Mode Classification in Cylindrical Dielectric Waveguides

Achint Kapoor and G. S. Singh

***Abstract*—We discuss an analytical approach which leads to a global scheme for mode classification in two- and three-layers step-profile cylindrical dielectric waveguides, based on the requirements of analytical continuity HE $\rightarrow$ TE and EH $\rightarrow$ TM in the limit such that the system under consideration becomes a circular metallic wave-guide. Technically, HE and EH hybrid modes correspond to the two roots of the problem's quadratic characteristic equation. It turns out that the mode designation has the universality in the sense that the equation obtainable from the root involving positive (negative) sign always describes HE (EH) mode.**

***Index Terms*—Dielectric rods, dielectric tubes, dielectric waveguides, mode classification, step-index fibers, W-type fibers**
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I. INTRODUCTION

THE information regarding the mode classification of the guided electromagnetic waves are of paramount significance in microwave and optical communications. The axially symmetric modes in cylindrical dielectric waveguides are transverse electric (TE) and transverse magnetic (TM) which preserve the condition $E_z = 0$ or $H_z = 0$ in course of their propagation. But the nonsymmetric modes are hybrid ones; they are superpositions of TE and TM fields, due to the fact that both Debye potentials E_z and H_z are needed to construct angular-dependent solutions in cylindrical dielectric waveguides. These superpositions are named as HE or EH according to whether the TE or TM term dominates in a suitable norm. A number of workers [1]–[6] have made attempts to search out the systematics that could be utilized to associate a characteristic equation with HE or EH mode and thereby to investigate the details of the corresponding mode theoretically.

Out of the various proposals in [1]–[4], Snitzer's scheme in [3] is the most satisfactory one. This is based on the values of an amplitude coefficient ratio P which essentially accounts for the relative amount of E_z and H_z in the hybrid modes. Snitzer found that $P = -1$ for HE_{11} far from cutoff and hence he proposed to designate the modes giving $P = -1$ $(+1)$ as HE (EH). This sign convention was further used by Kuhn [7] to designate hybrid modes in a cladded optical fiber. Although the Snitzer's criterion works well in a dielectric rod, the investigation by Safaai-Jazi and Yip [6] has revealed that the sign of P changes in an arbitrary fashion in the cladding mode region and hence this scheme cannot be utilized to classify unambiguously the hybrid modes of a cladded fiber.

Consequent upon reviewing the works in [1]–[4] and [7] and finding that there was not any precise, well defined, and global scheme for the classification of hybrid modes in cylindrical waveguides, authors in [6] attempted to propose a new suitable scheme. They obtained the characteristic equation for a three-layers structure in the form of a quadratic equation using the approach which is basically the same as that adopted by Kapany and Burke [5] for the mode designation in a cylindrical dielectric rod. The characteristic equations corresponding to two roots of the quadratic equation were used to describe two hybrid modes; equation which gives dominant HE_{11} mode corresponds to the HE mode while the other one corresponds to the EH mode. The analysis in [6] yields that the characteristic equation with positive (negative) sign of the root should be used for HE (EH) mode in step-index and W-type fibers whereas negative (positive) sign should be used in dielectric tubes. Hence their scheme too fails sort of being global because the reversal of sign convention is taking place in their analysis in designating HE and EH modes for the two classes of optical fibers. Moreover, in their approach, one has first to identify the equation containing dominant HE_{11} mode and, therefore, numerical calculation becomes an essential component even for mode classification.

The purpose of our work is to develop a global scheme which can be utilized to classify the hybrid modes through an analytical approach in two- and three-layers step-profile cylindrical dielectric waveguides. We take recourse to three steps in our endeavor. First, we separate the dispersion relation for a three-layers structure into two distinct equations corresponding to two roots of a quadratic equation in terms of the Bessel function appearing in the outermost medium. We then substitute the appropriate conditions in the dispersion relation for cladded fibers (step-index or W-type fibers) or dielectric tubes to arrive at the characteristic equation for a dielectric rod. Finally, we impose the condition to get the well known TE and TM mode equations, see for example in [8], corresponding to a circular metallic waveguide. The physical basis for mode classification in our approach is the fact that in the limiting situation HE (EH) mode equation would yield TE (TM) mode condition because of the dominance of the magnetic (electric) field. This is inherently analogous to the idea that the hybrid modes on a circular fiber can be visualized using the concept of helical ray path as described, e.g., in the classic book by Snyder and Love [9], and can be classified according to the dominant polarization properties of the rays at the core boundary. Our final step has, of

