Theoretical Basis of Polarization Mode Dispersion Equalization up to the Second Order

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***Abstract*—We introduce a theoretical basis of polarization mode dispersion (PMD) equalizers based on the operator representation of PMD using Taylor's expansion. The two types of configuration of PMD equalizers are derived as the inverse of diagonalization operators and delay time difference compensation. One is a type using physical rotation of quarter wave phase plates. The other is a type using variable phase shifters suitable for PLC integration. Waveform comparison algorithm was simulated to show the existence of multiple equivalent optimum points due to the symmetry and periodicity of optical circuits. The second order PMD equalization is discussed briefly on the case of cascading the first and the second PMD equalizing circuits with two different polarization state converters.**

***Index Terms*—Optical equalizers, optical fiber communication, optical fiber dispersion, optical fiber polarization, optical phase shifters, optical pulse shaping, optical transfer functions.**

I. Introduction

IN GLOBAL size photonic networks, polarization mode dispersion (PMD) of transmission lines can severely limit the information capacity due to pulse waveform degradation [1]. For overcoming this limitation, fiber cables and optical components are improved to reduce their PMD through the developments of transoceanic photonic networks such as TPC-5. Polarization scrambling technique was also implemented to reduce polarization hole burning effect [2]. On the other hand, in terrestrial networks, modern single-mode fibers (SMF's) exhibit negligible PMD, with average differential group delays (DGD's) of the order of 0.1 ps/km$^{1/2}$, for the practical highest bit rate of 10 Gb/s in these days. However, in the implementation of 40-Gb/s systems for the next generation, some of the older fiber cables were found to show large PMD effects with average DGD's of up to 2 ps/km$^{1/2}$. Moreover, the instantaneous DGD in such high-PMD fibers generally fluctuates randomly with time, and hence, can temporarily exceed values of more than 100 ps for transmission distances of only a few hundred kilometers, which may lead to a complete eye closure even in a 10-Gb/s signal [3].

Several methods for equalizing the PMD investigated previously are categorized as two approaches. One is the optical equalization of transfer function matrix for transmission line with PMD. The optical equalizer employs the inverse optical circuit of the transfer function matrix, adaptively synthesized by variable Mach–Zenhder lattice circuit using waveform comparison algorithm [4]. The second is the optical first-order PMD equalizer employing a polarization state converter to the principle polarization states and a delay time compensator [5]–[7]. Heismann demonstrated a fully automatic optical PMD equalizer that adaptively compensates for randomly fluctuating first-order PMD in optical transmission fibers. The performance of this compensator was tested in a 10-Gb/s transmission experiment employing a high-PMD fiber with an average DGD of 50 ps. Optical compensation of first-order PMD is accomplished by introducing a variable time delay between two adjustable orthogonal polarization states in the optical signal.

For the further, necessity of the equalizing second order PMD is recently discussed.

Unfortunately, the second-order PMD was defined only on the Poincaré sphere, where degrees of freedom to describe PMD of optical fibers are not sufficient [8]–[11]. We introduced a new formulation for describing second-order PMD using transfer function matrix [12]. As the result of this new formulation, it is found that the PMD equalizer can be derived as the inverse operator for PMD definition.

In this paper, we reformulate the PMD equalizer including second-order PMD equalization. We discuss the operation principle of PMD equalizer in Section II. In Section III, synthesis of optical circuits for diagonalizing unit is described. In Section IV, the second-order PMD equalization is discussed in a limited condition. In Section V, we conclude the discussion.

