Analysis of Repeated Unequally Spaced Channels for FDM Lightwave Systems

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***Abstract*—In long-haul optical frequency-division-multiplexing (FDM) systems, transmission characteristics are degraded by four-wave mixing (FWM). To overcome this problem, repeated unequally spaced (RUS) channels have been recently proposed as a new frequency allocation. In this paper, frequency distribution and intensity of generated FWM lights, and a total bandwidth of signal lights of RUS channels are compared with those of already known equally spaced (ES) and unequally spaced (US) channels. It is found that intensities of generated FWM lights of RUS are less than those of ES when the number of channels and a total bandwidth of signals are common in both channels. It is also revealed that RUS has a narrower total bandwidth than US when the number of channels and the minimum channel spacing are common in both channels. Since RUS simultaneously satisfies a low FWM light intensity and a narrow signal bandwidth, it is considered that RUS is suitable for FDM lightwave transmission systems.**

***Index Terms*—Four-wave mixing (FWM), frequency allocation of channels, frequency-division multiplexing (FDM).**

I. Introduction

TRANSMISSION characteristics in frequency-division multiplexing (FDM) lightwave communication systems with low-dispersion optical fibers such as dispersion-shifted fibers are limited by four-wave mixing (FWM) [1], [2]. Characteristics of FWM are closely related to frequency allocations of channels, and up to now equally spaced (ES) and unequally spaced (US) channels were proposed and examined. ES channels have a lot of FWM lights whose frequencies are coincident with those of signal lights. As a result, a signal-to-noise ratio for ES channels is heavily degraded by FWM. On the other hand, US channels do not have any FWM lights whose frequencies agree with those of the signal lights [3]. In US channels, however, a total bandwidth, which is occupied by all the signals, expands drastically with an increase in the number of channels. Thus, it is difficult to have a lot of channels in US. Especially in lightwave communication systems which use optical fiber amplifiers, it is important to achieve a total bandwidth of all the signals as narrow as possible, because the light frequency range, where light intensity is amplified, is limited. To overcome the problems described above, repeated unequally spaced (RUS) channels have been recently proposed and their effectiveness has been demonstrated [4]–[6].

In this paper, RUS channels are theoretically examined, and are compared with ES and US channels. It is found that intensities of generated FWM lights of RUS are less than those of ES when the number of channels and the total bandwidth are common in both channels. For example, when the number of channels is 12 with a total bandwidth of 1500 GHz, an oscillation wavelength of 1550 nm for a light source, an optical fiber length $L = 80$ km, a decay rate $\alpha = 0.2$ dB/km, and a derivative dispersion coefficient $dD_c/d\lambda = 0.07$ ps/km/nm^2, the intensities of generated FWM lights of RUS are less than half of those of ES. On the other hand, US channels do not have FWM lights whose frequencies are coincident with signal lights. However, RUS has an advantage in that its signal lights occupy a narrower total bandwidth than US when the number of channels and the minimum channel spacing are common in both channels. When the number of channels is 30, the total bandwidth of RUS is only 0.37 times as large as that of US.

This paper is structured as follows. In Section II, we briefly review FWM, which is followed by explanations of ES, US, and RUS channels. Section III compares the FWM characteristics of RUS with those of ES and US from the viewpoint of FWM light intensity and a total signal bandwidth. Obtained results are summarized in Section IV.

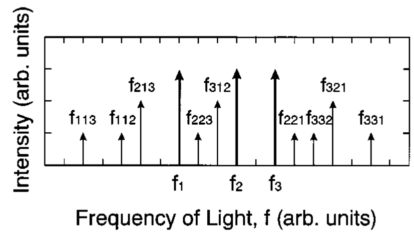

Fig. 1. Generation of FWM lights from three signal lights.

II. Fundamentals of Analysis

A. Four-Wave Mixing

A light frequency $f_{\rm FWM}$ of an FWM light, which is generated by third order nonlinear effect, is related to three signal lights' frequencies f_1, f_2, and f_3 as follows:

$$f_{\rm FWM} = f_{ijk} = f_i + f_j - f_k \qquad (i,\, j,\, k = 1,\, 2,\, 3). \quad (1)$$

Here, we exclude f_{ijk} with $i = k$ or $j = k$ where interruptions from other channels to signals do not happen. As a result, we will examine FWM lights with the frequency of f_{321}, f_{312}, f_{213}, f_{332}, f_{331}, f_{223}, f_{221}, f_{113}, and f_{112}, which are shown in Fig. 1. Note that the number of the FWM lights is enhanced drastically with an increase in the number of channels. For example, when the number of channels is 12, the number of FWM lights reaches to as many as 792.

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Intensity of an FWM light is given as follows: We assume that signal lights and FWM lights are monochromatic, while (1) is satisfied. It is also supposed that transverse modes are fundamental, the lights propagate along z-axis, and the electric fields are polarized along x-axis. Under these assumptions, nonlinear polarization $\boldsymbol{P}_{NL}$ is written as

$$\boldsymbol{P}_{NL} = \chi^{(3)} \vdots \boldsymbol{E_i E_j E_k} = (DX_{1111}) E_i(z) E_j(z) E_k^*(z) \quad (2)$$

where $\boldsymbol{E_i}$, $\boldsymbol{E_j}$, and $\boldsymbol{E_k}$ are electric fields of signal lights which propagate in optical fibers, and $\chi^{(3)}$ is a third-order nonlinear susceptibility which is a fourth-rank tensor. Nonlinear susceptibility X_{1111} introduced by Maker and Terhune [8] is written as $DX_{1111} \equiv 1/4\chi^{(3)}$. Here, D is a degenerate coefficient, and the values are $D = 1$ for $f_i = f_j = f_k$, $D = 3$ for $f_i = f_j \neq f_k$, and $D = 6$ for $f_i \neq f_j \neq f_k$. Note that optical fibers have $\chi^{(2)} = 0$ due to inversion symmetry of SiO_2 which constitutes optical fibers, and $\chi^{(3)}$ brings lowest-order nonlinear optical effects.

