits a unique minimum value, which is therefore the global optimum, that is achieved in a single point or in a connected (convex) subset of C [25].

It follows that any operations research procedure able to achieve a local optimum of constrained minimization problems provides excitation coefficients such as to produce the maximum field compatible with constraints, i.e., excitations that are globally optimal.

Nevertheless, the use of standard operations research procedures deserves some comment. Their use is relatively new in electromagnetic applications, and their apparent difficulty means that they tend to be used as black box routines without the features, mechanics, and limitations of the adopted methods being properly understood. Consequently, the solution procedures, which are not necessarily well tailored to the problem at hand, may turn out to be very cumbersome from the computational point of view or even not effective at all. Moreover, the causes and countermeasures regarding possible procedure convergence failures are not easily dealt with. This is the case of [24], where the present approach was numerically implemented by resorting to a "general purpose" procedure available from the NAG library [27]. Because of slow convergence problems, the resulting solution algorithm made it possible only to manage problems with up to a few dozen unknowns.

The need thus arises to develop new ad hoc solution approaches capable of overcoming the limitations of the previous approach through a full understanding of each single step of the procedure and exploiting the particular nature of the problem at hand, i.e., a linear objective function and convex (analytically known) quadratic constraints. These two circumstances well match the idea to let the optimization evolve inside the convex set, which renders the proposed point of view to synthesis profoundly different from other, more well-known projection-based approaches [28], [29]. In actual fact, the analytical knowledge of the boundary of the set in each violation point makes it possible to develop two different and somehow original optimization strategies. Moreover, the proposed point of view also makes it possible to identify and overcome the causes of possible slow convergence (or lack of convergence) to the actual solution. Finally, the approach naturally lends itself to the use of numerically efficient computer codes [which are in fact based on fast Fourier transform (FFT) routines for standard arrays] for the computation of the quantities at hand.

III. New Algorithms

Two new algorithms for solving the optimization problem, named "Hit and Set" and "Hit and Get," respectively, are herein described. Both of them are based essentially on the observation that

$$\text{Re }(E(\underline{r}_0)) = w \tag{4}$$

where w is a real constant, identifies parallel hyperplanes that are orthogonal to the gradient of the objective function in the space of the unknowns $(x_1, \ldots, x_N, y_1, \ldots, y_N)$. Let $\hat{i}_g$ be the unit vector identifying the direction of the gradient in such a space. It follows that the problem can be formulated as finding the hyperplane with a nonempty intersection with the feasible set and characterized by a minimum value of the real constant w, i.e., the hyperplane "tangent" to the set C.

In order to simplify the description of the two proposed algorithms, let us suppose that the space of the unknowns is bidimensional[2] and spanned by the usual x, y coordinates. Moreover, let us suppose that the feasible set is given by

$$f(x,y) = ax^2 + by^2 \leq c \tag{5}$$

where a, b, c are positive real constants. Equation (5) defines a convex set with an ellipsoidal boundary. Note that the normal unit vector $\hat{i}_n$ to this set in each boundary point, defined as the direction in which $f(x,y)$ varies at the maximum rate, can be easily evaluated as

$$\hat{i}_n = \nabla f(x,y) / ||\nabla f(x,y)|| \tag{6}$$

where ∇f denotes the gradient of $f(x,y)$.

Also note that (3b) does not need to be expressly enforced because it will be induced in any case by requirement (3a) through a phase shift of excitations.

A. The "Hit and Set" Approach

The first proposed algorithm entails the iteration of two different steps until a suitable stopping rule, defined in the following, is satisfied. Let $\mathbf{P}_{\rm int}$ be a point inside the feasible set that can be the null vector at the very first step of the iterative procedure.

In every "hit" step, the boundary of the feasible set is hit at the point $\mathbf{P}_b$ by moving from $\mathbf{P}_{\rm int}$ in the direction opposite to $\hat{i}_g$ [Fig. 1(a)]. The step amounts to moving along the steepest descent direction $(-\hat{i}_g)$, which is an obvious preferential direction. Because of the linearity of the objective function, $\hat{i}_g$ needs to be computed only once at the beginning of the whole minimization procedure. If we denote by β^+ the distance between $\mathbf{P}_{\rm int}$ and the boundary of the convex set in the $-\hat{i}_g$ direction, the step is also described by