II. Operation Principle of First-Order PMD Equalizer

Transfer function matrix of an optical fiber with PMD can be defined as the Fourier transform of its impulse response, which is not necessary to use principle state assumption [8]. The operator representation of PMD is derived from Taylor expansion around optical carrier frequency ω_c [12]

$$T(\omega) = \exp\left(D\delta\omega + \frac{1}{2}\frac{dD}{d\omega}\delta\omega^2\right) T(\omega_c) \tag{1}$$

where $D(\omega)$ is the frequency derivative operator for output polarization state given by

$$D(\omega) = \frac{dT(\omega)}{d\omega} \cdot T(\omega)^{-1}. \tag{2}$$

ω_c is the optical carrier frequency and $\delta\omega = \omega - \omega_c$. The transfer function matrix is given

$$T(\omega) = \begin{bmatrix} \cos\Theta \cdot \exp(-j\phi - j\psi), -\cos\Theta \cdot \exp(-j\phi + j\psi) \\ \sin\Theta \cdot \exp(+j\phi - j\psi), \cos\Theta \cdot \exp(+j\phi + j\psi) \end{bmatrix} \cdot \exp(-j\Phi) \tag{3}$$

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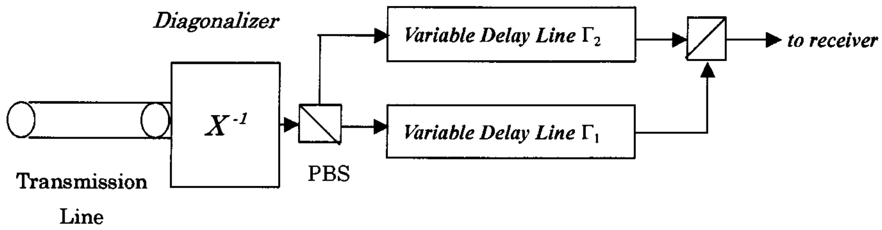

Fig. 1. Basic structure of PMD equalizer.

by a unitary matrix for a loss-less fiber: where Θ is the polarization angle, which describes power distribution between the two orthogonal modes. ϕ is the vertical phase difference, which denotes the phase difference between T_{11} and T_{21}. ψ is the horizontal phase difference, which denotes the phase difference between T_{11} and T_{12}.

The first-order PMD is given by imaginary part difference of the eigen value of $\mathbf{D}(\omega)$, and the second-order PMD is evaluated as the norm of $d\mathbf{D}/d\omega$ [12]. The output polarization state $\mathbf{E}(\omega)^{\text{out}}$ for the input polarization state $\mathbf{E}(\omega)^{\text{in}}$ is given by

$$\begin{aligned} E(\omega)^{\text{out}} &= T(\omega)E(\omega)^{\text{in}} \\ &= \exp\left(D\delta\omega + \frac{1}{2}\frac{dD}{d\omega}\delta\omega^2\right)T(\omega_c)E(\omega)^{\text{in}}. \end{aligned} \quad (4)$$

This is the basic expression of output polarization state of the fiber with PMD.

For the first, we concentrate our discussion on the first-order PMD, neglecting the second-order PMD operator. A matrix $\mathbf{X}$ is defined using the eigen states of operator $\mathbf{D}(\omega)$ for diagonalization

$$\begin{aligned} &X^{-1}E(\omega)^{\text{out}} \\ &\quad = X^{-1}\exp(D\delta\omega)X\cdot X^{-1}T(\omega_c)E(\omega)^{\text{in}} \\ &\quad = \begin{bmatrix} \exp(-j\Gamma_1\delta\omega) & 0 \\ 0 & \exp(-j\Gamma_2\delta\omega) \end{bmatrix}\cdot X^{-1}T(\omega_c)E(\omega)^{\text{in}} \end{aligned} \quad (5)$$

where Γ_1 and Γ_2 are imaginary parts of eigen values of $D(\omega)$.

Equation (5) implies that $\mathbf{X}^{-1}\mathbf{E}(\omega)^{\text{out}}$ is given by two orthogonal components of $\mathbf{X}^{-1}\mathbf{T}(\omega_c)\mathbf{E}(\omega)^{\text{in}}$ delayed by different group delay time Γ_1 and Γ_2, when we neglect the higher order terms at this instance. Γ_1 and Γ_2 are the maximum and minimum delay times, so that two orthogonal components of $\mathbf{X}^{-1}\mathbf{T}(\omega_c)\mathbf{E}(\omega)^{\text{in}}$ correspond to the principle polarization states [2].