When an electric field $E_i(z)$ of a signal light at a position z is given by

$$E_m(z) = E_m \exp\left(-\tfrac{1}{2}\alpha z - i k_m z\right) \qquad (m = i,\, j,\, k), \quad (3)$$

where α is a power decay coefficient and k_m is a wave number of a signal light, an electric field $E(z)$ of an FWM light is written as

$$E(z) = i \frac{2\pi\omega}{n_r c} (DX_{1111}) E_i E_j E_k^* \exp\left(-\tfrac{1}{2}\alpha z\right) \cdot \left(\frac{\exp(-\alpha z + i\Delta\beta z) - 1}{i\Delta\beta - \alpha} \right). \quad (4)$$

Here, $\omega = 2\pi f_{ijk}$ is an angular frequency of an FWM light, n_r is an effective refractive index of an optical fiber, and c is a speed of light in a vacuum. Also, a fiber end is placed at $z = 0$, and a difference in wave numbers $\Delta\beta$ is

$$\Delta\beta = k - k_i - k_j + k_k \quad (5)$$

where $k = n_r \omega / c$ is a wave number of an FWM light.

According to Boyd and Kleinman [9], time-averaged FWM light intensity I is given by

$$I = -\mathrm{Im}\left[\frac{\omega}{2} \iiint E^*(\omega,\, x,\, y,\, z) \cdot P_{NL}(\omega,\, x,\, y,\, z)\, dx\, dy\, dz \right] \quad (6)$$

where integration is performed all over an optical fiber. Using signal light intensities I_i, I_j, and I_k, (6) is rewritten as

$$I = \frac{256\pi^4\omega^2}{n_r^4 c^2} (DX_{1111})^2 \frac{I_i I_j I_k}{A_{\mathrm{eff}}} \frac{\exp(-\alpha z)}{\alpha^2 + \Delta\beta^2} \cdot \left\{ [1 - \exp(-\alpha z)]^2 + 4 \exp(-\alpha z) \sin^2 \frac{\Delta\beta z}{2} \right\}. \quad (7)$$

Here, A_{eff} is an effective core area which is defined as

$$A_{\mathrm{eff}} = \left[\int N^2(x,\, y)\, dx\, dy \right]^2 \left[\int N^4(x,\, y)\, dx\, dy \right]^{-1} \quad (8)$$

where $N(x,\, y)$ is a normalized mode profile.

Introducing an efficiency $\eta \, (0 \leq \eta \leq 1)$ defined as

$$\eta = \frac{I(z, \Delta\beta)}{I(z, \Delta\beta = 0)} = \frac{\alpha^2}{\alpha^2 + \Delta\beta^2} \cdot \frac{[1 - \exp(-\alpha z)]^2 + 4 \exp(-\alpha z) \sin^2 \dfrac{\Delta\beta z}{2}}{[1 - \exp(-\alpha z)]^2} \quad (9)$$

the time-averaged intensity of an FWM light I is given by

$$I = \frac{256\pi^4\omega^2}{n_r^4 c^2} (DX_{1111})^2 \frac{I_i I_j I_k}{A_{\mathrm{eff}}} \exp(-\alpha z) \cdot \frac{[1 - \exp(-\alpha z)]^2}{\alpha^2} \eta \equiv I_{ijk}. \quad (10)$$

Finally, from (5), a difference in wave numbers $\Delta\beta$ is expressed as

$$\Delta\beta = \beta(f) - \beta(f_i) - \beta(f_j) + \beta(f_k) \quad (11)$$

by using a wave number $\beta(f)$ at an FWM light frequency $f = f_{ijk}$. Here, f_i, f_j, and f_k are light frequencies of signals. When $\Delta\beta$ is small enough, $\beta(f)$ is expanded around a zero-dispersion frequency f_0, and $\Delta\beta$ is given by [7]

$$\Delta\beta = -\frac{\lambda^4 \pi}{c^2} \frac{dD_c}{d\lambda} [(f_i - f_0) + (f_j - f_0)] \cdot (f_i - f_k)(f_j - f_k) \quad (12)$$

where λ is a wavelength of an FWM light and $dD_c/d\lambda$ is a derivative dispersion coefficient of an optical fiber. In this paper, (9), (10), and (12) are used for analysis.

In the following, it is assumed that $I_i = I_j = I_k$ and $dD_c/d\lambda$ is constant. As a result, time-averaged intensity I is given by a product of an efficiency η and a degenerate coefficient D. For simplicity, time-averaged FWM light intensity $I = I_{ijk}$ in (10) is normalized as follows:

$$\begin{aligned} I_{iik} &= \eta/4 \qquad (i \neq k) \\ I_{ijk} &= \eta \qquad (i \neq j \neq k). \end{aligned} \quad (13)$$

B. Frequency Allocation of Channels and FWM

Frequency allocations of ES and US channels are briefly reviewed, which is followed by an explanation of a frequency allocation of RUS channels.

1) Frequency Allocation of ES Channels: A frequency allocation of ES channels has signal lights with equal frequency separations between adjacent signals as shown in Fig. 2(a). Using a channel separation Δf_c and the number of channels N_{ES}, a total bandwidth B_{ES} is written as

$$B_{\mathrm{ES}} = (N_{\mathrm{ES}} - 1)\Delta f_c. \quad (14)$$

Since Δf_c is constant for each channel, a lot of FWM lights, whose frequency are coincident with those of signal lights, are generated. From (1), frequencies of FWM lights generated within a signal bandwidth always agree with those of signals.

2) Frequency Allocation of US Channels: A frequency allocation of US channels has signal lights whose frequency separations are different in every two signals as shown in Fig. 2(b), and does not have FWM signals whose frequencies are coincident with those of signal lights. This reason is as follows: From (1), a frequency f_{ijk} of an FWM light is related to frequencies f_i, f_j, and f_k as

$$f_{ijk} - f_i = f_j - f_k. \quad (15)$$

In US frequency allocation, we have $f_j \neq f_k$ for every signal. As a result, we always obtain $f_{ijk} \neq f_i$ for all signals.

When we consider US channels, it is convenient to introduce a slot spacing Δf which is defined as a minimum frequency unit.