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A. Kapoor is with the Department of Physics, Ranchi College, Ranchi 834008, India.
G. S. Singh is with the Department of Physics, University of Roorkee, Roorkee 247667, India.
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course, the essential ingredient as in that of Lee et al. [10] in which modes of a metallic cylinder coated with a thin dielectric layer is identified through the limiting case of a vanishingly thin coating leading to $\mathrm{HE}_{mn} \rightarrow \mathrm{TE}_{mn}$ and $\mathrm{EH}_{mn} \rightarrow \mathrm{TM}_{mn}$.

II. Basic Equations and Solutions

We consider a concentric three media cylindrical dielectric waveguide with the outermost medium, region III, extending up to infinity. The innermost and the middle cylindrical media, regions I and II, have radii "a" and "b," respectively. The permittivity and permeability of the three media are (ϵ_j, μ_j), $j = 1, 2, 3$. We choose a cylindrical polar coordinate system (ρ, ϕ, z) with z-axis taken as the guide axis. The complete translational invariance of the waveguide allows us to single out the z-dependence giving the form of (z, t) dependence as $\exp[i(\beta z - \omega t)]$, where ω is the angular frequency and β is the axial propagation constant to be determined by the interface boundary conditions.

A. Step-Index and W-Type Fibers

We consider the discrete index profile such that the z-component of the field satisfies the scalar wave equation. The step-index fiber is characterized by $\epsilon_1\mu_1 > \bar{\beta}^2 > \epsilon_2\mu_2 > \epsilon_3\mu_3$ whereas in a W-type fiber we have $\epsilon_1\mu_1 > \bar{\beta}^2 > \epsilon_3\mu_3 > \epsilon_2\mu_2$ with $\bar{\beta} = \beta/k_0$ as the normalized propagation constant and k_0 as the free-space propagation constant. The expressions for the z-components of the fields in these cladded optical fibers are expressed in the forms

$$E_z^1 = A_m^1 J_m(\alpha_1\rho)F_m; \quad H_z^1 = B_m^1 J_m(\alpha_1\rho)G_m \tag{1}$$

in the core (region I; $0 \leq \rho < a$),

$$\begin{aligned} E_z^2 &= \left[A_m^2 I_m(\alpha_2\rho) + \bar{A}_m^2 K_m(\alpha_2\rho)\right] F_m; \\ H_z^2 &= \left[B_m^2 I_m(\alpha_2\rho) + \bar{B}_m^2 K_m(\alpha_2\rho)\right] G_m \end{aligned} \tag{2}$$

in the cladding (region II; $a < \rho < b$) and

$$E_z^3 = \bar{A}_m^3 K_m(\alpha_3\rho)F_m; \quad H_z^3 = \bar{B}_m^3 K_m(\alpha_3\rho)G_m \tag{3}$$

in the jacket (region III; $b < \rho \leq \infty$).

Here

$$F_m = \cos(m\phi + \phi_0)\exp[i(\beta z - \omega t)], \tag{4}$$

$$G_m = \sin(m\phi + \phi_0)\exp[i(\beta z - \omega t)] \tag{5}$$

and

$$\alpha_j = k_0|\epsilon_j\mu_j - \bar{\beta}^2|^{1/2} \tag{6}$$

with $j = 1, 2, 3$ and ϕ_0 as a phase constant. Also, $J_m(x)$ is the Bessel function of the first kind, and $I_m(x)$ and $K_m(x)$ are the modified Bessel functions of the first and second kinds, respectively. $A_m^j, \bar{A}_m^j, B_m^j$ and $\bar{B}_m^j$ are the constants of integrations.

