Effect of Velocity Mismatch and Microwave Attenuation on Time-Domain Response of Traveling-Wave Electrooptic Modulators

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***Abstract*—The dynamic response of a traveling-wave (TW) electrooptic phase modulator is treated using a Fourier transform technique. An integral expression for the induced phase shift which takes into account the optical-microwave velocity mismatch and the microwave attenuation is derived. In the case of a Gaussian modulating pulse and negligible microwave attenuation, the temporal dependence of the modulated pulse amplitude can be expressed in terms of error functions. Calculated pulse shapes showing the transition from a Gaussian to a flat top output pulse with increasing phase mismatch in the absence of microwave attenuation are presented. The effect of microwave attenuation on pulse shape, amplitude, and width is also explored. The method used to obtain these results is generally applicable to the time-domain analysis of TW modulators.**

***Index Terms*—Electrooptic, integrated optics, modulators, pulse, phase, traveling-wave, velocity matching.**

I. Introduction

INTERFEROMETRIC electrooptic modulators in lithium niobate (LN) are widely used in digital communication systems operating at 2.5 and 10 Gb/s and in analog systems for cable television. Not only are rise and fall times <10 ps achieved with this technology, but interferometric designs provide the chirp free performance needed for long-distance transmission. These devices utilize a traveling-wave (TW) configuration in which the modulating microwave signal propagates in a strip line or coplanar waveguide on the surface of the insulating substrate in the same direction as the modulated light wave [1], [2]. Present practice for the highest bandwidths ($\gg$1 GHz) is to use very thick ($\approx$15–30 μm) electrodes to achieve velocity matching by increasing the microwave propagation speed to match that of the optical carrier [3]–[5].

In spite of the widespread interest in such modulators and the recognized importance of velocity matching and microwave attenuation, however, it appears that a quantitative analysis of modulator response in the time domain has not been reported. This paper presents the results of such an analysis in which general expressions for the temporal dependence of the modulated pulse amplitude are derived. These results are applied in obtaining a closed form expression for the case of a Gaussian modulating pulse in the absence of microwave attenuation, and in exploring the effect of microwave attenuation on modulator response.

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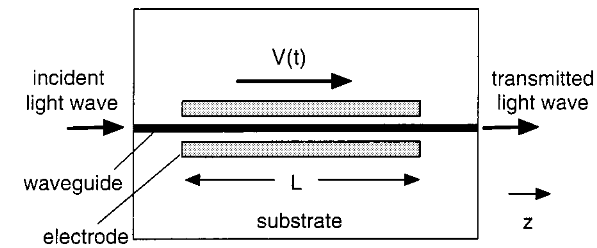

Fig. 1. Traveling-wave electrooptic phase modulator.

II. TW Modulator Response Model

The model assumes a uniform electrode and waveguide structure in the modulation region extending from $z = 0$ to $z = L$, as in Fig. 1. The TW modulating pulse produces a refractive index change $\Delta n(z,t)$, $0 \leq z \leq L$. Analysis of the phase change in the modulated light wave makes use of the Fourier transform of the input pulse $A(\omega_m)$, given by

$$A(\omega_m) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \Delta n(0,t) e^{-i\omega_m t}\, dt \tag{1}$$

with ω_m the radian frequency component of the modulating microwave signal. The refractive index change is given by the inverse Fourier transform of $A(\omega_m)$, which can be written

$$\Delta n(z,t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} A(\omega_m) e^{i(\omega_m t - \beta_m z) - \alpha'_m(\omega_m) z}\, d\omega_m \tag{2}$$

with β_m the propagation constant and $\alpha'_m(\omega_m)$ the microwave amplitude attenuation constant. Expressing the dependence of β_m on ω_m as

$$\beta_m = \omega_m / v_m \tag{3}$$

with v_m, the microwave group velocity, given by

$$v_m = \left(\frac{\partial \beta_m}{\partial \omega_m}\right)^{-1} \tag{4}$$

it follows that

$$\Delta n(z,t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} A(\omega_m) e^{i\omega_m \left(t - \frac{z}{v_m}\right) - \alpha'_m(\omega_m) z}\, d\omega_m. \tag{5}$$

Thus, the refractive index change propagates along the waveguide at the microwave group velocity.