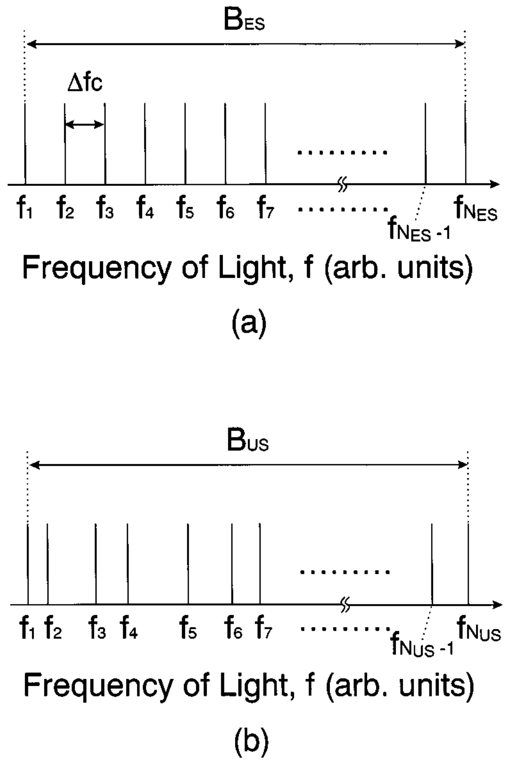

Fig. 2. Frequency allocations of: (a) equally spaced (ES) channels and (b) unequally spaced (US) channels. Here, B_{ES} and B_{US} are total bandwidths of signal lights for ES and US channels, respectively, and Δf_c is a channel spacing of ES. A frequency of a channel i is indicated as f_i.

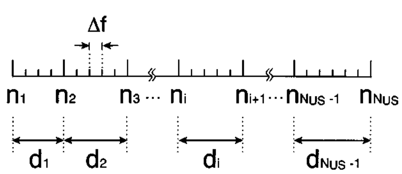

Fig. 3. Definition of a slot. A slot spacing Δf is defined as a minimum frequency unit. A frequency position of channel i is indicated by a slot $n_i(i = 1, 2, \ldots, N_{\mathrm{US}})$, and a spacing between the adjacent channels is written as $d_i = n_{i+1} - n_i(i = 1, 2, \ldots, N_{\mathrm{US}} - 1)$.

As shown in Fig. 3, a frequency position of channel i is indicated by a slot $n_i(i = 1, 2, \ldots, N_{\mathrm{US}})$. Similarly, a spacing between the adjacent channels is written as $d_i(i = 1, 2, \ldots, N_{\mathrm{US}} - 1)$. They are related as

$$d_i = n_{i+1} - n_i(i = 1, 2, \ldots, N_{\mathrm{US}} - 1). \tag{16}$$

An allocation $\tilde{n}$ is considered as a vector with the elements of $n_i(i = 1, 2, \ldots, N_{\mathrm{US}})$, and an allocation spacing $\tilde{d}$ as a vector with the elements of $d_i(i = 1, 2, \ldots, N_{\mathrm{US}} - 1)$, which are written as

$$\tilde{n} = \{n_1, n_2, \ldots, n_{N_{\mathrm{US}}} \tag{17}$$

$$\tilde{d} = [d_1, d_2, \ldots, d_{N_{\mathrm{US}}-1}]. \tag{18}$$

Frequency allocations of US channels are constructed as follows:

Step 1) When the number of channels is N_{US} and a minimum slot spacing is $\Delta n = n_2 - n_1$ as shown in Fig. 3, a test frequency spacing $\tilde{d}$ is taken as

$$\tilde{d} = [\Delta n, \Delta n + 1, \ldots, \Delta n + N_{\mathrm{US}} - 2]. \tag{19}$$

Step 2) If we put n_1 as an origin, a test allocation $\tilde{n}$ is obtained as

$$n_{i+1} = n_i + d_i \tag{20}$$

$$n_1 = 0 \tag{21}$$

where (16) is used.

Step 3) Slot spacings between any two channels in a test allocation $\tilde{n}$ are examined. If we have an equal slot spacing, we will go to Step 4). When every slot spacing is different each other, a test allocation itself is a US frequency allocation.

Step 4) Shuffle each element d_i of $\tilde{d}$ and make a new test allocation spacing $\tilde{d}$, then go to Step 3).

Note that if we have $N_{\mathrm{US}} = P+1$ where P is a prime number, the channel spacings are all distinct and the channel bandwidth is guaranteed to be optimal [10]. As a result, Step 4), which is basically a computer exhaustive search, can be eliminated by using an algorithm in [10].

Here, we consider an example with $N_{\mathrm{US}} = 6$ and $\Delta n = 5$. If a test allocation spacing

$$\tilde{d} = [5, 6, 7, 8, 9] \tag{22}$$

is taken, we have a US allocation

$$\tilde{n} = \{0, 5, 11, 18, 26, 35\} \tag{23}$$

from (20) and (21).

A total signal bandwidth B_{US} of US channels is given by

$$B_{\mathrm{US}} = n_N \cdot \Delta f = \sum_{i=1}^{N_{\mathrm{US}}-1} d_i \cdot \Delta f \tag{24}$$

where we put $n_1 = 0$.

Note that B_{US} has a minimum value $\min(B_{\mathrm{US}})$, and from a definition of US channels, $d_i \neq d_j\,(i \neq j)$ and $d_i \geq \Delta n = \Delta f_c/\Delta f$ must be simultaneously satisfied. As a result, $\min(B_{\mathrm{US}})$ is written as

$$\begin{aligned}\min(B_{\mathrm{US}}) &= (\Delta n + (\Delta n + 1) + \cdots + (\Delta n + N_{\mathrm{US}} - 2))\Delta f \\ &= (N_{\mathrm{US}} - 1)\left\{\Delta f_c + \left(\frac{N_{\mathrm{US}}}{2} - 1\right)\Delta f\right\}\end{aligned} \tag{25}$$

from (24).

When the number of channels N_{US} increases, it becomes hard to find a US frequency allocation. It is also difficult to achieve a narrow total bandwidth B_{US}, because $\min(B_{\mathrm{US}})$ is approximately proportional to N_{US}^2 for a large N_{US} as shown in (25).

3) Frequency Allocation of RUS Channels: As described above, ES channels have a lot of FWM lights with the frequencies coincident with those of signals, and US channels have a wide signal bandwidth for a large number of channels.

In contrast to these problems, RUS channels analyzed here can simultaneously achieve a few FWM lights with the signal frequencies and a narrow signal bandwidth even for a large number of channels.

A frequency allocation of RUS channels is formed by repeating a US frequency allocation which is a base unit, as shown in Fig. 4. Here, a maximum frequency of a base unit is the same as a minimum frequency of the following base unit.

Note that a base unit is not only a US frequency allocation, but also must satisfy the following conditions. Consider a US frequency allocation $\tilde{n}_{\mathrm{US}}$ with the number of channels N_{US}. When an allocation $\tilde{n}$ is formed by repeating this US frequency allocation, every constituent allocation, which has consecutive N_{US} channels, have to become a US frequency allocation, too.