When we rewrite (5) in the following form, the basic idea of PMD equalization can be understood clearly

$$\begin{aligned} &\begin{bmatrix} \exp(+j\Gamma_1\delta\omega) & 0 \\ 0 & \exp(+j\Gamma_2\delta\omega) \end{bmatrix} X^{-1}E(\omega)^{\text{out}} \\ &\quad = X^{-1}T(\omega_c)E(\omega)^{\text{in}}. \end{aligned} \quad (6)$$

For equalization, the received signal polarization state $\mathbf{E}(\omega)^{\text{out}}$ should be operated by a linear transformation $\mathbf{X}^{-1}$, then the delay time difference should be compensated. Finally, we obtain $\mathbf{X}^{-1}\mathbf{T}(\omega_c)\mathbf{E}(\omega)^{\text{in}}$, which is a transformation of input polarization state, without any distortion in waveform. This concept is the basis of first order PMD compensation methods [6], [7].

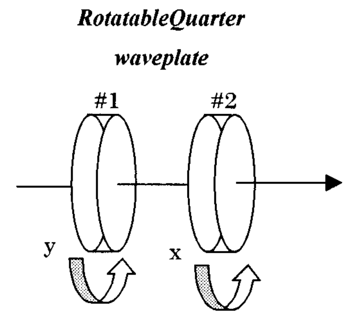

Fig. 2. Optical circuit type #1 for diagonallizer $\mathbf{X}^{-1}$.

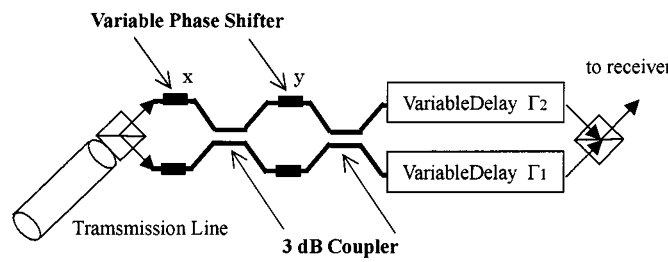

Fig. 3. Integrated first-order PMD equalizer on PLC.

III. Synthesis of PMD Equalizer Circuits

Fig. 1 illustrates the basic structure of the first-order PMD equalizer described by (9).

Here, we discuss on the optical circuit synthesis of operator X^{-1} using the basic PMD parameters $\{\alpha_1, \beta_1, \gamma_1\}$ for the first-order. The basic PMD parameters are defined as the Taylor expansion coefficients of $\{\Theta, \phi, \psi\}$

$$\begin{aligned} \Theta &= \Theta_0 + \alpha_1\delta\omega + \frac{1}{2}\alpha_2\delta\omega^2 \\ \phi &= \phi_0 + \beta_1\delta\omega + \frac{1}{2}\beta_2\delta\omega^2 \\ \psi &= \psi_0 + \gamma_1\delta\omega + \frac{1}{2}\gamma_2\delta\omega^2. \end{aligned} \quad (7)$$

The operator $\mathbf{X}^{-1}$ is determined using eigen vectors of the operator $\mathbf{D}(\omega)$

$$\begin{aligned} X^{-1} &= \frac{\sqrt{\alpha_1^2 + \gamma_1^2\sin^2 2\Theta_0}}{j\alpha_1 + \gamma_1 \sin 2\Theta_0} e^{-2j\phi} \\ &\times \begin{bmatrix} \sqrt{\frac{\Gamma+\beta_1+\gamma_1\cos 2\Theta_0}{2\Gamma}}, & -\frac{(j\alpha_1+\gamma_1\sin 2\Theta_0)e^{j2\phi_0}}{\sqrt{2\Gamma(\Gamma+\beta_1+\gamma_1\cos 2\Theta_0)}} \\ \sqrt{\frac{\Gamma-\beta_1-\gamma_1\cos 2\Theta_0}{2\Gamma}}, & \frac{(j\alpha_1+\gamma_1\sin 2\Theta_0)e^{j2\phi_0}}{\sqrt{2\Gamma(\Gamma-\beta_1-\gamma_1\cos 2\Theta_0)}} \end{bmatrix} \end{aligned} \quad (8)$$