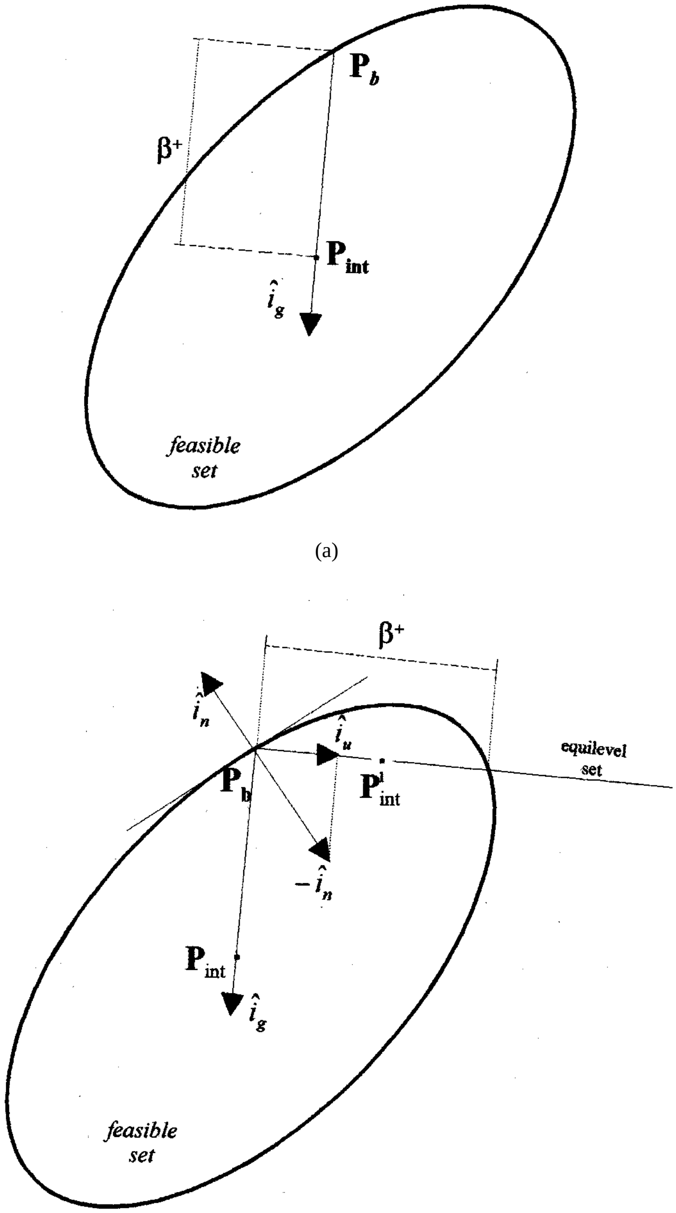
$$\mathbf{P}_b = \mathbf{P}_{\rm int} - \beta^+ \hat{i}_g \tag{7}$$

and the computation of β^+ is easily achieved (see the Appendix).

In every "set" step, an optimal setting for $\mathbf{P}^1_{\rm int}$ inside the set is identified in order to (hopefully) get the maximum gain in the subsequent "hit" step. A possible criterion that can be used to achieve this is to move as far away as possible from the boundary of the set. However, in doing that, the value of the objective function achieved in the previous hit step must not be increased. One way of satisfying these two requirements is to move along the projection of the normal $(\hat{i}_n)$ to the boundary (evaluated in $\mathbf{P}_b$) onto the current equilevel set. In fact, $-\hat{i}_n$ is the direction that (locally) maximizes the distance from the boundary, and its projection onto the equilevel set guarantees that the objective function does not increase in a set step. This direction can be easily computed by subtracting from $(-\hat{i}_n)$ its projection along $\hat{i}_g$, so that

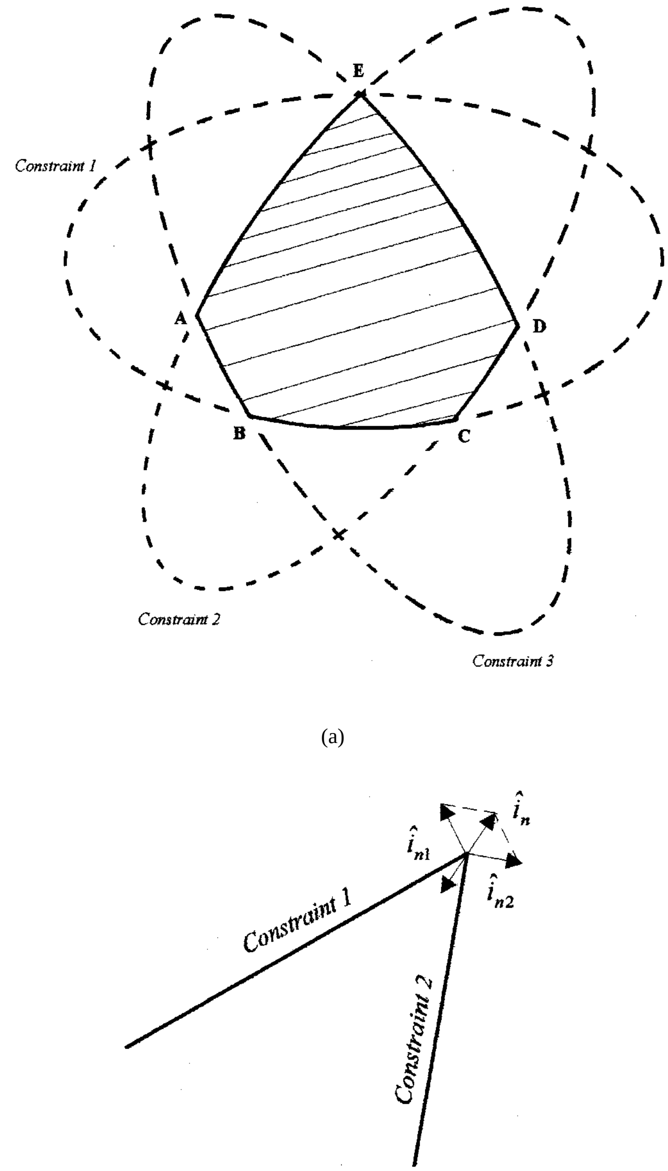
$$\hat{i}_u = \frac{-\hat{i}_n + (\hat{i}_n \cdot \hat{i}_g)\hat{i}_g}{||-\hat{i}_n + (\hat{i}_n \cdot \hat{i}_g)\hat{i}_g||} \tag{8}$$

[2] This hypothesis will be removed in the following.



(b)

Fig. 1. Representation of the "hit" and "set" steps. (a) In the hit step, the boundary point $\mathbf{P}_b$ is found by moving in the direction $-\hat{i}_g$. (b) In the "set" step, one ties to find the optimal setting in order to get the maximum gain in the subsequent step.



