Comments on "Optical Amplifier Noise Figure Reduction for Optical Single-Sideband Signals"

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***Abstract*—In a recent paper[1] it was demonstrated that a noise figure below the so-called "quantum limit" is possible using single-sideband (SSB) modulation. In this comment we discuss the interpretation of this result and its implications for optical communications systems.**

***Index Terms*—Optical amplifiers, optical communication, optical fiber amplifiers, optical noise.**

GRIFFIN *et al.* have demonstrated that a theoretical noise figure of $F = 1$ (0 dB) is possible for an EDFA by using an optical single- sideband (SSB) modulation format for the signal. This result is in apparent violation of the commonly accepted "quantum limit" of $F = 2$ (3 dB). Furthermore, as we show in this comment, it is misleading in that it does not imply that the amplification process is noiseless, as would be expected by analogy with the definition of noise figure commonly used in electrical systems [1]. There has already been some debate regarding the definition of noise figure commonly used for optical amplifiers [2]–[4], and this recent result is likely to encourage further discussion. The purpose of this comment is to provide some clarification of the interpretation and application of the result. In particular, we review earlier theoretical analyses which show that a minimum noise figure of 0 dB for an SSB signal does not violate any fundamental principles; we show that the 0-dB noise figure does not imply noiseless amplification, but does imply benefit in terms of link budget and transmission power; and we discuss the implications of this for optical transmission system design. We conclude by suggesting that no change is necessary to the conventional definition of optical noise figure, but rather that care should be taken when applying the noise figure to the evaluation of the signal-to-noise ratio (SNR) of spectrally efficient modulation formats such as SSB.

Following Heffner [5], we restrict our discussion to the case of linear optical amplifiers, by which we mean devices which are phase-preserving linear multipliers of photon number. It is commonly believed that the noise figure of a linear amplifier has a minimum value, typically referred to as the "quantum limit," of $F = 2$. The term implies that this is a fundamental result, and indeed it was argued in [2] that at large gain the simultaneous measurement of in-phase and quadrature components of the electric field necessitates a doubling of the Heisenberg uncertainty, as shown in [6]. However, the "quantum limit" is not fundamental, and in fact it is shown in [5, eq. (30)] that, in the limit of high gain, the uncertainty in the received photon number and phase at the amplifier output can in principle approach the minimum value possible at the amplifier input. Furthermore, an example given in [7, p. 101] illustrates that it is possible to achieve $1 \leq F \leq 2$ in a highly inverted, short length of Er-doped fiber (i.e., in the low-gain regime). This example alone is sufficient to show that the "quantum limit" is not a fundamental result, but the consequence of a specific analysis viewed in the high-gain limit.

The conventional derivation of the noise figure proceeds as described by Griffin *et al.* (see footnote 1) and in more detail in [9, pp. 78–100]. Assuming an intensity-modulated signal, the standard procedure is to consider a CW light source incident on an ideal photodetector and to determine the statistical fluctuations in photoelectrons observed over an integration period of T seconds, corresponding to an electrical bandwidth of $B_e = 1/2T$. Prior to amplification the origin of these fluctuations is shot noise, whereas after amplification the dominant noise source is signal-spontaneous beat noise. The SNR is defined to be the ratio of the mean photocurrent squared (i.e., the electrical signal power) to the mean-squared fluctuations in the photocurrent, and the noise figure is the ratio of the input SNR to the output SNR. In all previous analyses of which we are aware (e.g., [2], [7]–[9]), it has been assumed that the intensity-modulated signal has an optical spectrum which is symmetrical about the optical carrier frequency (i.e., consists of two sidebands) and that consequently the minimum optical filter bandwidth following the amplifier to eliminate excess amplified spontaneous emission (ASE) noise is $B_o = 2B_e$. This assumption that the ASE noise power spectrum falling on the detector has a spectral width of at least $2B_e$ centered on the carrier frequency results in the well-known expression with a high-gain limit of $F = 2$. Furthermore, in the case of double sideband (DSB) signals, the result is unchanged if the optical filter bandwidth is reduced to reject the ASE noise in one-half of the optical spectrum, because although there is a consequent reduction of 3 dB in the signal-spontaneous beat noise, the detected signal power is also reduced by 3 dB so that there is no change in received SNR. Hereafter, we refer to the noise figure derived using this established procedure as the *conventional (optical amplifier) noise figure*.