To relate the refractive index change in the modulating microwave signal to the phase change it induces in the optical wave, we start with the scalar wave equation

$$\frac{\partial^2 a}{\partial z^2} = \frac{n^2}{c^2}\frac{\partial^2 a}{\partial t^2} \tag{6}$$

with a the complex electric field amplitude for the waveguide mode, n the effective refractive index for that mode, and c the free-space speed of light. The solution to (6) can be written

$$a = a_0 e^{i(\omega t - \beta z) - i\phi} \tag{7}$$

with a_0 a real constant equal to the magnitude of the field, ω the radian frequency of the optical carrier, β the propagation constant of the optical wave, and ϕ the phase change in the optical wave due to the electrooptic modulation. The propagation constant is related to the radian frequency by

$$\beta = \frac{n_0 \omega}{c} \tag{8}$$

with n_0 the effective refractive index of the optical waveguide mode in the absence of modulation.

Since the refractive index change in (5) is expressed as an integral over spectral components, it is convenient to express the induced phase change as a function of frequency and position, i.e., $\phi(z, \omega + \omega_m)$. The modulated refractive index can then be expressed in terms of its spectral components as

$$n(z, t, \omega_m) = n_0 + \frac{A(\omega_m) e^{i\omega_m \left(t - \frac{z}{v_m}\right) - \alpha'_m(\omega_m) z}}{\sqrt{2\pi}}. \tag{9}$$

It will be assumed that the change in n is much less than n_0. Then, substituting (7) and (9) into (6), differentiating, neglecting $\partial^2\phi/\partial z^2$, and canceling common terms yields

$$\frac{\partial \phi(z, t, \omega + \omega_m)}{\partial z} = \frac{\omega A(\omega_m)}{\sqrt{2\pi} c} e^{i\omega_m (t - z/v_m) - \alpha_m(\omega_m) z} \tag{10}$$

where v, the optical group velocity, is given by

$$v = \left(\frac{\partial \beta}{\partial \omega}\right)^{-1} \tag{11}$$

and $t = z/v$. Equation (10) thus gives a spectral decomposition of the optical phase change. By analogy with (5), the net change in phase $\Delta\phi(L, t)$ for the modulated output is given by

$$\Delta\phi(L, t) = \frac{\omega}{\sqrt{2\pi} c} \int_{-\infty}^{\infty} B(\omega_m) e^{i\omega_m (t - L/v)}\, d\omega_m \tag{12}$$

where

$$B(\omega_m) = A(\omega_m) \int_0^L e^{-i\omega_m \mu z - \alpha'_m(\omega_m) z}\, dz \tag{13}$$

and the velocity mismatch factor μ is given by

$$\mu = \frac{1}{v_m} - \frac{1}{v}. \tag{14}$$

The integral in (13) is evaluated as

$$\int_0^L e^{-i\omega_m \mu z - \alpha'_m(\omega_m) z}\, dz = \frac{e^{-i\omega_m \mu L - \alpha'_m(\omega_m) z} - 1}{-i\omega_m \mu - \alpha'_m(\omega_m)}. \tag{15}$$

In the case that microwave attenuation can be neglected, $(\alpha'_m(\omega_m) \equiv 0)$, (12) becomes

$$\Delta\phi(L, t) = \frac{\omega}{\sqrt{2\pi} c} [G(t - t_1) - G(t - \Delta t - t_1)] \tag{16}$$

where $t_1 = L/v$, $\Delta t = \mu L$, and $G(t)$ is defined as

$$G(t) = \int_{-\infty}^{\infty} \frac{A(\omega_m) e^{i\omega_m t}}{i\omega_m \mu} d\omega_m. \tag{17}$$

To facilitate the evaluation of $G(t)$, it is noted that

$$\sqrt{2\pi} \Delta n(0, t) = \mu \frac{dG(t)}{dt} \tag{18}$$

where, from (2)

$$\Delta n(0, t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} A(\omega_m) e^{i\omega_m t}\, d\omega_m. \tag{19}$$

Integrating this expression from $-\infty$, where $\Delta n(0, t)$ vanishes under the assumption of a finite modulating pulse, yields

$$G(t) = \frac{\sqrt{2\pi}}{\mu} \int_{-\infty}^{t} \Delta n(0, t)\, dt. \tag{20}$$

The electrooptic phase shift can be determined using (16) and (20), for the case that microwave attenuation can be neglected. In the next section, these results are applied to the case of a Gaussian modulating pulse.

III. Modulator Response to a Gaussian Modulating Pulse

A Gaussian modulating pulse produces a refractive index change for $z = 0$ given by

$$\Delta n(0, t) = \Delta n_0 e^{-t^2/\tau^2} \tag{21}$$

where the maximum refractive index change is Δn_0 and the full width of the modulating microwave pulse to e^{-1} electric field amplitude is 2τ.