(b)

Fig. 2. (a) Pictorial view of a section of the feasible set, defined as the intersection of hypercylinders. (b) About the definition of "pseudonormal." Two constraints are simultaneously violated.

is the moving direction in a set step. Then $\mathbf{P}^1_{\text{int}}$ is set halfway between $\mathbf{P}_b$ and the other boundary point in $\hat{i}_u$ direction [see Fig. 1(b)]. So

$$\mathbf{P}^1_{\text{int}} = \mathbf{P}_b + \frac{\beta^+}{2}\hat{i}_u \tag{9}$$

wherein β^+ is the distance between $\mathbf{P}_b$ and the other violation point along $\hat{i}_u$. This choice agrees with the need to be as far as possible from the boundary in order to get the maximum gain in the subsequent hit step.

Then the procedure continues by iterating the hit and set steps until $\hat{i}_n$ becomes parallel to $\hat{i}_g$, which means that the hyperplane tangent to the convex set has been reached. This approach resembles the strategy of "interior points" methods for convex programming, which try to solve the optimization problem by moving along "central paths" of the feasible set [30].

In an actual synthesis problem, the feasible set has a nonregular boundary as is defined by the intersection of hypercylindrical sets [see Fig. 2(a)]. This circumstance introduces an additional difficulty since no "normal" vector can be defined at the nonregular point of the boundary. On the other hand, all that is needed is a vector directed as far as possible "outside" the convex set at hand. This consideration suggests introducing a "pseudonormal" vector, defined as the sum of $\nabla|E(\underline{r_i})|^2$, where $\nabla|E(\underline{r_i})|^2$ are the normal vectors to each hypercylinder whose boundary is being violated and evaluated in the violation

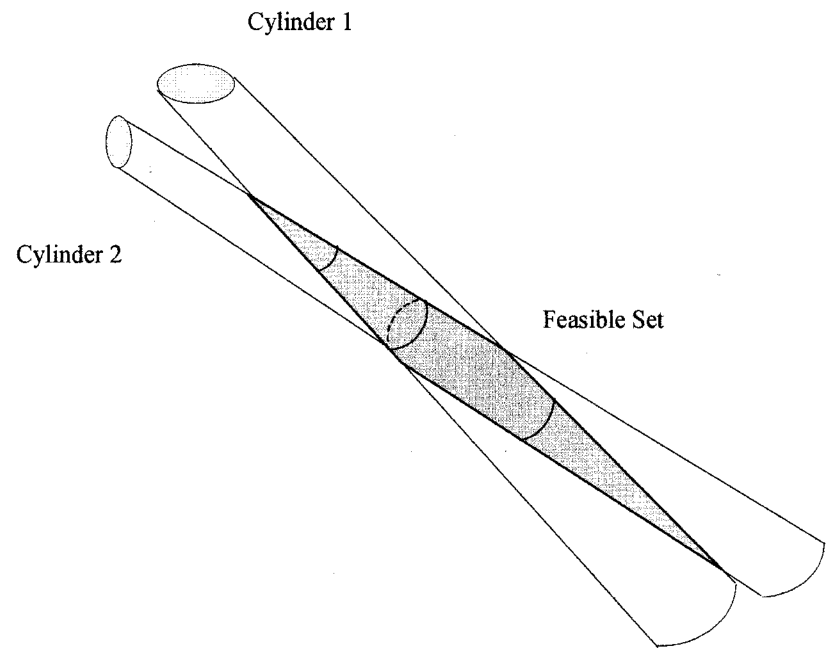


Fig. 3. Pictorial view (in $\mathcal{R}^3$) of the feasible set when the cylinders are nearly parallel.

point [see Fig. 2(b)]. If N is the number of violation points, the pseudonormal unity vector $\hat{i}_n$ is therefore given by

$$\hat{i}_n = \frac{\sum_{i=1}^{N} \nabla |E(r_i)|^2}{\left\| \sum_{i=1}^{N} \nabla |E(r_i)|^2 \right\|} \tag{10}$$

and plays almost the same exact role as $\hat{i}_n$ in all the set steps. The only significant difference is in the stopping rule, which needs to be redefined. A simple choice consists of iterating the two steps until no significant difference is observed on the point $\mathbf{P}_b$ in two consecutive hit steps.

The two iterated steps (7) and (9) imply that the successive values of the objective function do not increase. As $\mathrm{SLL}(\underline{r})$ is bounded (provided $\mathrm{SLL}(\underline{r})$ in the pencil beam region is enforced to a fixed although very large value), the feasible set C is also compact. As, in a compact set, a convergent succession can always be extracted from a non increasing one [31], convergence of the proposed procedure is demonstrated.

B. The "Hit and Get" Approach

The "hit and set" algorithm gives much better optimization results than our previous approach [24], allowing us to deal successfully with problems having hundreds of unknowns. In some cases, however, slow-convergence problems could still keep the optimization procedure far from the actual solution. In this respect, the "geometrical" point of view we use makes it possible to identify the possible causes of the slow convergence and helps to develop suitable countermeasures.

It may happen, for example, that the hypercylinders (whose intersection constitutes C) are nearly parallel to one another, so that the overall feasible set extends greatly along a few directions, while being relatively narrow along all of the many other coordinates (see Fig. 3). Now, whenever the boundary of the feasible set and the equilevel set are nearly parallel to one another while the feasible set is very narrow along the normal direction, only a negligible lowering of the objective function is possible in the two consecutive steps (see Fig. 4). Moreover, the same reasoning may be applied to the following pair of iterations, and so on, thus leading to an overall slow convergence (or even premature stopping) of the procedure.