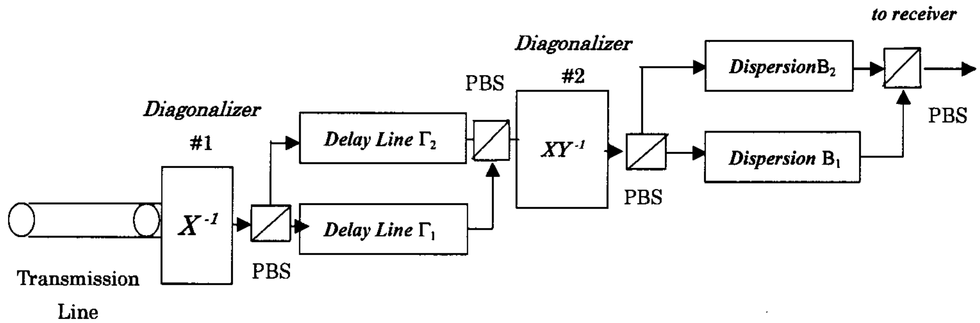

Fig. 4. Second-order PMD equalizer.

where Γ is the half of PMD or DGD given by

$$\Gamma = \Gamma_1 = -\Gamma_2 = \sqrt{\alpha_1^2 + \beta_1^2 + \gamma_1^2 + 2\beta_1\gamma_1 \cos 2\Theta_0}. \quad (9)$$

The operator X^{-1} is special unitary, so that, using Euler's generalized rotation [13], we can generally write X^{-1} as

$$X^{-1} = \begin{bmatrix} e^{-jz} & 0 \\ 0 & e^{+jz} \end{bmatrix} \cdot \begin{bmatrix} \cos y & -\sin y \\ \sin y & \cos y \end{bmatrix} \cdot \begin{bmatrix} e^{-jx} & 0 \\ 0 & e^{+jx} \end{bmatrix} \quad (10)$$

where $-\pi < x, y, z < \pi$ This Euler's generalized rotation representation corresponds to the physical circuit synthesis of the operator $\mathbf{X}^{-1}$. Comparing (8) and (10), we find $z = 0$. Then, we can synthesize the operator $\mathbf{X}^{-1}$ in two forms, using the following relation:

$$\begin{bmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{bmatrix} = C^{-1} \cdot \begin{bmatrix} e^{-jx} & 0 \\ 0 & e^{+jx} \end{bmatrix} \cdot C$$

where

$$C = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \cdot \begin{bmatrix} e^{+j\frac{\pi}{4}} & 0 \\ 0 & e^{-j\frac{\pi}{4}} \end{bmatrix}. \quad (11)$$

The first type is a circuit synthesis using physical rotation of phase plates: in other words, the variable phase-shifter in (10) is replaced by physical rotation using (11). The operator $\mathbf{X}^{-1}$ can be equivalently rewrite as follows:

$$X^{-1} = \begin{bmatrix} e^{-j\frac{\pi}{4}} & 0 \\ 0 & e^{+j\frac{\pi}{4}} \end{bmatrix} \cdot \begin{bmatrix} \cos x' & -\sin x' \\ \sin x' & \cos x' \end{bmatrix} \cdot \begin{bmatrix} e^{+j\frac{\pi}{4}} & 0 \\ 0 & e^{-j\frac{\pi}{4}} \end{bmatrix} \cdot \begin{bmatrix} \cos y' & -\sin y' \\ \sin y' & \cos y' \end{bmatrix}. \quad (12)$$

Equation (13) indicates the operations as follows. First, rotates the quarter-wave phase-plate by y', then secondly rotates the second quarter-wave phase-plate by x'. Fig. 2 illustrates the first type optical circuit corresponding to $\mathbf{X}^{-1}$. This circuit is not practical because it requires resetting at the edge of periodic cycle for automatically adaptive operation. Then, Heismann used a polarization controller with three stages, which is equivalent to this type but does not require resetting [7].