An example of steps to form an RUS frequency allocation is as follows.

Step 1) As a test base unit, form a US frequency allocation $\tilde{n}_{\mathrm{US}}$ with the number of channels N_{US} and an adjacent channel spacing $\tilde{d}_{\mathrm{US}}$.

Step 2) A spacing $\tilde{d}$ between base units has $(N-1)$ elements, and is obtained from $\tilde{d}_{\mathrm{US}}$ as

$$\tilde{d} = [d_1,\, d_2,\, \ldots,\, d_{N_{\mathrm{US}}-1},\, d_1,\, d_2,\, \ldots,\, d_{N_{\mathrm{US}}-1},\, d_1,\, \ldots]. \quad (26)$$

Step 3) Construct a new allocation spacing by extracting consecutive $(N_{\mathrm{US}}-1)$ elements from $\tilde{d}$. Among them, the number of spacings, which are independent each other, is $(N_{\mathrm{US}}-1)$, and its spacings are

$$\tilde{d}^{(1)} = [d_1,\, d_2,\, d_3,\, \ldots,\, d_{N_{\mathrm{US}}-1}] \quad (27)$$

$$\tilde{d}^{(2)} = [d_2,\, d_3,\, d_4,\, \ldots,\, d_1]$$

$$\vdots \quad (28)$$

$$\tilde{d}^{(N_{\mathrm{US}}-1)} = [d_{N_{\mathrm{US}}-1},\, d_1,\, d_2,\, \ldots,\, d_{N_{\mathrm{US}}-2}]. \quad (29)$$

Following this step, an allocation $\tilde{n}^{(i)}(i = 1, 2, \ldots, N_{\mathrm{US}}-1)$ is obtained from $\tilde{d}^{(i)}(i = 1, 2, \ldots, N_{\mathrm{US}}-1)$ where we used (20) and (21).

Step 4) If all allocations $\tilde{n}^{(i)}(i = 1, 2, \ldots, N_{\mathrm{US}}-1)$ in Step 3) are US, $\tilde{n}_{\mathrm{US}}$ in Step 1) is used as a base unit, and the allocation $\tilde{n}$ becomes an RUS frequency allocation.

Step 5) If $\tilde{n}^{(i)}(i = 1, 2, \ldots, N_{\mathrm{US}}-1)$ is not US, form another US allocation which is different from $\tilde{n}_{\mathrm{US}}$, and repeat Steps 2)–4).

If we use the algorithm in [10], Step 5) is not needed, because the unequal-spaced condition is automatically satisfied.

Here, we will form an RUS frequency allocation with the number of channels $N = 12$. As a test base unit with $N_{\mathrm{US}} = 6$, we take

$$\tilde{n}_{\mathrm{US}} = \{0,\, 5,\, 11,\, 18,\, 26,\, 35\}. \quad (30)$$

If we put a spacing between the base units as

$$\tilde{d} = [5,\, 6,\, 7,\, 8,\, 9,\, 5,\, 6,\, 7,\, 8,\, 9,\, 5] \quad (31)$$

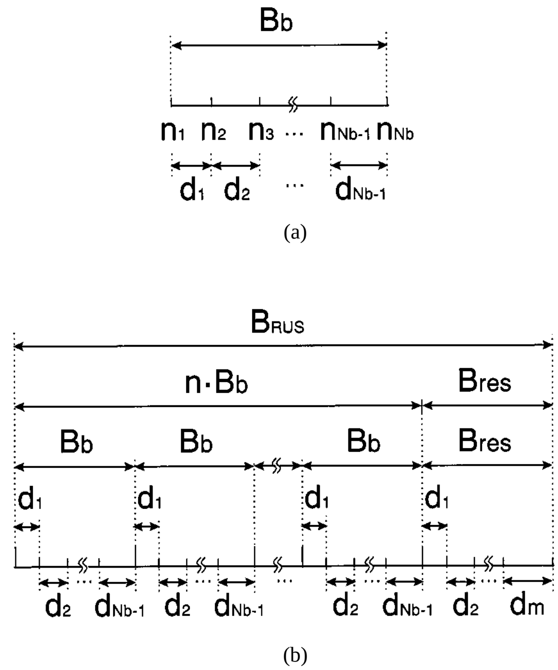

Fig. 4. Frequency allocation of RUS channels. (a) A US allocation which is used as a base unit with a bandwidth of B_b and the number of channels N_b and (b) RUS channels consisting of n base units shown in (a) and additional m channels. A total bandwidth of signal lights is given by $B_{\mathrm{RUS}} = nB_b + B_{res}$ where B_{res} is a bandwidth of additional m channels.

new unit spacings $\tilde{d}^{(i)}$ and allocations $\tilde{n}^{(i)}$ are as follows:

$$\begin{aligned} \tilde{d}^{(1)} &= [5,\, 6,\, 7,\, 8,\, 9], & \tilde{n}^{(1)} &= \{0,\, 5,\, 11,\, 18,\, 26,\, 35\} \\ \tilde{d}^{(2)} &= [6,\, 7,\, 8,\, 9,\, 5], & \tilde{n}^{(2)} &= \{0,\, 6,\, 13,\, 21,\, 30,\, 35\} \\ &\vdots \\ \tilde{d}^{(5)} &= [9,\, 5,\, 6,\, 7,\, 8], & \tilde{n}^{(5)} &= \{0,\, 9,\, 14,\, 20,\, 27,\, 35\}. \end{aligned} \quad (32)$$

Since they are all US frequency allocations, $\tilde{n}_{\mathrm{US}}$ is used as a base unit. From $\tilde{d}$, an RUS frequency allocation

$$\tilde{n} = \{0,\, 5,\, 11,\, 18,\, 26,\, 35,\, 40,\, 46,\, 53,\, 61,\, 70,\, 75\} \quad (33)$$

was obtained.

Next, we consider an RUS frequency allocation which is formed by repeating base units by n times and adding m channels. The number of channels N_{RUS} is given by

$$N_{\mathrm{RUS}} = n(N_b - 1) + 1 + m \quad (34)$$

where N_b is the number of channels for the base unit, and n and m are written as

$$n = \left\lfloor \frac{N_{\mathrm{RUS}} - 1}{N_b - 1} \right\rfloor \quad (35)$$

$$m = (N_{\mathrm{RUS}} - 1) \bmod (N_b - 1). \quad (36)$$

A total bandwidth of signal lights B_{RUS} is given by

$$B_{\mathrm{RUS}} = nB_b + B_{\mathrm{res}} \quad (37)$$

as shown in Fig. 4. Here, B_b is a signal bandwidth of a base unit, and B_{res} is a bandwidth of additional m channels.