The cylindrical geometry of the system allows us to express [9] the transverse components of the fields in terms of E_z^j and H_z^j as

$$\begin{aligned} E_\rho^j &= \frac{i}{\alpha_j^2}\left(\beta\frac{\partial E_z^j}{\partial\rho} + \frac{\omega\mu_j}{\rho}\frac{\partial H_z^j}{\partial\phi}\right); \\ H_\rho^j &= \frac{i}{\alpha_j^2}\left(\beta\frac{\partial H_z^j}{\partial\rho} - \frac{\omega\epsilon_j}{\rho}\frac{\partial E_z^j}{\partial\phi}\right); \\ E_\phi^j &= \frac{i}{\alpha_j^2}\left(\frac{\beta}{\rho}\frac{\partial E_z^j}{\partial\phi} - \omega\mu_j\frac{\partial H_z^j}{\partial\rho}\right); \\ H_\phi^j &= \frac{i}{\alpha_j^2}\left(\frac{\beta}{\rho}\frac{\partial H_z^j}{\partial\phi} + \omega\epsilon_j\frac{\partial E_z^j}{\partial\rho}\right). \end{aligned} \tag{7}$$

The continuity of the tangential components $E_\rho^j, H_\rho^j, E_\phi^j$ and H_ϕ^j at the interface boundaries at $\rho = a$ as well as at $\rho = b$ leads to a set of eight linear homogeneous equations in unknown $A_m^j, \bar{A}_m^j, B_m^j$ and $\bar{B}_m^j$. The nontrivial solution of these equations gives us a determinantal equation whose zeros yield the cutoff values of the system under consideration. Now evaluating the determinant and rearranging the various terms we ultimately get, after a large amount of algebra, a quadratic equation in the form

$$R_1\eta_4^2 + R_2\eta_4 + R_3 = 0 \tag{8}$$

where

$$\begin{aligned} R_1 = \mu_3\epsilon_3 \big[&\gamma_3\alpha_2 b P_1(P_2 + P_3) \\ &+ \gamma_3^2\alpha_2^2 b^2(P_2P_3 - P_4P_5) + P_1^2\big] \end{aligned} \tag{9}$$

$$\begin{aligned} R_2 = \gamma_3\gamma_4\alpha_2 b&(\mu_2\epsilon_3 + \mu_3\epsilon_2)(P_2P_3 - P_4P_5) \\ &- P_1P_3(\mu_3\epsilon_2\eta_3\gamma_3\alpha_2 b - \mu_2\epsilon_3\gamma_4) \\ &- P_1P_2(\mu_2\epsilon_3\eta_3\gamma_3\alpha_2 b - \mu_3\epsilon_2\gamma_4) \\ &- \eta_3P_1^2(\mu_2\epsilon_3 + \mu_3\epsilon_2) \end{aligned} \tag{10}$$

and

$$\begin{aligned} R_3 = \mu_2\epsilon_2 &\left[\gamma_4^2(P_2P_3 - P_4P_5) - \gamma_4\eta_3P_1(P_2 + P_3) + \eta_3^2P_1^2\right] \\ &+ \xi_- P_1(\gamma_4 + \eta_3P_6)(\epsilon_2P_5 + \mu_2P_4) \\ &- \xi_-^2\left[P_1^2 + P_1P_6(P_2 + P_3)\right. \\ &\left.+ P_6^2(P_2P_3 - P_4P_5)\right] \end{aligned} \tag{11}$$

with

$$\begin{aligned} P_1 = (\mu_2\gamma_2 + \mu_1\eta_1\gamma_1\alpha_2 a)&(\epsilon_2\gamma_2 + \epsilon_1\eta_1\gamma_1\alpha_2 a) \\ &- (\chi\gamma_1\alpha_2 a)^2 \end{aligned} \tag{12}$$

$$P_2 = \chi^2\gamma_1\alpha_2 a + (\epsilon_2\eta_2 - \epsilon_1\eta_1)(\mu_2\gamma_2 + \mu_1\eta_1\gamma_1\alpha_2 a) \tag{13}$$

$$P_3 = \chi^2\gamma_1\alpha_2 a + (\mu_2\eta_2 - \mu_1\eta_1)(\epsilon_2\gamma_2 + \epsilon_1\eta_1\gamma_1\alpha_2 a) \tag{14}$$

$$P_4 = \chi[(\epsilon_2\gamma_2 + \epsilon_1\eta_1\gamma_1\alpha_2 a) + \gamma_1\alpha_2 a(\epsilon_2\eta_2 - \epsilon_1\eta_1)] \tag{15}$$