The key point to be drawn from the above review is that there is no fundamental physical limit invoked by the "quantum limit" of the conventional optical noise figure. It is a simple conse-

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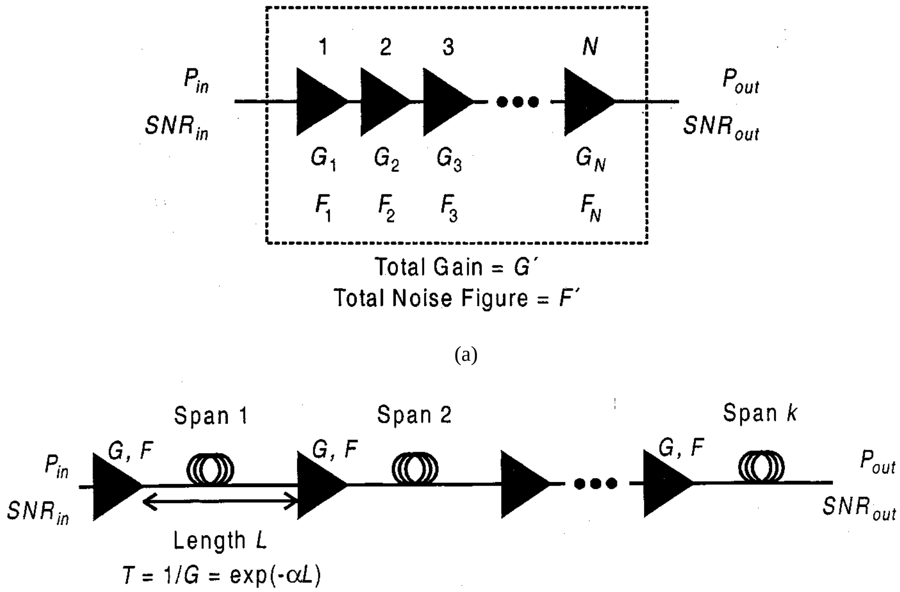

(b)

Fig. 1. Applications of the cascading formulas for the conventional optical noise figure. (a) A high-gain amplifier constructed from a cascade of individual lower gain amplifiers. (b) A "Type A" transparent transmission link constructed from k identical amplifiers of gain G and noise figure F connected by fiber spans with transmission $T = 1/G = \exp(-\alpha L)$.

quence of the particular configuration chosen for the analysis under which the noise figure is defined. The fundamental physical limit lies, in fact, at $F = 1$ which can be achieved in the high-gain limit if the amplifier is properly matched to the receiver and a suitable modulation scheme is used [5]. However, the conventional definition has proven useful because in most circumstances of practical interest it obeys cascading formulas similar to those of the standard electrical noise figure [4], [7]. In addition, it is a useful parameter for describing the quality of an optical amplifier in a signal-independent manner. However, the results presented by Griffin *et al.* (see footnote 1) highlight the fact that there exist situations in which the direct application of the conventional optical noise figure produces apparently anomalous results. In their example of SSB modulation, application of the conventional noise figure produces an output SNR which may be up to 3 dB below the actual SNR observed at the receiver. This illustrates the fact that application of the noise figure in systems employing novel and/or spectrally efficient modulation schemes (and, indeed, nonintensity-modulated schemes in general) should be approached with extreme caution.

The suggestion that a noise figure of $F = 1$ is theoretically possible is misleading because in order to obtain this result it is necessary to implicitly redefine the optical noise figure. As discussed above, the conventional definition is based on the assumption that the signal is intensity-modulated and therefore consists of an optical carrier and two symmetric sidebands. Performing the calculation for an SSB signal thus implicitly alters the definition, since the optical spectrum is no longer that of an intensity-modulated signal. Similarly, anomalous results could be obtained for other nonintensity-modulated signals, such as FM, PM, or AM signals with full or partial suppression of the optical carrier. The other reason why the unity minimum noise figure is misleading is that, by analogy with the electrical domain, the commonly understood implication of unity noise figure is that the amplifier adds no noise. This is clearly not the case for the optical amplifier—a "floor" of ASE noise power accompanies the signal at the output of the amplifier, regardless of the bandwidth or modulation format. Indeed, it is a fundamental result that a totally noiseless linear optical amplifier would violate Heisenberg's Uncertainty Principle [5].