One way of avoiding this problem is to try to move along the boundary of the feasible set in a descent direction. Unfortunately, the projection of $(-\hat{i}_g)$ on the tangent plane is generally directed outside C because of the fact that the boundary is not a linear manifold. A viable alternative is the direction $\hat{i}_d$ given by a linear combination of the (every time) descent direction $-\hat{i}_g$ and the direction $\hat{i}_u$ pointing inside the convex set, i.e.,

$$\hat{i}_d = \frac{-\hat{i}_g + K\hat{i}_u}{\|-\hat{i}_g + K\hat{i}_u\|} \tag{11}$$

wherein K is a positive real constant. If K is large enough, then $\hat{i}_d$ is directed inside C, so that such a choice simultaneously matches both the requirement of moving in the objective function descent direction and far away from the violated constraints. In addition, a proper choice of K may provide a direction $\hat{i}_d$ nearly parallel to the boundary of the feasible set. This new strategy and its capability of successfully tackling the above-discussed problem is pictorially represented in Fig. 5.

Note that the higher the value of K, the lower the objective function decreases at each step, while the lower the value of K, the more $\hat{i}_d$ is directed along the boundary of the feasible set, with beneficial effects on convergence rate. Therefore, K has

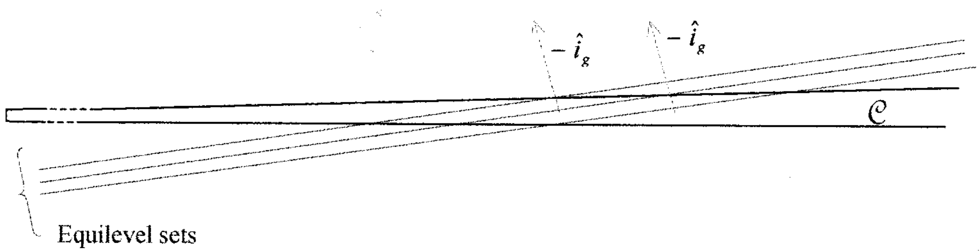


Fig. 4. Two steps of the "hit and set" strategy in a case where the boundary of $\mathcal{C}$ and the equilevel set are nearly parallel.

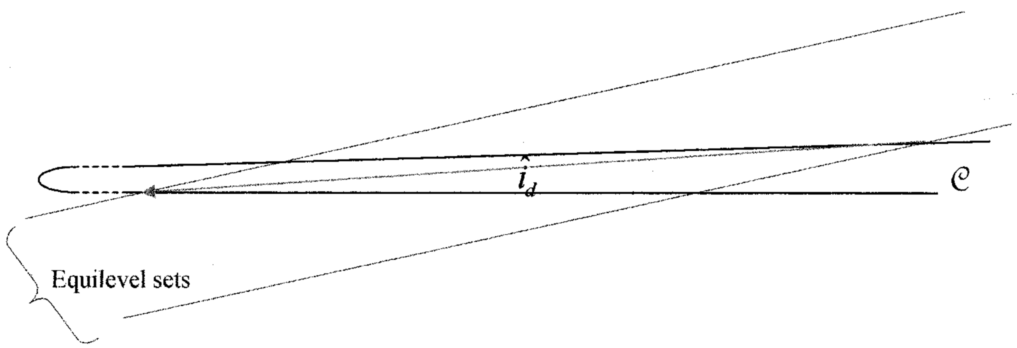


Fig. 5. A step of the "hit and get" algorithm in a case where boundary of $\mathcal{C}$ and equilevel set are nearly parallel.

to be selected from several trials by making a tradeoff between accuracy and convergence rate.

As moving along the successive $\hat{i}_d$ directions gives rise to a nonincreasing behavior of the objective function, the same convergence proof as in Section III-A applies here.

IV. Numerical Results

In this section, numerical results showing the effectiveness of the proposed approaches are reported. This section is divided into three parts, each one regarding a different geometrical configuration. The first concerns linear arrays, the second planar arrays, and the third conformal (circular arc) arrays. For planar arrays, both constraints in near zone and in far zone are considered. All the results have been obtained with a Pentium II 350-MHz processor.

A. Linear Arrays

The radiation pattern $E(u)$ of a linear array with N isotropic elements uniformly spaced by d can be written as [12]

$$E(u) = \sum_{n=0}^{N-1} a_n e^{jnu} \tag{12}$$

with $u = k_0 d \cos(\varphi)$, wherein k_0 is the propagation constant in free space and φ is the angle between the direction of observation and the array line. The gradient of the objective function is therefore the vector

$$i_g = [1, \ldots, \cos(nu_0), \ldots, \cos((N-1)u_0), 0, \ldots, \sin(nu_0), \ldots, \sin((N-1)u_0)] \tag{13}$$

where u_0 is the desired direction of maximum field intensity.

The first example is concerned with the synthesis of a broadside linear array of 16 isotropic elements uniformly spaced by half-wavelength with equal-level sidelobes and a beam width (BW) of 19°. This sidelobe configuration is useful for the validation of the algorithms, since an analytical solution can be easily calculated by Dolph–Chebyshev methods. The result, obtained by the hit-and-set algorithm in less than 1 s, agrees perfectly with the analytical result.[3] The relative difference between excitations, defined as $|(Ia_i - In_i)/Ia_i|$—where In_i and Ia_i are the excitations of the ith element evaluated numerically and analytically, respectively—is on the order of 10^{-3} for all the elements in the array.

When a beam width of 20° or greater is required for this array, the hit-and-set algorithm is no longer able to find the optimal solution due to slow-convergence problems and premature stopping.[4] This is consistent with the very rapid growth of

[3]In order to perform the comparison accurately, the analytical solution is evaluated by imposing the maximum value of the array factor as in the numerical synthesis. Then, beamwidths and excitation coefficients are compared.

[4]The critical value of the beamwidth decreases when the number of elements in the array increases.

