

# Polarization-Dependent Loss-Induced Pulse Narrowing in Birefringent Optical Fiber with Finite Differential Group Delay

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**Abstract**—A combination of polarization-mode dispersion (PMD) and polarization-dependent losses in optical fiber may lead to anomalous pulse narrowing despite the existence of finite differential group delay. It raises the issue for more complete assessments when studying the pulse propagation in single-mode fiber (SMF) in the presence of polarization-dependent loss.

**Index Terms**—Optical fiber dispersion, optical fiber polarization, polarization-dependent loss, pulse propagation.

## I. INTRODUCTION

SINGLE-MODE optical fibers support two polarization modes. These two modes (the birefringence) result from either intrinsic noncylindrical symmetric core shape or from the environmental perturbations such as bending and twisting. As a result when the optical pulse propagates in the single mode fiber potentially there exist two physical effects, namely, the polarization mode dispersion (PMD), and the polarization-dependent loss (PDL). With the increasing of the bit rate in telecommunication, both PMD and PDL are recognized more importantly as a limiting factor. Poole and Wagner [1] have introduced the concept of principal states of polarization (PSP's) to characterize the differential group delays (DGD) for a given optical fiber when PDL is zero. For a fiber without PDL and a pulse with very narrow band these two PSP's are orthogonal, and they represent the slowest and fastest propagating pulses. Any other pulses can be decomposed in terms of the PSP's and will be broadened during propagation. The maximum broadening is given by the DGD, which is the differential time delay between the two PSP's. Recently, Gisin and Huttner [2] have extended the concept of PSP's to include the elements of PDL in the optical fiber. One of the main difference is that PSP's are no longer necessarily orthogonal. Furthermore they have shown some rather surprising results involving interactions of PMD and PDL. For example, they showed that DGD of a concatenation of birefringence fibers and elements with PDL can produce a global DGD that is larger than the sum of the DGD's of all the trunks [3]. They also showed that even with zero DGD for a concatenation of birefringence fibers with PDL element there still exist finite pulse spreading [4].

In this letter we show another surprising consequence of the interaction between PMD and PDL. Following Huttner *et al.* [4], we analyze a concatenation of three elements, one element with only PDL (element 2) sandwiched between two HiBi fibers (elements 1 and 3). We make the assumptions that the attenuation of the PDL element and the birefringence of the HiBi fibers are both independent of the wavelength. The formalism to treat such a concatenation was done by Huttner *et al.* [4] so we directly quote their results for the square of the complex PSP vector  $\mathbf{W} \cdot \mathbf{W} = \chi^2$

$$\chi^2 = \beta_1^2 + \beta_3^2 + 2\beta_1\beta_3[(1 - \cosh \alpha)(\vec{e}_1 \cdot \vec{e}_2)(\vec{e}_2 \cdot \vec{e}_3) + \cosh \alpha(\vec{e}_1 \cdot \vec{e}_3) - i \sinh \alpha \vec{e}_1 \cdot (\vec{e}_2 \times \vec{e}_3)] \quad (1)$$

where

$\vec{e}_i$  is the axis of birefringence or of the PDL of element number  $i$  on the Poincaré sphere;  
 $\beta_1$  and  $\beta_3$  are the modal birefringence of elements 1 and 3;  
 $\alpha$  gives the PDL of element 2.

The real part of the  $\chi$  is interpreted as the DGD for the whole system. As one can see whenever  $\vec{e}_1$ ,  $\vec{e}_2$ , and  $\vec{e}_3$  are in the same plane  $\chi^2$  is real. However, it can either be positive or negative [4]. Moreover when it is positive it can be independent of the PDL value  $\alpha$ . For example, if we choose

$$(\vec{e}_1 \cdot \vec{e}_3) = (\vec{e}_1 \cdot \vec{e}_2)(\vec{e}_2 \cdot \vec{e}_3) \quad (2)$$

then we get

$$\chi^2 = \beta_1^2 + \beta_3^2 + 2\beta_1\beta_3(\vec{e}_1 \cdot \vec{e}_3) > 0. \quad (3)$$

Equation (3) corresponds to finite DGD over the full wavelength range in which our approximations (i.e., wavelength-independent birefringence and PDL) are valid. Equation (2) can be satisfied by taking the PDL direction parallel with either of the PMD elements 1 and 3. In the following, we will directly calculate the output pulse shape as a function of the PDL value, and show that for certain values of  $\alpha$  the output pulse can be narrower than the input pulsewidth.

