A Versatile Method for Analyzing Paraxial Optical Propagation in Dielectric Structures

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***Abstract*—This paper presents a fast and accurate quasi-analytic model for studying optical field propagation in weakly guiding dielectric structures. The proposed efficient and versatile computational scheme is obtained by merging the Hermite–Gauss (HG) total field expansion with the numerical collocation method and is particularly suited for longitudinally nonuniform structures. By means of a quasilinearization scheme, the same procedure has also been successfully applied to the analysis of field propagation in Kerr-nonlinear media. The latter achievement gives an indication of the great potentialities offered by this straightforward method. Several examples are discussed in the paper and in all cases the results computed by the proposed method favorably compare with those from alternative methods.**

***Index Terms*—Expansion methods, Kerr nonlinearity, longitudinally nonuniform structures, optical propagation, orthogonal functions.**

I. Introduction

THERE are only very few cases of optical propagation in dielectric structures that admit to analytic solutions. The situation is even more acute if longitudinal nonuniformity or nonlinearity is included. On the other hand there is now an increasing number of practical situations, particularly pertinent to the development of photonic devices and circuits, that require the reliable analysis of optical propagation in "awkward" geometries. One such geometry of considerable current interest is the tapered dielectric structure.

Interest in achieving fast and accurate analyzes of optical field propagation in longitudinally nonuniform and/or nonlinear structures has continuously been driving the search for convenient and efficient computation schemes. The most commonly used method for the analysis of field propagation is probably the beam propagation method (BPM). However, several other schemes have been proposed and applied, [1], [2] and references therein, ranging from those dominantly numerical [e.g., BPM, finite difference (FD), finite element (FE)] to the semi-analytic ones (e.g., variational, perturbation, expansion methods). Methods of the latter category, wherever applicable, are of particular interest because they tend to produce convenient and computationally simplified procedures.

The quasianalytic scheme presented in this paper falls under the category of field expansion methods, [3]–[5], and entails expanding the (total) optical field in terms of an appropriate and convenient basis set. Specifically, the Hermite–Gauss (HG) functions have been found to be particularly well suited for the analysis of optical propagation in dielectric (open) structures, as described in Section II. The actual execution of the HG field expansion requires computations which are most effectively and efficiently done by the collocation method (CM). The HG and CM are thus combined to obtain the composite computation scheme, referred to as the HGCM, which is presented in Section III. Finally, in Sections IV and V the HGCM is used to solve several cases of optical field propagation in various dielectric (open) structures and the results compared with those obtained from other models. The success of the HGCM to solve for a wide range of cases clearly demonstrates the versatility and accuracy of this scheme.

II. The Function Expansion Method

This paper deals with the analysis of optical field propagation in dielectric structures that have a relatively small spatial variation of the refractive index, i.e., in the context of weakly guiding structures, [3], for which it is appropriate to use a quasiscalar formulation and linearly polarized fields. The nondegenerate case can indeed be analyzed by the HGCM if the electromagnetic field equations are accordingly modified, [6]. Further, slab waveguide-like structures are considered so that the field is assumed to be independent of the transverse (y) coordinate, i.e., $(\partial/\partial y) \equiv 0$. For harmonic time dependent fields a component of the electric or magnetic field $F(x,z)$ will, with reference to the above conditions, approximately satisfy the reduced wave equation

$$\left\{\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}+k_0^2\varepsilon(x,z)\right\}F(x,z)=0 \tag{1}$$

where

$k_0=(2\pi/\lambda_0)$	free-space propagation constant;
λ_0	wavelength in vacuum;
$\varepsilon(x,z)=n^2(x,z)$	(relative) dielectric distribution, with $n(x,z)$ the corresponding refractive index profile.