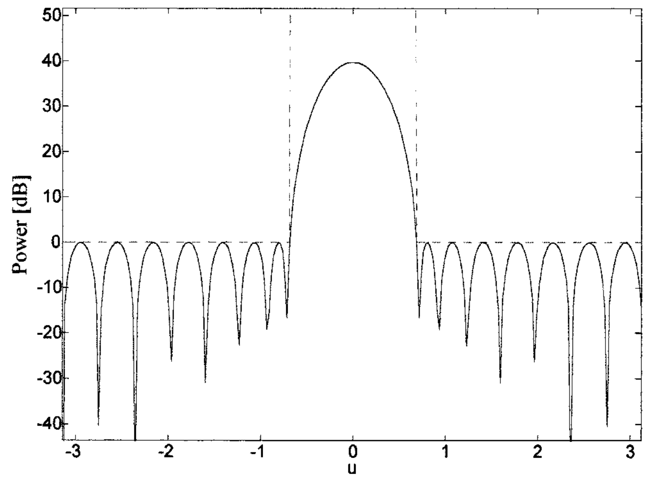


Fig. 6. Radiation pattern of the broadside linear array with 16 elements uniformly spaced of half-wavelength.

the maximum for increasing BW, which rapidly makes the feasible set an elongated one, and the problem becomes ill conditioned. However, the synthesis is carried out successfully by the hit-and-get approach, as shown in Fig. 6, wherein the synthesis result for a BW $= 25°$ is reported.

In the performed examples with linear arrays, it has been observed that a value of the constant K equal to 20 furnishes the best compromise in terms of convergence time and accuracy. However, the algorithm is not very sensitive to this value. A large number of examples on linear arrays with equal sidelobes level have been performed using different values of K in the range from ten to 100, and for all of them the numerical and analytical results are in agreement, with a relative difference between excitations of about 10^{-3}.

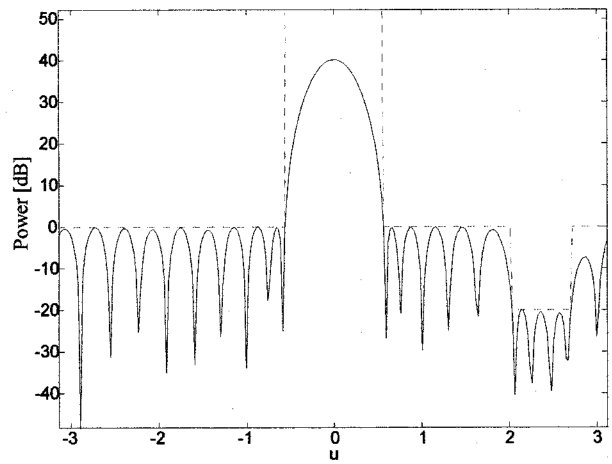
In order to show the flexibility and the effectiveness of the proposed approaches in overcoming limitations of the analytic methods, the synthesis of beams with a nonuniform sidelobes topography has been considered by using the hit-and-get approach.

The results of a shaped sidelobes synthesis are shown in Fig. 7. The linear array is made up of 20 isotropic elements uniformly spaced by half a wavelength. The design constraints prescribe an asymmetrical pattern with a beamwidth of about 20° and maximum field intensity in the broadside direction. The sidelobes are enforced to be equally leveled on one side of the main lobe and further reduced by about −20 dB for $2.01 \leq u \leq 2.72$ on the other side. Note that although the field intensity is reduced by 20 dB for $2.01 \leq u \leq 2.72$, the maximum value is reduced by only 1 dB with respect to the case of uniform sidelobes, thus confirming the effectiveness of the procedure.

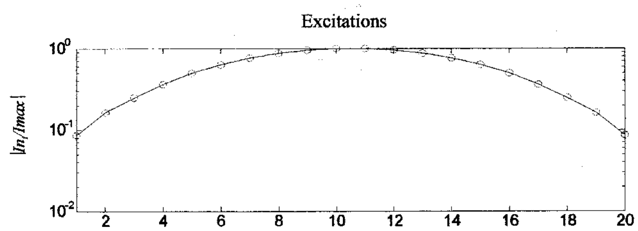
In all the examples performed on linear arrays, the hit-and-get approach has revealed itself to be from five to 100 times faster than the general purpose optimization tool of MATLAB.

B. Planar Arrays

The hit-and-get algorithm preserves its effectiveness even if the array consists of a large number of elements. For a planar array, synthesis algorithms can deal with several hundreds or even thousands of unknown excitations.



(a)



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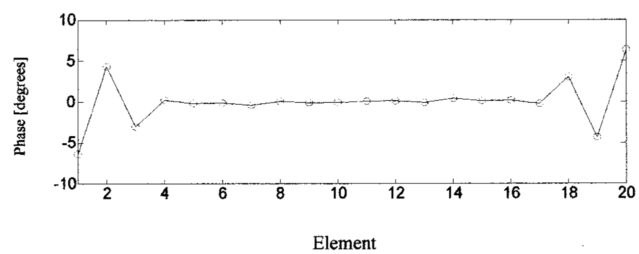


(c)

Fig. 7. (a) Radiation pattern of a linear array with 20 isotropic elements uniformly spaced half-wavelength, synthesized with an asymmetric mask function (dashed line). (b) Normalized amplitude. (c) Phase of the evaluated excitation coefficients.

The radiation pattern $E(u, v)$ of a planar array with $N \times M$ isotropic elements placed on the xy plane and uniformly spaced in the x- and y-directions by d_x and d_y, respectively, can be written as

$$E(u, v) = \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} a_{n,m} e^{j(nu+mv)} \qquad (14)$$

where $a_{n,m}$ is the excitation coefficient of the n, mth element, $u = k_0 d_x \sin(\theta) \cos(\varphi)$, $v = k_0 d_y \sin(\theta) \sin(\varphi)$.