We now present our theoretical analysis. We start to calculate the transfer matrix for the three section system, this matrix of size  $2 \times 2$  can be written as

$$T(\omega) = \exp(-|\alpha|/2)R(-\theta_3) \cdot U(i\beta_3\omega) \cdot R(\theta_3 - \theta_2) \cdot U(\alpha) \cdot R(\theta_2 - \theta_1) \cdot U(i\beta_1\omega) \cdot R(\theta_1) \quad (4)$$

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where

- $\theta_i$  describes the angle for the direction of the  $i$ th element measured with respect to some fixed direction in two dimensional space;
- $\beta_1$  and  $\beta_3$  are the modal birefringence of elements 1 and 3;
- $\alpha$  gives the PDL of element 2;
- $R$  and  $U$  are both  $2 \times 2$  matrices describing the usual rotation, and the PMD and PDL.

To directly analyze the pulsewidth, we use Gaussian electric field as the input which has the form

$$E_{\text{in}}(t) = \frac{A}{(2\pi)^{1/4} \sqrt{\tau}} \exp \left[ -i\omega_0 t - \frac{t^2}{4\tau^2} \right] \quad (5)$$

where

- $\tau$  Gaussian pulsewidth;
- $\omega_0$  optical frequency;
- $A$  normalization constant.

As a result the input power is in Gaussian form

$$P_{\text{in}}(t) = |E_{\text{in}}(t)|^2 = \frac{|A|^2}{\sqrt{2\pi} \tau} \exp \left[ -\frac{t^2}{2\tau^2} \right]. \quad (6)$$

Defining the pulsewidth square  $\sigma^2$  as the time square averaged over the power

$$\sigma^2 = \frac{\int_{-\infty}^{\infty} t^2 P(t) dt}{\int_{-\infty}^{\infty} P(t) dt} - \left[ \frac{\int_{-\infty}^{\infty} t P(t) dt}{\int_{-\infty}^{\infty} P(t) dt} \right]^2. \quad (7)$$

Let us now consider a linearly polarized input Gaussian pulse so that the output electric field can be written as

$$\mathbf{E}_{\text{out}}(\omega) = T(\omega) \cdot \begin{pmatrix} \cos \theta_{\text{in}} \\ \sin \theta_{\text{in}} \end{pmatrix} E_{\text{in}}(\omega). \quad (8)$$

Here  $\theta_{\text{in}}$  is the input angle describing the input direction of the linear polarization. Now, we give the output power as a function of time for the following special case  $\theta_1 = 0$ ,  $\theta_2 = \theta_3 = 3\pi/8$ ,  $\theta_{\text{in}} = -\pi/4$ ,  $\beta_1 = \beta_3 = \beta$ :

$$\begin{aligned} P_{\text{out}}(t) = & \exp[-(|\alpha| + \alpha)] \left[ \left( \sqrt{2} + 1 \right) P_{\text{in}}(t) \right. \\ & - 2 \cos(\omega_0 \beta) \exp(-\beta^2/8\tau^2) P_{\text{in}}(t + \beta/2) \\ & \left. + \left( \sqrt{2} - 1 \right) P_{\text{in}}(t + \beta) \right] / 4\sqrt{2} \\ & + \exp[-(|\alpha| - \alpha)] \left[ \left( \sqrt{2} + 1 \right) P_{\text{in}}(t) \right. \\ & + 2 \cos(\omega_0 \beta) \exp(-\beta^2/8\tau^2) P_{\text{in}}(t - \beta/2) \\ & \left. + \left( \sqrt{2} - 1 \right) P_{\text{in}}(t - \beta) \right] / 4\sqrt{2}. \end{aligned} \quad (9)$$

In order to compare the output pulsewidth and the input pulsewidth, we define the following effective squared pulsewidth difference:

$$\sigma_{\text{eff}}^2 = \sigma_{\text{out}}^2 - \sigma_{\text{in}}^2. \quad (10)$$

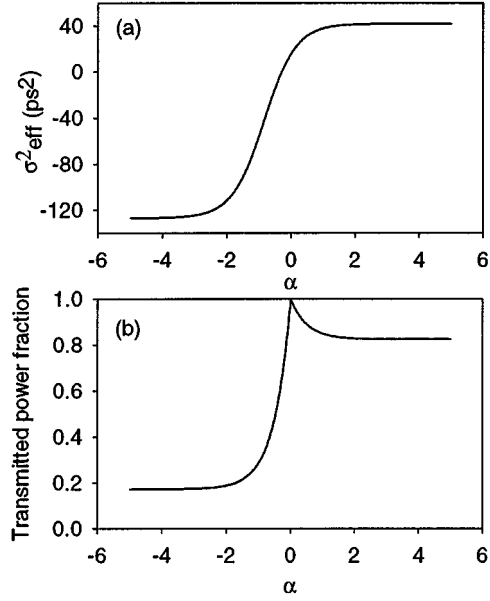


Fig. 1. The results for the three section system are shown for the parameters:  $\theta_1 = 0$ ,  $\theta_2 = \theta_3 = 3\pi/8$ ,  $\theta_{\text{in}} = -\pi/4$ ,  $\beta_1 = \beta_3 = 20$  ps,  $\tau = 25$  ps, and  $\omega_0 = 400\pi$  rad/ps: (a) The effective pulsewidth square difference between output and input pulse is plotted as a function of the PDL value  $\alpha$  of the mid-section. Note that it becomes negative when  $\alpha$  is below certain negative value, and it corresponds to pulse narrowing and (b) the corresponding power transmission fraction is shown as a function of PDL value  $\alpha$ .