For the case that microwave attenuation can be neglected, it follows from (20), with the substitution $\xi = t/\tau$, that

$$G(t) = \frac{\sqrt{2\pi} \tau \Delta n_0}{\mu} \int_{-\infty}^{t/\tau} e^{-\xi^2}\, d\xi. \tag{22}$$

The integral in (22) can be written as

$$\int_{-\infty}^{t/\tau} e^{-\xi^2}\, d\xi = \frac{\sqrt{\pi}}{2} [1 + \mathrm{erf}(t/\tau)] \tag{23}$$

where the error function $\mathrm{erf}(x)$ is defined as

$$\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-y^2}\, dy. \tag{24}$$

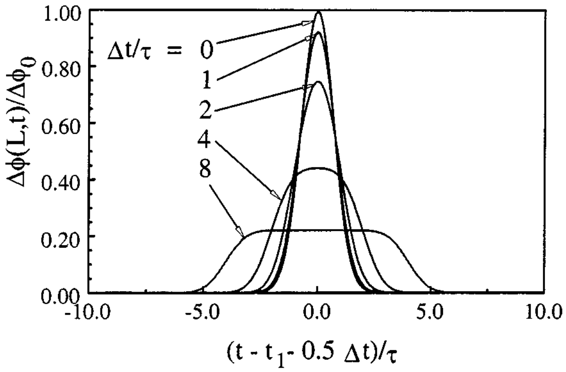

Fig. 2. Temporal dependence of modulated phase shift for $\Delta t/\tau = 0, 1, 2, 4$, and 8 and no microwave attenuation. The plot is normalized with respect to the maximum phase shift for perfect velocity matching and no microwave attenuation $\Delta\phi_0$. The horizontal axis (time) variable $(t - t_1 - 0.5\Delta t)/\tau$ is offset so that the plots will be symmetric about the vertical axis.

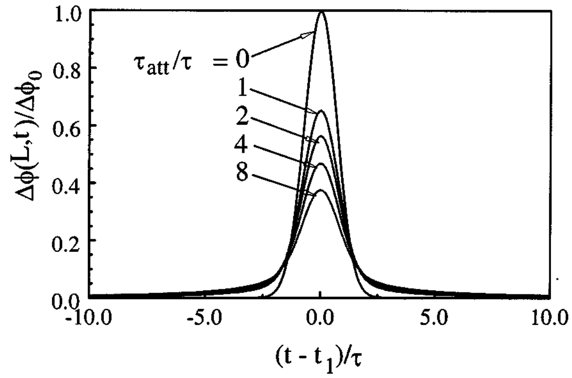

Fig. 3. Temporal dependence of modulated phase shift with no velocity mismatch for values of the microwave attenuation factor $\tau_{\mathrm{att}}/\tau = 0, 1, 2, 4$, and 8.

Finally, combining the results in (16), (22), (23), and (24), it follows that

$$\Delta\phi(L) = \frac{\sqrt{\pi}\tau\Delta\phi_0}{2\mu L}\left[\mathrm{erf}\left(\frac{t - t_1}{\tau}\right) - \mathrm{erf}\left(\frac{t - \Delta t - t_1}{\tau}\right)\right] \tag{25}$$

with $\Delta\phi_0$, the phase shift which would occur in the case of perfect velocity matching ($\mu = 0$), given by $\Delta\phi_0 = \omega\Delta n_0 L/c$.

Plots of the temporal modulator response calculated from (25) are given in Fig. 2 for the cases that $\Delta t/\tau = 0, 1, 2, 4$ and 8, where $\Delta t = \mu L$ is the microwave/optical delay difference over the modulator length L due to velocity mismatch.

If microwave attenuation cannot be neglected, the closed-form result of (25) is no longer applicable and the temporal response of the modulator must be calculated numerically from (12), (13), and (15), with $A(\omega_m)$, the Fourier transform of (21), given by

$$A(\omega_m) = \frac{\Delta n_0 \tau}{\sqrt{2}} e^{-\omega_m^2 \tau^2/4}. \tag{26}$$

The product of microwave attenuation and modulator length, which is taken into account via (15), can be written

$$\alpha'_m(\omega_m)L = \sqrt{\tau_{\mathrm{att}}|\omega_m|} \tag{27}$$

for the case that the microwave attenuation is proportional to the square-root of the microwave frequency (skin-effect loss)—generally a good approximation in the frequency range from a few tens of megahertz to a few tens of gigahertz. The factor τ_{att} can be evaluated from a known value of the microwave attenuation coefficient α_0 using (32) in the Appendix.