As a first test case, let us consider an array of 31×31 isotropic elements uniformly spaced by half a wavelength in both the x and y directions. The beam is constrained to have a circular shape in the u-v plane with a beamwidth (measured at 0-dB level) of about 13.5° along the two main cuts $(u = 0, v = 0)$ (Fig. 8). The numerical solution is equal

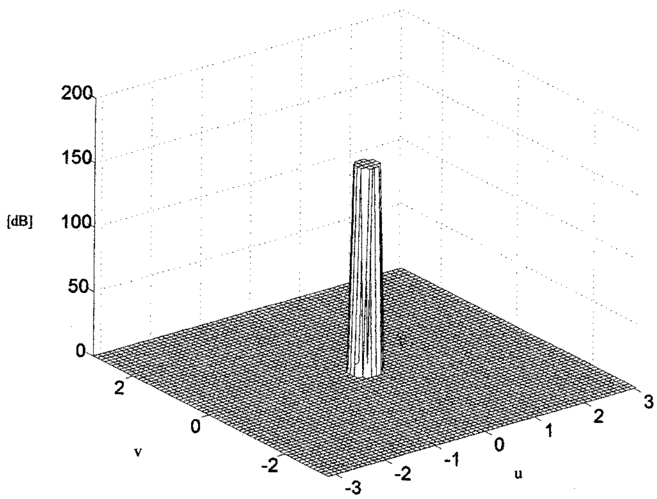


Fig. 8. "Mask" function with circular shape fixing the constraints for the planar array. The prescribed beam width at the two principal sections $(u = 0, v = 0)$ is about 13.5°.

to the expected Tseng–Cheng (Chesbyshev) one along the principal cuts [Fig. 9(a)], but it appears to be much better along the $u = v$ cut [Fig. 9(b)], wherein sidelobes decay much faster without any visible enlargement of the main beam. Also note that this optimal solution for the considered mask constrained problem does not exhibit equal sidelobes.

In all these cases, $K = 12$ has been used, and the same comments as for the linear array case apply.

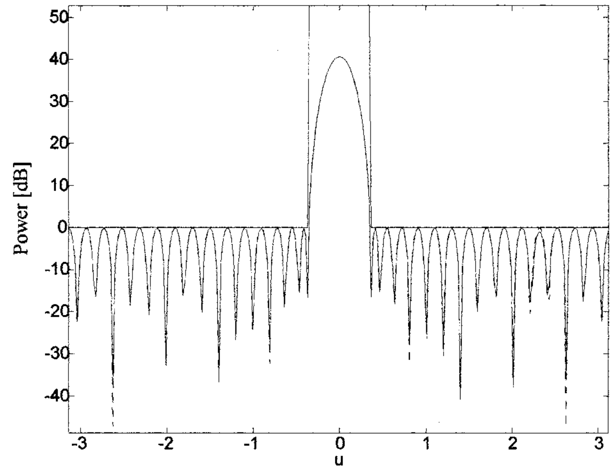
In order to show the flexibility of the approach, near-field constraints have been added to the synthesis problem. In particular, a reduction of the near-field intensity has been enforced outside the square (centered on the z-axis) of 9λ side located over a plane at a distance of 5λ from the array.

Evaluation of the near field, which is herein a scalar function, and enforcement of the constraints are efficiently performed thanks to 2-D FFT codes. In particular, a (256 × 256) grid has been assumed to evaluate both the far and near field and to enforce the corresponding constraints. The results of the synthesis (by the hit-and-get approach) show a reduction of the maximum of the far-field intensity equal to 1 dB (Fig. 10), as compared to a reduction of 10 dB of the near-field intensity in the constrained zone (Fig. 11), thus confirming also in this case the effectiveness of the developed procedures.

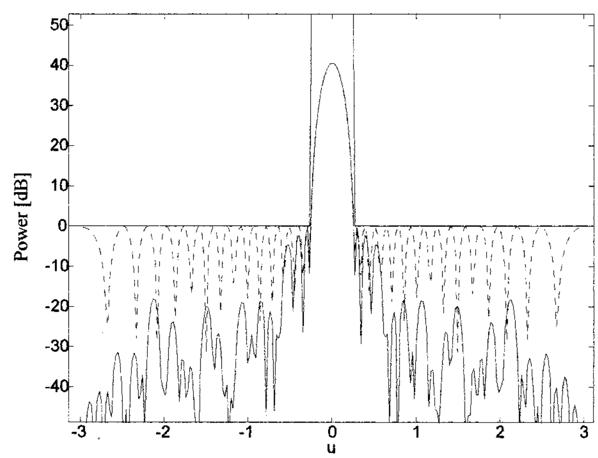
The required computation time for these examples, involving the solution of a nonlinear programming problem with almost 2000 real unknowns and from 65 536 to 131 072 constraints, varies from 13 min for the former cases to about 2 h for the case including near-field constraints. It has to be noted that because of the large number of unknowns and constraints, it has not been at all possible to perform the synthesis using standard general-purpose optimization tools.

C. Conformal Arrays

In order to show the effectiveness of the approach for conformal arrays, let us consider the case of a circular arc array.



(a)



(b)

Fig. 9. Radiation pattern at the section (a) $v = 0$ and (b) $u = v$ of a planar array with 31 × 31 isotropic elements uniformly spaced of half-wavelength. Numerical result (solid line) obtained by hit-and-get method is compared with analytical one (dotted line) obtained by Tseng and Cheng method.