We now discuss the practical interpretation and application of the result of Griffin *et al.* (see footnote 1). To illustrate our discussion, we consider the two specific examples illustrated in Fig. 1. The first is the case of a high-gain amplifier constructed from a cascade of N amplifiers, with individual gain $G_1, G_2, \ldots, G_N$ and noise figure $F_1, F_2, \ldots, F_N$, shown in Fig. 1(a). In the high-signal power regime, it is well known that the usual "microwave cascade" formula applies [4], [7, pp. 112–113], and thus the gain G' and noise figure F' of the cascade are given by

$$G' = G_1 G_2 \ldots G_N \tag{1}$$

$$F' = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} + \cdots + \frac{F_N - 1}{G_1 G_2 \ldots G_N}. \tag{2}$$

Equation (2) is derived simply by accumulating the ASE noise through the amplifier cascade, and thus for an optimally-filtered SSB signal the output SNR will be given by

$$\mathrm{SNR_{out}} = \frac{F'}{2}\,\mathrm{SNR_{in}} \tag{3}$$

where $\mathrm{SNR_{in}}$ is the shot-noise-limited input SNR. We note that the correct result for SSB modulation is thus obtained by first determining the overall conventional noise figure according to (2), and *then* dividing by two (subtracting 3 dB), and that dividing the noise figure of each amplifier by two first, and *then* applying (2) will give the wrong result.

Our second example, illustrated in Fig. 1(b), is the case of a "Type A" cascade of k identical amplifiers and transmission spans [7, p. 115] configured as a transparent link such that the gain G of each amplifier exactly compensates the transmission T of each span. If each amplifier has conventional high-signal noise figure F, then in the high-signal approximation, the overall noise figure of the link is [4]

$$F'(k) = kF - (k - 1). \tag{4}$$

If we apply the result of Griffin *et al.* to each individual amplifier, then for fully inverted amplifiers, we would obtain an overall noise figure for the link of $F'(k) = 1$, independent of k. This result is implausible, as there is no net gain and each amplifier adds an equal quantity of ASE noise in the signal bandwidth, and thus the SNR must degrade in each span. The correct result for SSB signals is obtained, as before, by

$$\mathrm{SNR_{out}} = \frac{F'}{2}\,\mathrm{SNR_{in}} = \frac{kF - (k-1)}{2}\,\mathrm{SNR_{in}} \tag{5}$$

where F is the conventional noise figure of each amplifier. Equation (5) shows that there are real advantages to be gained by using SSB modulation, as suggested by Griffin *et al.* (see footnote 1). The simplest way of generating an SSB signal is to filter out one sideband after conventional intensity modulation. While this does not improve the link budget for single-span transmission (since the 3-dB improvement in receiver sensitivity is spent in the 3-dB loss in signal power incurred in filtering out one sideband), it allows the signal to be launched into the transmission span with 3 dB lower power for the same received SNR as compared with conventional DSB modulation. In systems for which nonlinear propagation effects within and between channels may be significant, this presents a considerable advantage. Alternatively, if more sophisticated techniques for SSB generation can be implemented which do not intrinsically discard signal power (e.g., optimized versions of those described in [10], [11]), the link budget may be increased by up to 3 dB. Furthermore, other advantages of SSB transmission have been identified, such as improved immunity to chromatic dispersion in millimeter-wave distribution systems (see footnote 1) [11] and the ability to compensate for fiber dispersion in the signal after detection, in the electrical domain [10]. Clearly, SSB modulation also reduces the optical spectrum required by the signal, potentially allowing a greater number of more closely spaced channels for improved spectral efficiency in WDM systems.

In conclusion, we have argued that the recent result of Griffin *et al.* (see footnote 1), while of great interest to the optical communications community, does not necessitate any re-evaluation of the definition of noise figure for optical amplifiers, nor does it invalidate the widely accepted result that in the high-gain limit the conventional noise figure has a minimum value of $F = 2$. We hope that this comment will help to clarify the assumptions behind the conventional definition of optical noise figure and to illustrate the particular care which must be taken when applying and interpreting the noise figure in systems using modulation schemes other than standard intensity modulation. In particular, we have argued that there are real benefits to be gained by using modulation schemes, such as SSB, which make more efficient use of optical bandwidth.

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