In Fig. 1(a), we plotted the  $\sigma_{\text{eff}}^2$  as a function of PDL value  $\alpha$  of the midsection for the special case of (9) with parameters  $\beta = 20$  ps,  $\tau = 25$  ps, and  $\omega_0 = 400\pi$  rad/ps (i.e., the wavelength is about  $1.5 \mu\text{m}$ ). It is clear that the output pulsewidth becomes narrower than the input pulsewidth when PDL value  $\alpha$  is less than some critical value. This result is very surprising when one recalls (3) which gives a finite DGD ( $\sim 33.26$  ps for the case studied here) that is independent of the PDL value  $\alpha$ . Also, as expected, at zero PDL we have positive  $\sigma_{\text{eff}}^2$  (i.e., pulse spreading). Furthermore it is noted from the Fig. 1(a) that output pulsewidth is only about 90.6% of the input pulsewidth at  $\alpha = 2$  (hence it has around 10% reduction in pulsewidth). Fig. 1(b) shows the corresponding power transmission fraction. As expected there is 100% transmission when PDL value is zero. On the other hand the power transmission fraction is not symmetric with respect to the sign of  $\alpha$ . This should not be a surprise since in the limit of  $\alpha$  being either positive or negative infinity the PDL element becomes a perfect polarizer with corresponding passing axes perpendicular to each other. For the given special choice, in the limit of  $\alpha \rightarrow -\infty$  there is only 17.363% power transmission, and in the limit of  $\alpha \rightarrow \infty$  the fraction of power transmitted is about 82.637%. Using the same parameters as in Fig. 1, Fig. 2 plots the normalized output power shape for  $\alpha = -2$  (solid line) as well as the input pulse shape (the dotted line). It is clear that the output pulse shape is narrower. This gives direct support of our calculation displayed in Fig. 1(a), namely narrowing output pulsewidth corresponds to negative  $\sigma_{\text{eff}}^2$ . In addition to the results reported in this letter we have also explored different arrangements of the three sections. They also produce pulsewidth narrowing for certain values of PDL. Furthermore it is noticed that our results sensitively depend on input frequency  $\omega_0$ .

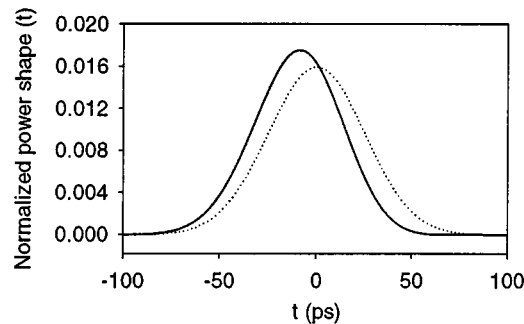


Fig. 2. Using the same parameters as those in Fig. 1 the normalized output power (solid line) is plotted as a function of time for PDL value  $\alpha = -2$ . It is clear that the output pulse is narrower than the input pulse (dotted line).

To understand the anomalous pulse narrowing, let us take a close look at the (9) in which there are terms involving input frequency  $\omega_0$ . Those terms are resulting from the interference. And they are responsible for the pulsewidth narrowing. What we have demonstrated in this letter shows that in the presence of PDL element a finite DGD does not necessarily mean pulse spreading. Whether the pulse shall spread or not depends on how one launches the input pulse, i.e., the input polarization. One can take the advantage of the interference to reduce the pulsewidth by special PDL value as well as by arranging the input polarization. The interference phenomena between two nonorthogonal PSP's in the presence of PDL has also been discussed in [2], however, the authors did not explicitly recognize the anomaly.

Our counter intuitive example shows that to fully understand the pulse propagation under both PMD and PDL it is necessary to include the input polarization as well as its pulse spectrum. Very recently Shieh [7] has formulated a new definition of principal states of polarization that includes the information of the optical pulse. It is unfortunate, however, that this new formula-

tion excludes the possibility of PDL. Therefore an extension of the theory is necessary.

In conclusion, we have shown that in the presence of finite PDL and due to the interference, the output pulse can be narrowed despite a finite DGD. This effect can be potentially used to overcome the adverse effect of normal pulse spreading with finite DGD. It also points the need of a more reliable measurement technique that is directly sensitive to the pulse shape when one evaluates the PMD of a fiber network [5], [6].

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