Plots of the temporal modulator response calculated numerically are given in Fig. 3 for the case of perfect velocity matching ($\Delta t = 0$), for $\tau_{\mathrm{att}}/\tau = 0, 1, 2, 4$, and 8. Fig. 4(a) and (b) plots the response for cases in which velocity mismatch and microwave attenuation are both present.

Dependence on $\Delta t/\tau$ of peak modulation amplitude, and pulse width at 10% of peak amplitude, are plotted in Figs. 5 and 6, respectively, for $\tau_{\mathrm{att}}/\tau = 0, 1, 2, 4$, and 8.

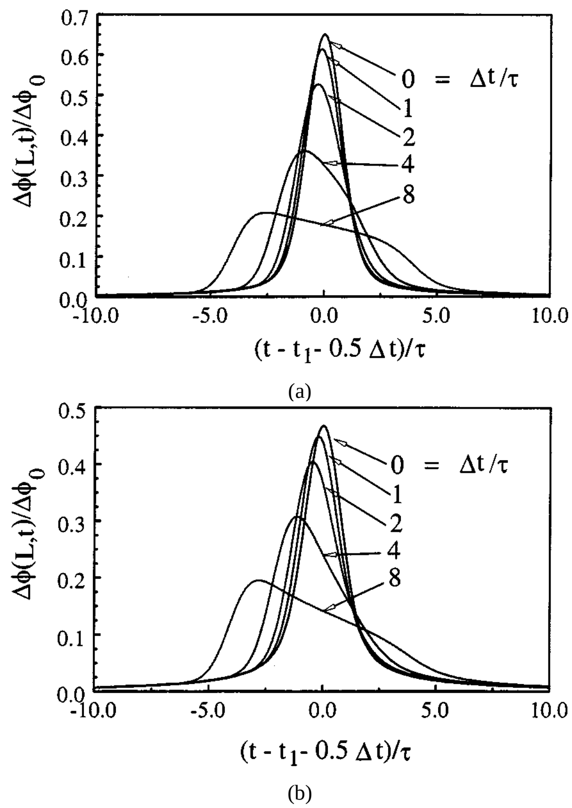

Fig. 4. Temporal dependence of modulated phase shift for $\Delta t/\tau = 0, 1, 2, 4,$, and 8: (a) $\tau_{\mathrm{att}}/\tau = 1$ and (b) $\tau_{\mathrm{att}}/\tau = 4$.

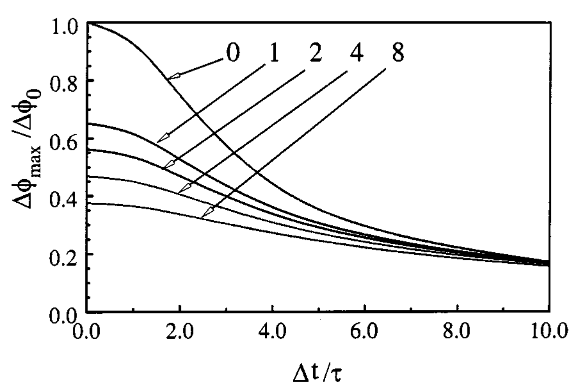

Fig. 5. Dependence on $\Delta t/\tau$ of the peak amplitude of the modulated pulse $\Delta\phi_{\max}$, normalized with respect to the maximum phase shift for perfect velocity matching and no microwave attenuation $\Delta\phi_0$, for $\tau_{\mathrm{att}}/\tau = 0, 1, 2, 4$, and 8.

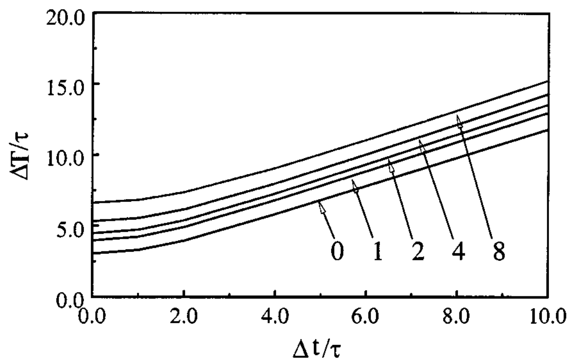

Fig. 6. Dependence of the temporal width ΔT of the modulated pulse on $\Delta t/\tau$, with ΔT the full width measured at a modulation depth of 10% of the peak amplitude, for $\tau_{\rm att}/\tau = 0, 1, 2, 4$ and 8.