$$P_5 = \chi[(\mu_2\gamma_2 + \mu_1\eta_1\gamma_1\alpha_2 a) + \gamma_1\alpha_2 a(\mu_2\eta_2 - \mu_1\eta_1)] \tag{16}$$

and

$$P_6 = \alpha_2\gamma_3 b. \tag{17}$$

Also

$$\eta_1 = \frac{1}{\alpha_1 a}\frac{J'_m(\alpha_1 a)}{J_m(\alpha_1 a)};\quad \eta_2 = -\frac{1}{\alpha_2 a}\frac{I'_m(\alpha_2 a)}{I_m(\alpha_2 a)};$$
$$\eta_3 = -\frac{1}{\alpha_2 b}\frac{I'_m(\alpha_2 b)}{I_m(\alpha_2 b)};\quad \eta_4 = -\frac{1}{\alpha_3 b}\frac{K'_m(\alpha_3 b)}{K_m(\alpha_3 b)} \tag{18}$$

and

$$\gamma_1 = -\frac{1}{\alpha_2 a}\frac{K_m(\alpha_2 a)}{I_m(\alpha_2 a)};\quad \gamma_2 = -\frac{1}{\alpha_2 a}\frac{K'_m(\alpha_2 a)}{I_m(\alpha_2 a)};$$
$$\gamma_3 = -\frac{1}{\alpha_2 b}\frac{K_m(\alpha_2 b)}{I_m(\alpha_2 b)};\quad \gamma_4 = -\frac{1}{\alpha_2 b}\frac{K'_m(\alpha_2 b)}{I_m(\alpha_2 b)} \tag{19}$$

where

$$\chi = \frac{m\beta}{\omega a^2}\left(\frac{1}{\alpha_1^2}+\frac{1}{\alpha_2^2}\right);\quad \xi_\pm = \frac{m\beta}{\omega b^2}\left(\frac{1}{\alpha_2^2}\pm\frac{1}{\alpha_3^2}\right). \tag{20}$$

The characteristic equation for a cladded optical fiber is now obtained from (8) by solving the quadratic equation in η_4. We thus get

$$2R_1\eta_4 = -R_2 \pm \left(R_2^2 - 4R_1R_3\right)^{1/2}. \tag{21}$$

These two roots lead to two separate equations which correspond separately to the hybrid modes HE and EH of the system.

B. Dielectric Tubes

A dielectric tube is characterized by the condition that $\epsilon_2\mu_2 > \bar\beta^2 > \epsilon_1\mu_1 \geq \epsilon_3\mu_3$ or $\epsilon_2\mu_2 > \bar\beta^2 > \epsilon_3\mu_3 \geq \epsilon_1\mu_1$. The solutions for the three regions are now expressed in the forms

$$E_z^1 = C_m^1 I_m(\alpha_1\rho)F_m;\quad H_z^1 = D_m^1 I_m(\alpha_1\rho)G_m \tag{22}$$

in region I

$$E_z^2 = \left[C_m^2 J_m(\alpha_2\rho) + \bar C_m^2 Y_m(\alpha_2\rho)\right]F_m;$$
$$H_z^2 = \left[D_m^2 J_m(\alpha_2\rho) + \bar D_m^2 Y_m(\alpha_2\rho)\right]G_m \tag{23}$$

in region II and

$$E_z^3 = \bar C_m^3 K_m(\alpha_3\rho)F_m;\quad H_z^3 = \bar D_m^3 K_m(\alpha_3\rho)G_m \tag{24}$$

in region III.

Here the resultant equation obtained after substitution of the boundary conditions is expressed in the quadratic form as

$$S_1\eta_4^2 + S_2\eta_4 + S_3 = 0, \tag{25}$$

where

$$S_1 = \mu_3\epsilon_3\left[\delta_3\alpha_2 b Q_1(Q_2+Q_3) + \delta_3^2\alpha_2^2 b^2(Q_2Q_3+Q_4Q_5) - Q_1^2\right] \tag{26}$$