Assuming the optical field to be varying predominantly along the longitudinal z-direction, i.e., considering essentially paraxial propagation, the lateral field profile is taken to be changing slowly with z, in which case it is convenient to write

$$F(x,z)=f(x,z)\exp(\pm ipz) \tag{2}$$

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where p is a suitably chosen constant and $f(x,z)$ represents the slowly varying lateral field profile. For the assumed paraxial propagation the field satisfies the following inequalities:

$$\left|\frac{\partial^2 f(x,z)}{\partial z^2}\right| \ll p\left|\frac{\partial f(x,z)}{\partial z}\right| \tag{3.a}$$

and

$$\left|\frac{\partial^2 f(x,z)}{\partial z^2}\right| \ll \left|\frac{\partial^2 f(x,z)}{\partial x^2}\right| \tag{3.b}$$

hence, the scalar wave equation (1) can be reduced to the form, [7], [8],

$$-2ip\frac{\partial f(x,z)}{\partial z} + \frac{\partial^2 f(x,z)}{\partial x^2} + \left\{k_0^2\varepsilon(x,z) - p^2\right\} f(x,z) = 0 \tag{4}$$

An important category of solutions to (1) that correspond to open (dielectric) structures and are particularly relevant to photonic devices and circuits, result from applying the following lateral boundary condition, [9]

$$|F(x,z)| \rightarrow 0 \quad \text{for } |x| \rightarrow \infty. \tag{5}$$

Under these conditions the field $F(x,z)$ is square integrable and, therefore, so is $f(x,z)$.

The objective here is to solve (4) by expanding $f(x,z)$ in terms of a convenient set of (known) basis functions. By this procedure (4) is reduced to a manageable set of coupled differential equations for evaluating the expansion coefficients. For this purpose it is necessary to choose an expansion set which is complete in the appropriate (square integrable) function space, [10].

A. Function Expansion Set

Although in principle any complete set of functions may be used to expand the optical field, $f(x,z)$, the overall computation process is more effective and efficient if a judicious choice of the basis set is made. One such choice is the set of local (dielectric) waveguide modes—the set is complete for the pertinent function space. This approach is known as the local mode expansion (LME) method, [3], [9], and compared to all other methods it is possibly the closest to a description of the actual, physical evolution of the field along the structure. However, the LME is generally difficult to implement because the complete set includes the radiation modes which form the continuous part of the eigenvalues spectrum, [3], [9]. To practically use the LME it is thus essential to discretise the continuous spectrum, a process which poses considerable difficulties, [11].

An alternative approach is to work with basis sets that are not only complete in the appropriate function space, but also discrete, [5], [12], [13]. The solution to (4) is thus written in the following general form:

$$f(x,z) = \sum_{k=1}^{M} a_k(z)\varphi_k(x) \tag{6}$$

where $\varphi_k(x)$ are the basis functions and $a_k(z)$ the z-dependent, complex expansion coefficients. [Note that the expansion coefficients are constants for modal solutions of longitudinally uniform dielectric structures.] In (6) M is the number of expansion terms; for the idealised case $M = \infty$, but for practical situations $M < \infty$. The accuracy of the field analysis based on the total field expansion (6) is thus limited only by the number of expansion terms.

B. The HG Basis Set

For the present formulation the set of HG functions are considered to be most suitable for the structures to be analyzed. The expression for the kth order HG function is

$$\varphi_k(x) = C_k H_k\left(\frac{x}{w_0}\right)\exp\left(-\frac{x^2}{2w_0^2}\right) \tag{7}$$

where C_k is a normalization constant; $H_k(x/w_0)$ the Hermite polynomial of order k and w_0 the constant HG waist parameter, [3].

One of the advantages of using the HG set in open structures is that, since each HG function tends to zero for $|x| \rightarrow \infty$, the total field (6) naturally satisfies the required boundary conditions (5). Note also that the HGs are the eigensolutions of the following second order differential equation, [14]

$$\frac{d^2\varphi_k(x)}{dx^2} + (2k+1-x^2)\varphi_k(x) = 0 \tag{8}$$

with $k = 1, 2, \ldots$, which will be used in the derivation below. In the present context (8) may be recognized as the characteristic equation for a slab waveguide with (ideal) parabolic dielectric distribution. Finally, it is important to mention that the HG functions also form an orthonormal set satisfying the orthogonality condition

$$\int_{-\infty}^{+\infty} \varphi_k^*(x)\varphi_q(x)\,dx = \delta_{kq} \tag{9}$$

where δ_{kq} is the Kronecker delta.