A total bandwidth $B_{\rm RUS}$ approximately increases linearly with the number of channels $N_{\rm RUS}$, because RUS is formed by repeating base units periodically. The first term of (37) is in proportion to $(N_b - 1)$, and the second term never exceeds a bandwidth B_b of a base unit. If $N_{\rm RUS} \gg N_b$, the first term is dominant in (37), and we have $B_{\rm RUS} \simeq nB_b$.

III. Calculated Results and Discussions

Frequency allocation, intensity of FWM lights, and a signal bandwidth of ES and US are compared with those of RUS quantitatively.

A. Comparison with ES

1) Frequency Allocation: ES channels have the simplest frequency allocation. The parameters of the frequency allocation are only the number of channels $N_{\rm ES}$ and a channel spacing Δf_c. When the number of channel is increased by one, we have only to add another channel at a frequency separated by Δf_c from the previous allocation.

Fig. 5 shows an example of ES frequency allocation with the number of channels $N = 12$ and a channel spacing $\Delta f_c = 140$ GHz. The total bandwidth of signal lights $B_{\rm ES}$ is 1540 GHz. Each channel frequency f_i is summarized in Table I as $F_i = f_i - f_1$.

A frequency allocation of RUS channels is also simply formed, once a base unit is obtained. When we add a channel, we only have to follow the allocation of a base unit. The key parameters are the number of channels $N_{\rm RUS}$, a total bandwidth $B_{\rm RUS}$, and a base unit. When we construct a base unit, a minimum channel spacing Δf_c and a slot spacing Δf have to be considered as well as N_b and B_b. Note that any consecutive N_b channels must construct US frequency allocations.

In this paper, Allocation 1 in Table IV is used as a base unit, which has $N_b = 6$, $\Delta f_c = 100$ GHz, $\Delta f = 20$ GHz, and $B_b = 700$ GHz. Fig. 6 shows an example of RUS frequency allocation with the number of channels $N_{\rm RUS} = 12$. Each channel frequency $F_i = f_i - f_1$ and a frequency spacing $\Delta f_i = f_{i+1} - f_i$ are summarized in Table II. This allocation consists of two equal base units and one additional channel, and the total bandwidth $B_{\rm RUS}$ is 1500 GHz.

2) Intensity of FWM Lights: An FWM light with the same frequency f_n as that of a channel n becomes a noise. In general, there exist plural FWM lights which have the same frequency as that of a signal light, and an FWM light frequency is expressed as

$$f_n = f^n_{\rm FWM(i)} \qquad (i = 1, 2, \ldots, m) \tag{38}$$

where m is a positive integer.

A total intensity $I^n_{\rm FWM}$ of FWM lights with a light frequency f_n is given by

$$I^n_{\rm FWM} = \sum_{i=1}^{m} I^n_{\rm FWM(i)} \tag{39}$$

where $I^n_{\rm FWM(i)} (i = 1, 2, \ldots, m)$ is an intensity of each FWM light.

Relations between a total intensity and a light frequency of FWM lights for RUS and ES frequency allocations are shown in Figs. 7 and 8, respectively. Here, the numbers of channels are equal ($N_{\rm ES} = N_{\rm RUS} = 12$), and the total signal bandwidths are approximately equal ($B_{\rm ES} \cong B_{\rm RUS} = 1500$) GHz. It is assumed that a wavelength of a channel 1 is $\lambda_1 = 1550$ nm, an optical fiber length is $L = 80$ km, a decay rate of an optical fiber is $\alpha = 0.2$ dB/km, and a derivative dispersion coefficient of an optical fiber is $dD_c/d\lambda = 0.07$ ps/km/nm^2. The horizontal line shows a difference in light frequencies $F = f_{\rm FWM} - f_0$ where f_0 is a zero-dispersion frequency, which is set at a midpoint of a total bandwidth, and the vertical line shows a total FWM light intensity $I^n_{\rm FWM}$. Closed circles correspond to FWM lights whose frequency are coincident with those of signals, and

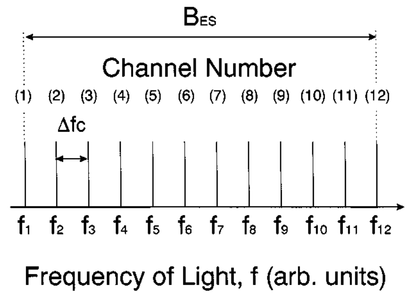

Fig. 5. An example of ES frequency allocations. Each channel frequency f_i is summarized in Table I as $F_i = f_i - f_1$. The number of channels is $N = 12$, and a channel spacing is $\Delta f_c = 140$ GHz. The total bandwidth of signal lights $B_{\rm ES}$ is 1540 GHz.

TABLE I
An Example of ES Frequency Allocations

Channel	1	2	3	4	5	6	7	8	9	10	11	12
F_i (GHz)	0	140	280	420	560	700	840	980	1120	1260	1400	1540

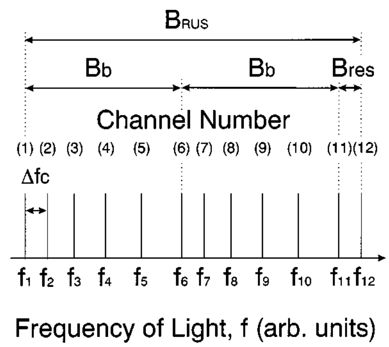

Fig. 6. An example of RUS frequency allocations with the number of channels $N_{\rm RUS} = 12$. This allocation consists of two equal base units and one additional channel. Each channel frequency $F_i = f_i - f_1$ and a frequency spacing $\Delta f_i = f_{i+1} - f_i$ are summarized in Table II. The total bandwidth $B_{\rm RUS}$ is 1500 GHz.