$$\begin{aligned}S_2 = &-\delta_3\delta_4\alpha_2 b(\mu_2\epsilon_3+\mu_3\epsilon_2)(Q_2Q_3+Q_4Q_5)\\ &- Q_1Q_3(\mu_3\epsilon_2\zeta_3\delta_3\alpha_2 b + \mu_2\epsilon_3\delta_4)\\ &- Q_1Q_2(\mu_2\epsilon_3\zeta_3\delta_3\alpha_2 b + \mu_3\epsilon_2\delta_4)\\ &+ \zeta_3 Q_1^2(\mu_2\epsilon_3+\mu_3\epsilon_2)\end{aligned} \tag{27}$$

and

$$\begin{aligned}S_3 = &\,\mu_2\epsilon_2\left[\delta_4^2(Q_2Q_3+Q_4Q_5) + \delta_4\zeta_3Q_1(Q_2+Q_3)\right.\\ &\left. - \zeta_3^2Q_1^2\right] + \xi_+Q_1(\delta_4-\zeta_3Q_6)(\epsilon_2Q_5+\mu_2Q_4)\\ &+ \xi_+^2\left[Q_1^2 - Q_1Q_6(Q_2+Q_3)\right.\\ &\left. + Q_6^2(Q_2Q_3-Q_4Q_5)\right]\end{aligned} \tag{28}$$

with

$$Q_1 = (\chi\delta_1\alpha_2 a)^2 - (\mu_2\delta_2 - \mu_1\zeta_1\delta_1\alpha_2 a) \times (\epsilon_2\delta_2 - \epsilon_1\zeta_1\delta_1\alpha_2 a) \tag{29}$$

$$Q_2 = \chi^2\delta_1\alpha_2 a - (\epsilon_2\zeta_2 - \epsilon_1\zeta_1)(\mu_2\delta_2 - \mu_1\zeta_1\delta_1\alpha_2 a) \tag{30}$$

$$Q_3 = \chi^2\delta_1\alpha_2 a - (\mu_2\zeta_2 - \mu_1\zeta_1)(\epsilon_2\delta_2 - \epsilon_1\zeta_1\delta_1\alpha_2 a) \tag{31}$$

$$Q_4 = \chi[(\epsilon_2\delta_2 - \epsilon_1\eta_1\gamma_1\alpha_2 a) - \delta_1\alpha_2 a(\epsilon_2\zeta_2 - \epsilon_1\zeta_1)] \tag{32}$$

$$Q_5 = \chi[(\mu_2\delta_2 - \mu_1\zeta_1\delta_1\alpha_2 a) - \delta_1\alpha_2 a(\mu_2\zeta_2 - \mu_1\zeta_1)] \tag{33}$$

and

$$Q_6 = \alpha_2\delta_3 b. \tag{34}$$

Also

$$\zeta_1 = -\frac{1}{\alpha_1 a}\frac{I'_m(\alpha_1 a)}{I_m(\alpha_1 a)};\quad \zeta_2 = \frac{1}{\alpha_2 a}\frac{J'_m(\alpha_2 a)}{J_m(\alpha_2 a)};$$
$$\zeta_3 = \frac{1}{\alpha_2 b}\frac{J'_m(\alpha_2 b)}{J_m(\alpha_2 b)} \tag{35}$$

and

$$\delta_1 = \frac{1}{\alpha_2 a}\frac{Y_m(\alpha_2 a)}{J_m(\alpha_2 a)};\quad \delta_2 = \frac{1}{\alpha_2 a}\frac{Y'_m(\alpha_2 a)}{J_m(\alpha_2 a)};$$
$$\delta_3 = \frac{1}{\alpha_2 b}\frac{Y_m(\alpha_2 b)}{J_m(\alpha_2 b)};\quad \delta_4 = \frac{1}{\alpha_2 b}\frac{Y'_m(\alpha_2 b)}{J_m(\alpha_2 b)}. \tag{36}$$

The counterpart of (21) is now obtained on simply replacing R_j by S_j.