IV. Discussion

The closed form expression in (25) is, to the best of our knowledge, the first such result for the response of a TW electrooptic modulator. This result is useful in cases where the effect of velocity mismatch dominates over microwave attenuation and dispersion. As an example, we consider a modulator in LN with thin electrodes operating at a wavelength near 1.5 μm. In that case, the optical and microwave group velocities are given, respectively, by $v = c/N$ and $v_m = c/N_m$, with N and N_m the optical and microwave group refractive indices. It follows from (14) that

$$\mu = \frac{N_m - N}{c}. \tag{28}$$

With $N = 2.18, N_m = 4.22$, and $c = 3.0 \times 10^{10}$ cm/s, it is calculated that $\mu = 68$ ps/cm. For an incident pulse with $\tau = 25$ ps, the curves in Fig. 2 correspond, respectively, to $L = 0, 0.37$ cm, 0.74 cm, and 1.47 cm, and 2.94 cm.

To account for microwave attenuation, the value of $\tau_{\rm att}$ can be calculated from (32) of the Appendix if the microwave power attenuation coefficient α_0 is known. For example, with $\alpha_0 =$ 0.6 dB/cm-$\sqrt{\rm GHz}$ [3] and $L = 1$ cm, it follows from (32) that $\tau_{\rm att} = 0.76$ ps. Thus, for $\tau = 25$ ps, $\tau_{\rm att}/\tau = 1$ for a modulator length $L = 5.7$ cm. Using the same value of α_0 but with τ decreased to 5 ps, $L = 2.56$ cm for $\tau_{\rm att}/\tau = 1$.

From Figs. 2 and 3, it is evident that, for a Gaussian modulating pulse, the output pulse shape remains symmetric for the case of velocity mismatch with no microwave attenuation, or microwave attenuation with no velocity mismatch. However, the output pulse becomes asymmetric when both velocity mismatch and microwave attenuation are present. The asymmetry results from the larger modulating voltage amplitude near $z = 0$, where the leading edge of the pulse is modulated the strongest, than near $z = L$, where the effect is greatest on the trailing edge.

V. Conclusion

A general method for treating time-domain response of TW electrooptic modulators has been presented. The method is applied in the derivation of closed-form expressions for the temporal dependence of modulated pulse amplitude for the case of a Gaussian modulating pulse in the absence of microwave attenuation. The pulse broadening has been expressed in terms of $\Delta t/\tau$, the ratio of the velocity mismatch delay to the e^{-1} halfwidth of the incident pulse amplitude. The temporal plots illustrate the transition in the modulated pulse shape from Gaussian to flat top with increasing velocity mismatch. The effect of microwave attenuation has also been treated using the general method by means of numerical integration.

Appendix
Relation Between $\tau_{\rm att}$ and Microwave Power Attenuation Coefficient α_0

It is assumed that the microwave power attenuation in the modulator electrodes is proportional to the product of the square root of the microwave frequency f_m and propagation distance along the electrodes z. The power attenuation constant α_0 has units dB/cm-$\sqrt{\rm GHz}$. In calculating the effect of microwave attenuation on modulator response, it is necessary to relate α_0 to the constant $\tau_{\rm att}$ used in the model calculations. If we write

$$V_m(z) = V_0 e^{-\alpha'_m(\omega_m)z} \tag{29}$$

for the dependence of the modulating voltage $V_m(z)$ on distance along the modulator, with V_0 a constant, it follows that the

$$\alpha_0 \sqrt{f_m} z = -20 \log_{10}[V_m(z)/V_m(0)]. \tag{30}$$

It follows from (29) and (30) that

$$\alpha'_m(\omega_m) z = \alpha_0 \sqrt{f_m} z / 8.68. \tag{31}$$

If we now recall

$$\alpha'_m(\omega_m) L = \sqrt{\tau_{\rm att}|\omega_m|} \tag{27}$$

with L in cm and $|\omega_m| = 2\pi f_m$, it follows from (31) that

$$\tau_{\rm att} = 2.11(\alpha_0 L)^2 \text{ ps}. \tag{32}$$

Using this expression in conjunction with (12), (13), (15), and (26), a quantitative calculation can be made of the effect of microwave attenuation on modulator response.

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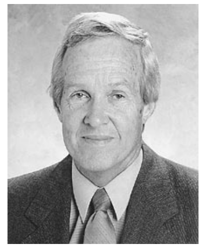

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