TABLE II
An Example of RUS Frequency Allocations

Channel	1	2	3	4	5	6	7	8	9	10	11	12
F_i (GHz)	0	100	220	360	520	700	800	920	1060	1220	1400	1500
Δf_i (GHz)	100	120	140	160	180	100	120	140	160	180	100	

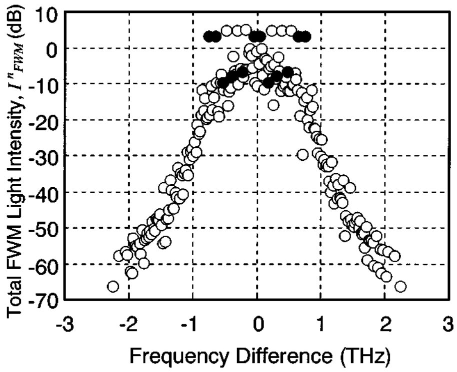

Fig. 7. Relation between a total intensity and a light frequency of FWM lights in an RUS frequency allocation.

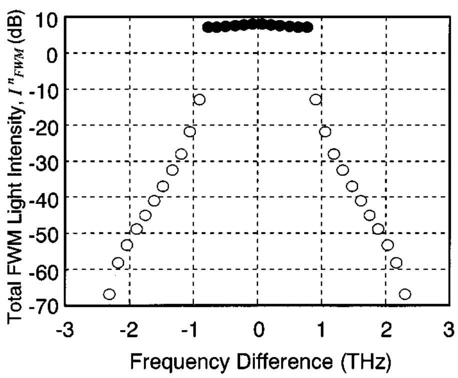

Fig. 8. Relation between a total intensity and a light frequency of FWM lights in an ES frequency allocation.

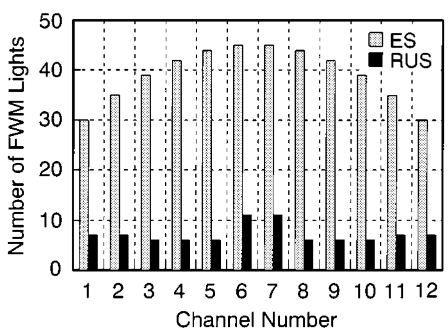

Fig. 9. The number of FWM lights generated at channel positions in RUS and ES frequency allocations. Gray bars correspond to ES, and black ones to RUS, respectively.

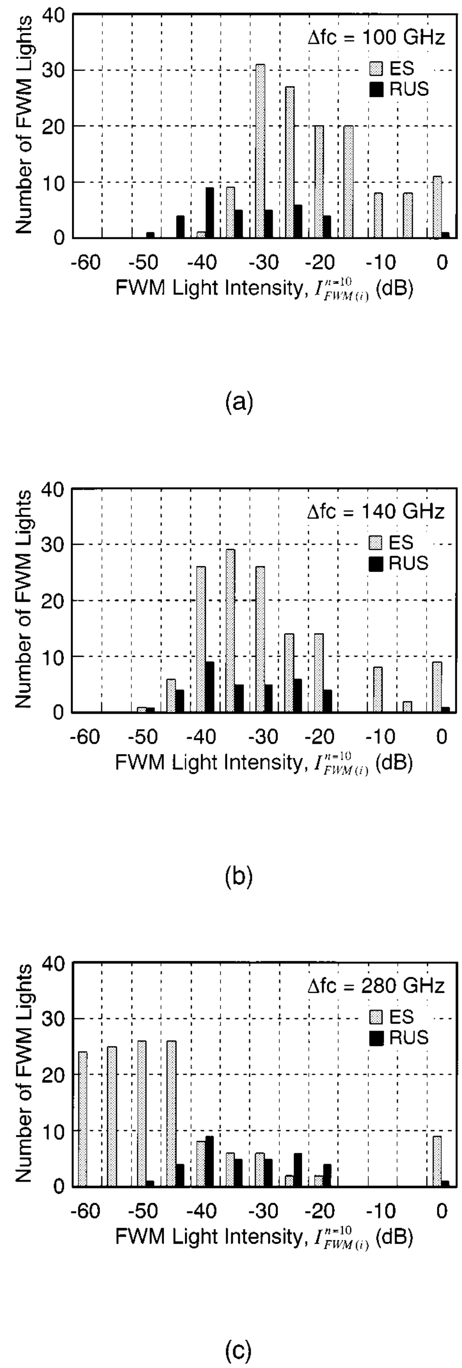

Fig. 10. Relations between the number and a total light intensity of FWM lights in ES and RUS frequency allocations. Here, the number of channels is 20, and FWM lights generated at a position of channel 10, which is a midpoint of frequency allocations, are considered. Gray bars correspond to ES, and the black ones to RUS in Table II, respectively: (a) $\Delta f_c = 100$ GHz, (b) $\Delta f_c = 140$ GHz ($B_{\rm ES} \simeq B_{\rm RUS}$), and (c) $\Delta f_c = 280$ GHz in ES frequency allocations.

the light frequencies of open circles do not agree with those of signals.

It is found that $I^n_{\rm FWM}$'s of RUS are less than half of those of ES from Figs. 7 and 8. This is because the number of FWM lights which satisfy $f_n = f^n_{\rm FWM(i)}(i = 1, 2, \ldots, m)$ and each intensity $I^n_{\rm FWM(i)}(i = 1, 2, \ldots, m)$ of RUS are both smaller than those of ES, which will be explained in the following. The number of FWM lights generated at signal light frequencies are shown in Fig. 9. The horizontal line shows a channel number n, and the vertical line shows the number of FWM lights. Gray bars correspond to ES, and black ones to RUS, respectively. It is easily found that RUS has a smaller number of FWM lights at channel positions than ES.

Fig. 10 shows a relation between a total light intensity and the number of FWM lights in ES and RUS frequency allocations. Here, the number of channels is 20, and an FWM light which has a light frequency f_{10} of a channel 10 is considered. The reason why we consider channel 10, which is located at a midpoint of frequency allocations, is that a midpoint channel has the largest number of FWM lights among all the constituent channels. The horizontal line shows a FWM light intensity $I^{n=10}_{\rm FWM(i)}$. The vertical line shows the number of FWM lights. Gray bars

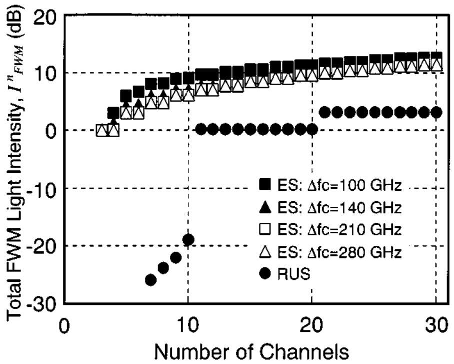

Fig. 11. Dependence of total intensity of FWM lights on the number of channels in ES and RUS frequency allocations. Closed squares, closed triangles, open squares, and open triangles correspond to $\Delta f_c = 100$, 140, 210, and 280 GHz, respectively, in ES frequency allocations. Closed circles show the calculated results for the RUS frequency allocation in Table II.