III. Mode Classification

The classification into HE or EH mode equation of the two equations corresponding to any three-layers dielectric waveguide as contained in (8) or (25) is being done through a process in which at first we put the condition to get the characteristic equation of a dielectric rod. In case of cladded optical fibers, this can be achieved from (8) if one considers the limit $a \to b$ or $b \to a$ implying that the regions 1 and 2 become identical. The former limit was followed in [6] but the latter one turns out to be mathematically simpler approach and we adopt it here. We put $b = a$, $\epsilon_2 = \epsilon_1$ and $\mu_2 = \mu_1$. Hence we have $\alpha_2 = i\alpha_1$ giving $\gamma_3 = \gamma_1$, $\gamma_4 = \gamma_2$ and $\eta_3 = \eta_2 = \eta_1$. Thus each of χ and P_j $(j = 2, 3, 4, 5)$ vanishes, and (8) reduces to

$$(\mu_1\eta_1 - \mu_3\eta_4)(\epsilon_1\eta_1 - \epsilon_3\eta_4) = \left(\frac{m\beta}{\omega a^2}\right)^2\left(\frac{1}{\alpha_1^2}+\frac{1}{\alpha_2^2}\right)^2 \tag{37}$$

which is the well-known characteristic equation [3] for a dielectric rod.

In case of the dielectric tubes, we substitute $\epsilon_1 = \epsilon_2$ and $\mu_1 = \mu_2$, and thereby $\alpha_1 = i\alpha_2$. We then put $b = a$ and write α_1 in place of α_2. This procedure leads to $\zeta_1 = \zeta_2 = \zeta_3 = \eta_1$, $\delta_3 = \delta_1$ and $\delta_4 = \delta_2$. Thus, if we replace now μ_2 by μ_1 and ϵ_2 by ϵ_1, (37) is immediately retrieved from (25).

In order to proceed further, we find that (37) can be regarded as a quadratic equation in η_4 and gives

$$2\mu_3\epsilon_3\eta_4 = (\mu_3\epsilon_1 + \mu_1\epsilon_3)\eta_1 \pm \left[(\mu_3\epsilon_1 - \mu_1\epsilon_3)^2\eta_1^2 + 4\mu_3\epsilon_3\left(\frac{m\beta}{a\omega}\right)^2\left(\frac{1}{\alpha_1^2}+\frac{1}{\alpha_3^2}\right)^2\right]^{1/2}. \quad (38)$$

It is to be noted here that the above equations with $\pm$ signs can be obtained directly from those with $\pm$ signs in (21).

We now consider the condition such that a dielectric rod would become a circular metallic waveguide. When ϵ_3 is very large, the equation with positive $(+)$ sign of the second term on the right hand side in (38) yields

$$\mu_3\eta_4 = \mu_1\eta_1 \quad (39)$$

whereas that with negative $(-)$ sign gives

$$\epsilon_3\eta_4 = \epsilon_1\eta_1. \quad (40)$$

If we let $\epsilon_3 \to -\infty$ so that $\alpha_3 \to i\infty$, we have $\eta_4 \to 0$ and $\eta_4\epsilon_3 \to \infty$. Thus, (39) and (40) give, respectively, $\eta_1 \to 0$ and $\eta_1 \to \infty$. Now using the expression for η_1 given in (18), we accordingly find that

$$J'_m(\alpha_1 a) = 0 \quad (41)$$

and

$$J_m(\alpha_1 a) = 0 \quad (42)$$

which in turn are the well-known conditions [8] for TE and TM modes, respectively. Thus we observe that in the limiting situation the characteristic equation, which is obtained with the positive sign corresponding to the roots of (21) or (38), yields the condition for TE modes whereas that with the negative sign gives the condition for TM modes. This establishes dominance of magnetic (electric) field enshrined in the characteristic equation obtainable from the root with $+(-)$ sign.

IV. Summary

We have presented an approach which has resulted into a global scheme for classification of hybrid modes in all types of three-layers dielectric waveguides, like step-index fibers, W-type fibers and dielectric tubes together with that in dielectric rods. In this scheme, the characteristic equation with the positive (negative) sign corresponding to two roots of a quadratic equation always yields TE (TM) mode condition in the limiting case of a circular metallic waveguide and corresponds to HE (EH) mode equation due to dominance of the magnetic (electric) field. The beauty of our approach lies in the fact that there is uniqueness in the mode designation related to the sign convention for various types of waveguides while analytical continuity is preserved throughout the process of mode designation. Moreover, it has its physical basis and is not merely based on mathematical nicety.

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G. S. Singh, photograph and biography not available at the time of publication.