On the plane of the antennas $(\theta = 90°)$, the radiation pattern is given by

$$E(\varphi) = \sum a_n h(\varphi - \alpha_n) e^{jk_0 R \cos(\varphi - \alpha_n)} \tag{15}$$

where α_n is the angular position of the nth radiator, $h(\varphi)$ is the radiation pattern on the plane $\theta = 90°$ of the generic element located at $\varphi = 0$, and R is the radius of the circle [12]. In order to provide a comparison with [18], a circular array with 25 elements angularly spaced by 5.625° along a circular arc with radius $R = 6.72\lambda$ has been considered, with an element pattern $h(\varphi)$ given by

$$h(\varphi) = \max(\cos(\varphi), 0). \tag{16}$$

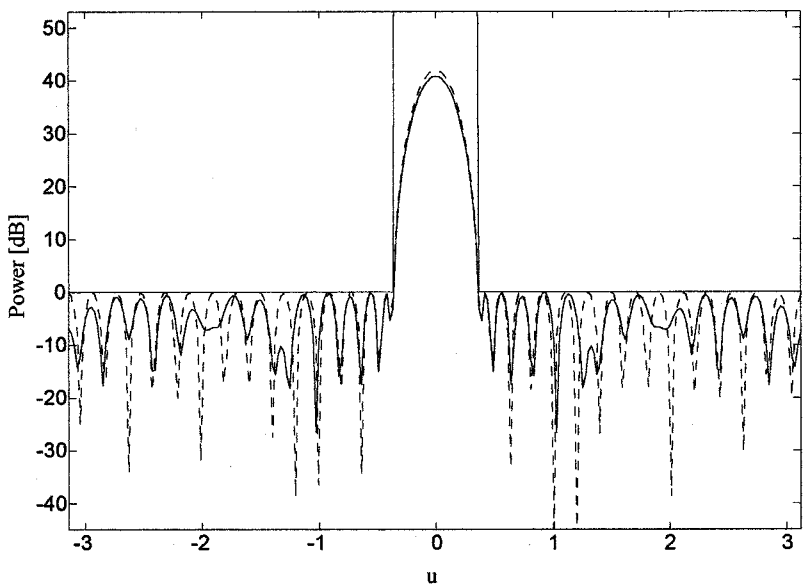


Fig. 10. Radiation pattern synthesized with near-field constraints (solid line) and without near-field constraints (dashed line) at the section $v = 0$.

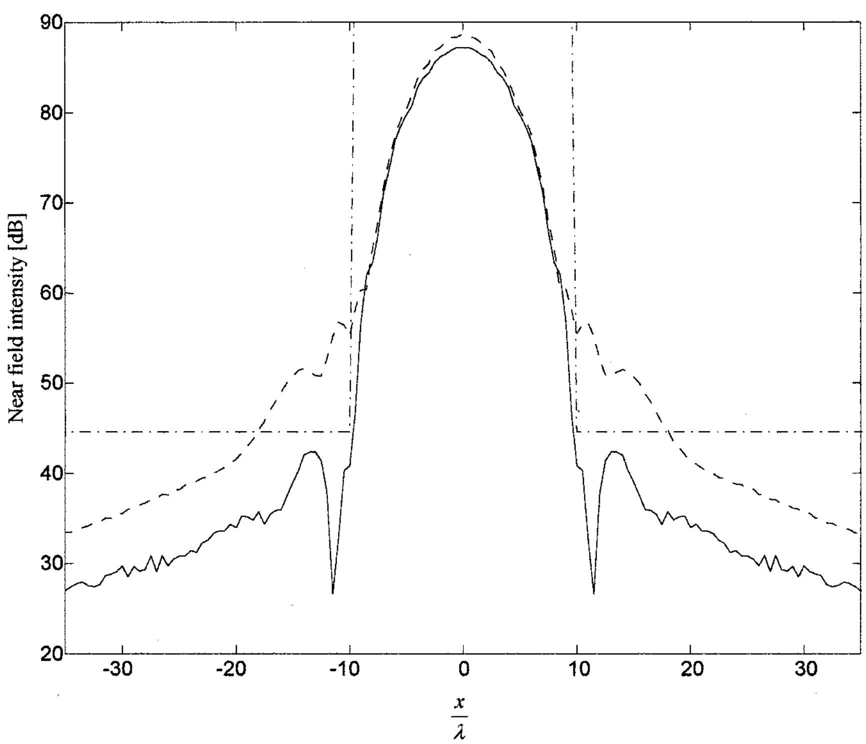
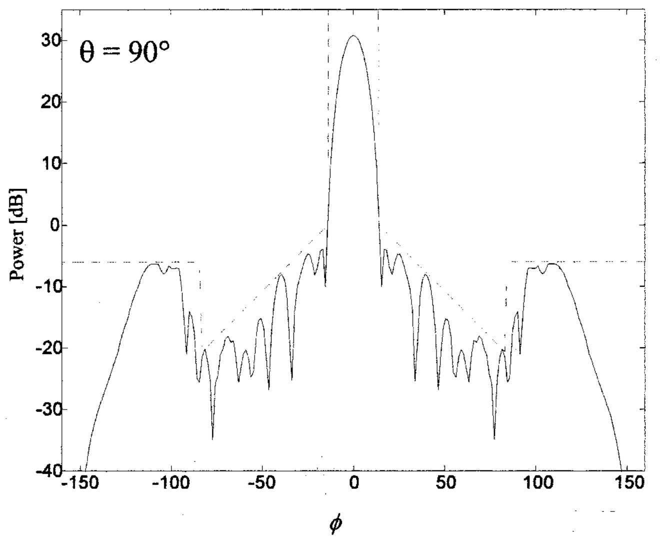
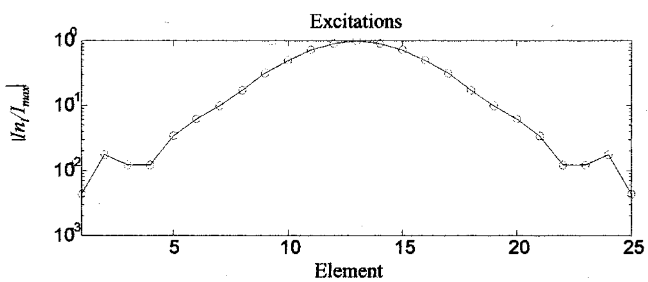


Fig. 11. Near-field intensity without near-field constraints (dashed line) and with near-field constraints (solid line) at the cut plane $y = 0$. The dash and dotted line represent the near-field constraints.