The second type is a circuit synthesis employing variable phase-shifters, which are suitable for PLC integration: the physical rotation in (10) is replaced by variable phase-shifter.

$$X^{-1} = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \cdot \begin{bmatrix} e^{-jy''} & 0 \\ 0 & e^{-jy''} \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \cdot \begin{bmatrix} e^{-jx} & 0 \\ 0 & e^{+jx} \end{bmatrix}. \quad (13)$$

Fig. 3 illustrates the operator $\mathbf{X}^{-1}$ in the form of PLC integration. Firstly, a variable phase-shifter "x" is inserted and a 3 dB coupler follows to mix the mode amplitudes. Second, the first operation is repeated in the same way. The couplers is equivalent to the physical rotation, in this case [14]. The variable delay lines may employ combination of 2×2 switches and delay line units.

IV. Second-Order PMD Equalization

Since $\mathbf{D}(\omega)$ and $d\mathbf{D}/d\omega$ are not commuting, in general, the following formula of operator algebra gives us the way of second-order equalization:

$$\exp\left(D\delta\omega + \frac{1}{2}\frac{dD}{d\omega}\delta\omega^2\right) = \exp(D\delta\omega) \cdot \exp\left(\frac{1}{2}\frac{dD}{d\omega}\delta\omega^2\right) \cdot \exp\frac{1}{2}\left[D\delta\omega, \frac{1}{2}\frac{dD}{d\omega}\delta\omega^2\right] \quad (14)$$

where the last bracket is zero, only when two operators are commuting. When the bracket is negligibly small, (4) can be rewritten as

$$\begin{aligned} X^{-1}E(\omega)^{\text{out}} &= X^{-1}\exp(D\delta\omega)X \cdot X^{-1} \cdot Y \cdot Y^{-1} \\ &\quad \times \exp\left(\frac{1}{2}\frac{dD}{d\omega}\delta\omega^2\right) Y \cdot Y^{-1}T(\omega_c)E(\omega)^{\text{in}} \\ &= \begin{bmatrix} \exp(-j\Gamma_1\delta\omega) & 0 \\ 0 & \exp(-j\Gamma_2\delta\omega) \end{bmatrix} \cdot X^{-1} \cdot Y \\ &\quad \cdot \begin{bmatrix} \exp\left(-j\frac{1}{2}B_1\delta\omega^2\right) & 0 \\ 0 & \exp\left(-j\frac{1}{2}B_2\delta\omega^2\right) \end{bmatrix} \\ &\quad \cdot Y^{-1} \cdot T(\omega_c)E(\omega)^{\text{in}} \end{aligned} \quad (15)$$

where Y is a matrix for diagonalization of $d\mathbf{D}/d\omega$. B_1 and B_2 is imaginary parts of eigenvalues of $d\mathbf{D}/d\omega$. We obtain

$\mathbf{Y^{-1}T}(\omega_c)\mathbf{E}(\omega)^{\mathbf{in}}$, which is a liner transformation of input polarization state, without any distortion in waveform.

Here, we have a second-order PMD equalizer as shown in Fig. 4.

Comparing with the first-order PMD equalizer, we find the PMD equalizer as cascading the first- and second-PMD equalizing optical circuits. The variable dispersion B_1 and B_2 can be realized by Mach–Zehnder lattice circuit on PLC reported previously [14], [15].

V. Conclusion

We introduce a theoretical basis of PMD equalizers based on the PMD operator representation of transfer function matrix approximation using Taylor expansion. The two types of configuration of PMD equalizers are shown as the inverse of PMD operators. One is a type of using physical rotation of quarter-wave phase plates. The other is a type using variable phase-shifters suitable for PLC integration. The second-order PMD equalization is discussed briefly on the cases of cascading the first and the second PMD equalizing circuits with two different polarization state converters.

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