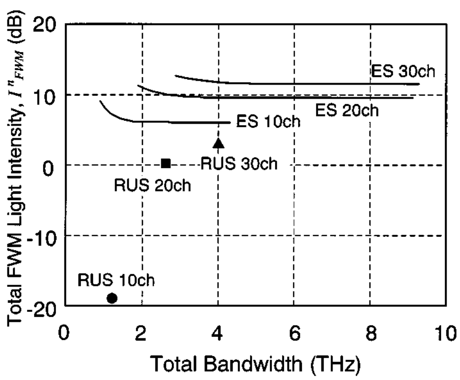

Fig. 12. Relation between a total intensity of FWM lights and a total bandwidth of signals. Three solid lines correspond to $N_{\rm ES} = 10, 20$, and 30 in ES frequency allocations. Closed circles, squares, and triangles correspond to $N_{\rm RUS} = 10, 20$, and 30, respectively, in RUS frequency allocations.

correspond to ES, and the black ones to the RUS in Table II, respectively. Fig. 10(a) corresponds to $\Delta f_c = 100$ GHz, (b) $\Delta f_c = 140$ GHz ($B_{\rm ES} \simeq B_{\rm RUS}$), and (c) $\Delta f_c = 280$ GHz in ES frequency allocations.

As shown in Fig. 10(a), the number and intensities of FWM lights of RUS are lower than those of ES. As a result, RUS has a smaller total FWM light intensity than ES. With an increase in a channel spacing, both $I_{{\rm FWM}(i)}^{n=10} (i = 1, 2, \ldots, m)$ and $I_{\rm FWM}^{n=10}$ in ES frequency allocation decrease as shown in Fig. 10(b). However, even when $B_{\rm ES} \simeq B_{\rm RUS}$, an FWM light intensity of ES is larger than that of RUS. If a channel spacing is still larger as shown in Fig. 10(c), ES has a lower FWM light intensity than RUS. In this case, an ES bandwidth $B_{\rm ES}$ is approximately twice as large as an RUS bandwidth $B_{\rm RUS}$.

When the number of channels increases, the number m of generated FWM lights also becomes large. Thus, a total FWM light intensity $I_{\rm FWM}^n$ is enhanced according to (39), as shown in Fig. 11. The horizontal line shows the number of channels, and the vertical line shows a total FWM light intensity $I_{\rm FWM}^n$ at a mid channel in each frequency allocation. Here, closed squares, closed triangles, open squares, and open triangles correspond to $\Delta f_c = 100$ GHz, 140 GHz, 210 GHz, and 280 GHz, respectively, in ES frequency allocations. Closed circles show the calculated results for the RUS frequency allocation in Table II.

When a total bandwidth of RUS is equal to or less than that of ES, a total FWM light intensity $I_{\rm FWM}^n$ of RUS is lower than that of ES. When Δf_c equals the minimum channel spacing of RUS such as $\Delta f_c = 100$ GHz, $I_{\rm FWM}^n$'s of RUS are less than one-seventh of those of ES. When total bandwidths are approximately common in ES and RUS, i.e., $B_{\rm ES} \cong B_{\rm RUS}$ ($\Delta f_c = 140$ GHz in ES), $I_{\rm FWM}^n$'s of RUS are less than one-fifth of those of ES.

Both in ES and RUS frequency allocations, total FWM intensities saturate with an increase in the number of channels, because light intensities of FWM lights, which are generated by newly added channels, are lower than those of the old channels. This reason is that a frequency difference $|f_{\rm new} - f|$ between a center frequency f and a frequency $f_{\rm new}$ of a newly added channel is larger than those for old channels.

3) Total Bandwidth of Signal Lights: A total bandwidth of ES and that of RUS are compared by taking a total FWM light intensity into account. Fig. 12 shows a relation between a total FWM light intensity $I_{\rm FWM}^n$ and a total bandwidth of signals. The horizontal line shows a total bandwidth of signal lights, and the vertical line shows a total FWM light intensity. Three solid lines correspond to $N_{\rm ES} = 10$, 20, and 30 in ES frequency allocations. Here, a total bandwidth is changed by altering a channel spacing. Closed circles, squares, and triangles correspond to $N_{\rm RUS} = 10$, 20, and 30, respectively, in RUS frequency allocations.

When the number of channels is common in ES and RUS frequency allocations, both FWM light intensity $I_{\rm FWM}^n$ and total bandwidth of ES are larger than those of RUS. When the total bandwidths of signal lights are common in ES and RUS frequency allocations, ES has more than 7 times as large a total FWM light intensity as RUS. Thus, it is concluded that RUS is superior to ES both in a total bandwidth of signal lights and a total intensity of FWM lights.

B. Comparison with US

1) Frequency Allocation: For a frequency allocation of US channels, it is important to design the number of channels $N_{\rm US}$, a minimum channel spacing Δf_c, a total bandwidth of signal lights $B_{\rm US}$, and a slot spacing Δf. A total bandwidth $B_{\rm US}$ has a minimum value $B_{\rm US}^{\rm min}$, which is a guideline for a most suitable US frequency allocation.

Fig. 13 shows an example of US frequency allocations with $N_{\rm US} = 12$, $\Delta f_c = 100$ GHz, and $\Delta f = 20$ GHz. The total bandwidth of signal lights is 2200 GHz which equals $B_{\rm US}^{\rm min}$. Each channel frequency $F_i = f_i - f_1$ and a frequency spacing $\Delta f_i = f_{i+1} - f_i$ are summarized in Table III.

Since RUS is formed by repeating a US base unit with a relatively narrow B_b, a total bandwidth can be less than that of US.

2) Intensity of FWM Lights: US does not have FWM lights whose frequencies are coincident with those of signal lights. On the contrary, RUS has FWM lights with the frequencies of signal lights. However, with an increase in a base unit bandwidth B_b, intensity of FWM lights is reduced.

3) Total Bandwidth of Signal Lights: As shown in (25), a minimum total bandwidth $B_{\rm US}^{\rm min}$ has a term proportional to $N_{\rm US}^2$. Thus, a total bandwidth of US drastically increases with an increase in the number of channels in contrast to ES and RUS frequency allocations.