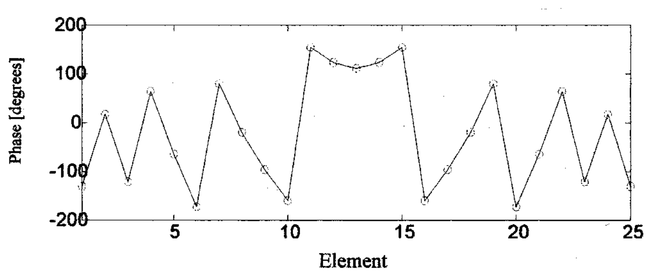
Design specifications require a beamwidth of about 20° around the maximum in $\varphi = 0°$, and the constraining mask has also been fixed in order to make a comparison with [18]. The results of the synthesis (Fig. 12) show that a reduction of about 5 dB on the sidelobe level with respect to the solution presented in [18] is possible by using this approach, thus



(a)



(b)



(c)

Fig. 12. (a) Synthesised radiation pattern (solid line) of a circular arc array of 25 elements, radius 6.72λ and interelement angular distance $5.625°$; the mask function is depicted by a dashed line. (b) and (c) Normalized amplitude and phase of the evaluated excitation coefficients.

confirming once again its validity and that of the developed solution procedures.

V. Conclusion

In this paper, a new solution approach to optimal focusing of scalar fields subject to arbitrary upper bounds is presented. The approach recognizes that the problem pertains to the class discussed in [32], thus ensuring global optimality by "local" methods. Then a conceptually simple geometrical point of view to the problem is introduced, which leads to two different easily implementable "interior point" methods for its solution. The introduced point of view also allows one to understand and somehow tackle possible slow-convergence problems, which are indeed neglected in all previous approaches.

The effectiveness of the proposed approaches is discussed through a series of examples in all cases of linear, planar, and conformal arrays. Even if the first presented solution procedure (hit and set) shares some similarity to the "central path" algorithms often adopted in convex optimization problems [30], the second proposed procedure (hit and get) has been shown to perform better.

It is worth noting that the approach naturally lends itself to the consideration of other convex constraints without impairing global optimality. Such constraints may include specifications such as upper bounds on the invisible part of the spectrum (in order to control superdirectivity), the maximum value of the excitation coefficients, and the maximum absolute variation between the coefficients of adjacent elements in an array.

The presented form of the approach does not take mutual coupling into account, but this can be easily achieved in two different ways: first, voltage excitations may be computed using the mutual admittance matrix, and, secondly, the active element patterns of [33] can be directly used in the synthesis so that no approximation is required on mutual coupling. Comparison between the two approaches shows advantages of the second proposed procedure [34].

As a final remark, we would like to note that the approach is also a natural candidate to perform as a powerful elementary brick in other more complicated focusing problems, such as optimal focusing of vectorial fields, synthesis of multibeam antennas, and synthesis of uniformly (or even nonuniformly) spaced arrays with unknown spacing.

Appendix

The distance $\beta^{+}(\mathbf{P}, \hat{i}_k)$ along the generic direction i_k between the feasible point $\mathbf{P}$ and the boundary of the feasible set can be easily evaluated in this way

$$\beta^{+}(\mathbf{P}, \hat{i}_k) = \min_{i=1,N_\nu} (\beta_i^{+}(\mathbf{P}, \hat{i}_k)) \tag{A.1}$$

wherein N_ν is the total number of constraints and $\beta_i^{+}(\mathbf{P}, \hat{i}_k)$ denotes the distance along $\hat{i}_k$ between the boundary of the set defined by the ith constraint and $\mathbf{P}$. Let $S_i(\mathbf{P})$ be the linear operator which relates set of excitations given by $\mathbf{P}$ to the field in $\underline{r}_i$.

The distances of $\mathbf{P}$ from the boundary defined by the ith constraint can be determined by solution of the equations

$$|S_i(\mathbf{P} + \beta_i^{+} i_k)|^2 = \mathrm{SLL}(\underline{r}_i). \tag{A.2}$$

If

$$\begin{aligned} a_i &= |S_i(i_k)|^2, \quad b_i = 2\ \mathrm{Real}\ (S_i(\mathbf{P}) S_i^{*}(i_k)) \\ c_i &= |S_i(\mathbf{P})|^2 - \mathrm{SLL}(\underline{r}_i) \end{aligned}$$

then β_i^+ can be evaluated solving the following quadratic equation:

$$a_i \beta_i^{+^2} + b_i \beta_i^+ + c_i = 0. \quad \text{(A.3)}$$

It is worth noting that in order to tackle roundoff errors, which may lead outside C, it proves convenient to use a value of β^+ slightly smaller (let us say one for each thousand) than (A.1).

ACKNOWLEDGMENT

T. Isernia thanks Prof. O. M. Bucci for his useful discussions.

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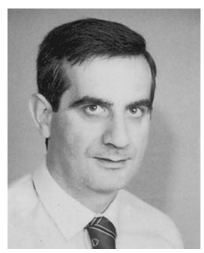

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