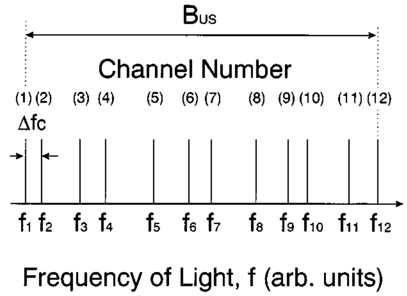

Fig. 13. An example of US frequency allocation. Here, $N_{\rm US} = 12$, $\Delta f_c = 100$ GHz, and $\Delta f = 20$ GHz. The total bandwidth of signal lights is 2200 GHz which equals $B_{\rm US}^{\rm min}$. Each channel frequency $F_i = f_i - f_1$ and a frequency spacing $\Delta f_i = f_{i+1} - f_i$ are summarized in Table III.

TABLE III
AN EXAMPLE OF US FREQUENCY ALLOCATIONS

Channel	1	2	3	4	5	6	7	8	9	10	11	12
F_i (GHz)	0	100	340	500	800	1020	1160	1440	1640	1760	2020	2200
Δf_i (GHz)	100	240	160	300	220	140	280	200	120	260	180	

Fig. 14 shows dependence of a total bandwidth on the number of channels in US and RUS. The horizontal line shows the number of channels. The vertical line shows a bandwidth expansion factor which is defined as $B/B_{\rm ES}$. Here, B is a total bandwidth of US or RUS, and $B_{\rm ES}$ is that of ES. Open circles corresponds to US frequency allocation. Closed circles, squares, and triangles correspond to Allocations 1, 2, and 3 whose frequency F_i (GHz) of channel i are shown in Table IV of RUS frequency allocations, respectively. Note that the channel spacing of ES equals that of the minimum channel spacing of US and RUS where $\Delta f_c = 100$ GHz. When a total bandwidth of signals is given, the number of channels increases as $B/B_{\rm ES}$ approaches 1.

In US frequency allocation, $B_{\rm US}^{\rm min}/B_{\rm ES}$ is linearly dependent on the number of channels N, because a term proportional to N^2 is dominant in $B_{\rm US}^{\rm min}$ and $B_{\rm ES}$ is in proportion to N.

In RUS frequency allocation, $B_{\rm US}^{\rm min}/B_{\rm ES}$ saturates with an increase in N, and the saturated value is given by

$$\lim_{N_{\rm RUS}\to\infty} \frac{B_{\rm RUS}}{B_{\rm ES}} = \frac{B_b}{(N_b - 1)\Delta f_c}. \tag{40}$$

When Allocation 1 is used as a base unit, this saturated value is only 1.4. When the number of channels increases up to 30, US has $B/B_{\rm ES}$ of 3.75. Thus, RUS has only 0.37 times as large a total signal bandwidth as US for 30 channels. This result indicates that a total bandwidth of RUS is relatively narrow, which is suitable for lightwave transmission systems even for a large number of channels, when compared with that of US.

Finally, a relation between a total bandwidth and a base unit is explained by using Allocations 1–3 as examples. When the base units have a common slot spacing Δf_c, a bandwidth expansion factor $B_{\rm RUS}/B_{\rm ES}$ is in proportion to N_b, which is the number of channels of a base unit. This reason is that B_b and $B_{\rm ES}$ are proportional to N_b^2 and N_b, respectively for a large N_b. As a result, Allocation 1 with the minimum N_b among these three base units has the lowest $B_{\rm RUS}/B_{\rm ES}$.

It is concluded that RUS is superior to US in a total bandwidth of signal lights for a large number of channels.

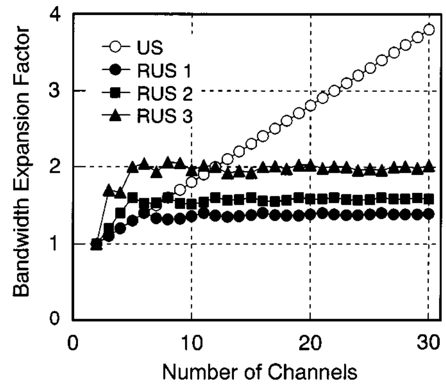

Fig. 14. Dependence of a total bandwidth on the number of channels. RUS 1, 2, and 3 use Allocation 1, 2, and 3, whose frequencies are summarized in Table IV, as a base unit, respectively.

TABLE IV
EXAMPLE OF BASE FREQUENCY ALLOCATIONS

Channel	1	2	3	4	5	6	7	8	9	10	11	12
Allocation 1	0	100	220	360	520	700						
Allocation 2	0	100	240	420	640	760	920	1120				
Allocation 3	0	100	340	500	800	1020	1160	1440	1640	1760	2020	2200

TABLE V
COMPARISON OF ES, US, AND RUS FREQUENCY ALLOCATIONS

Frequency allocation	ES	US	RUS
Total intensity of FWM lights	high	none	low
Total bandwidth of signal lights	narrow	wide	narrow

IV. SUMMARY

RUS frequency allocation was theoretically examined, and compared with ES and US.

By comparing the characteristics of RUS with those of ES, it is found that RUS is superior to ES both in a total bandwidth of signal lights and a total intensity of FWM lights. For example, in a lightwave transmission system with the number of channels of 12, a total bandwidth of 1500 GHz, an oscillation wavelength of 1550 nm, an optical fiber length $L = 80$ km, a decay rate of $\alpha = 0.2$ dB/km, and a derivative dispersion coefficient $dD_c/d\lambda = 0.07$ ps/km/nm^2, RUS has total FWM light intensity less than half of that of ES. When the total bandwidths of signal lights are common in ES and RUS frequency allocations, RUS has a small total FWM light intensity, which is less than one-seventh of that of ES.

By comparing the characteristics of RUS with those of US, it is revealed that RUS is superior to US in a total bandwidth of signal lights for a large number of channels. When the number of channels increases up to 30, there exist RUS which has only 0.37 times as large a total signal bandwidth as US.

The results obtained in this paper are qualitatively summarized in Table V. Since FDM lightwave transmission systems require low FWM light intensity and narrow bandwidth simultaneously, RUS frequency allocation is considered to be a good candidate for FDM channels.

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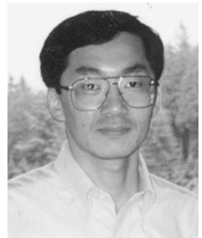

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