III. The HG Collocation Method

To compute the expansion coefficients, $a_k(z)$, (6), it is necessary to solve the set of coupled differential equations obtained by substituting the field expansion (6) into the paraxial wave equation (4) and orthogonalising using (9). Although exact, this formulation is cumbersome to solve because it entails evaluating integrals.

An alternative and very efficient computational procedure is to use the collocation method (CM), [6], [15], [16], which is here combined with the HG-expansion technique to derive the quasianalytic HGCM computation scheme. With the CM, the numerically laborious integrals are discretized by the application of the Gaussian quadrature formula (GQF)

$$\int_{-\infty}^{\infty} g(x)e^{-x^2}\,dx \approx \sum_{j=1}^{P} h_j g(x_j) \tag{10}$$

where $g(x)$ is any square integrable function; $h_j = (2^{P-1}P!\sqrt{\pi}/P^2[H_{P-1}(x_j)]^2)$ are the weighting functions for the Hermite polynomials, [14], and x_j are the P sampling points which are also referred to as collocation points. When using the GQF with the Hermite polynomials, the most suitable collocation points are the zeros of the Hermite polynomial of order P, (10).

The discretization of the lateral x-axis that will be used to solve the problems of interest is thus decided by the use of the GQF. In addition, by choosing the number P of sampling points, (10), to be equal to the number M of expansion terms, (6), it is possible to make use of the matrix formalism, thus simplifying the derivation.

By applying the collocation method to (4) it is implied that the paraxial wave equation is exactly satisfied at each collocation point x_j, i.e.

$$-2ip\left.\frac{\partial f(x,z)}{\partial z}\right|_{x=x_j}+\left.\frac{\partial^2 f(x,z)}{\partial x^2}\right|_{x=x_j} + \left(k_0^2\varepsilon(x_j,z)-p^2\right)f(x_j,z)=0 \quad (11)$$

with $f(x_j,z)=\sum_{k=1}^{M}a_k(z)\varphi_k(x_j)$ and $j=1,2,\ldots,M$.

Since the solution to (4) is expanded as in (6), (11) actually represents a set of M ordinary first order differential equations in z for the expansion coefficients $a_k(z)$, which can be compactly written in the following form:

$$-2ip\frac{da_k(z)}{dz}-(2k+1+p^2)a_k(z) + \sum_{q=1}^{M}B_{kq}(x_j,z)a_q(z)=0 \quad (12)$$

where $B_{kq}(x_j,z)=\varphi_k^*(x_j)(k_0^2\varepsilon(x_j,z)+x_j^2)\varphi_q(x_j)$ is the matrix element, and use has been made of the differential equation (8). After solving (12) the field solution is determined using (6).

IV. Propagation in Passive, Linear Media

Three quite different examples of application of the HGCM to solve for propagation in inhomogeneous, but linear dielectrics are presented in this section to demonstrate the scope of the method. Note that the HGCM has previously been shown to be effective in computing Gaussian Beam propagation in a homogeneous medium, [6]. The latter is an important test because the excellent agreement between the HGCM and the analytical solution, [6], confirms that the HG function set used in the computation can be effectively applied to analyze radiating (diffracting) fields, an aspect which is particularly important for representing radiation at waveguide discontinuities and in longitudinally nonuniform structures.

A. Waveguide Junctions

The step discontinuity between two slab dielectric waveguides, WG1 and WG2, of different widths, Fig. 1, may be viewed as an abruptly nonuniform structure. The net input field, specified as one particular discrete (bound) mode of WG1, is propagated across the junction into WG2. Because of the discontinuity it is expected that only a part of the input field will couple to the guided modes of WG2 while the rest is radiated into the surrounding (homogeneous) cladding medium.

The HGCM is used to solve for the propagating field distribution in such a structure and the results are then compared with those from an approximate plane wave spectrum method which

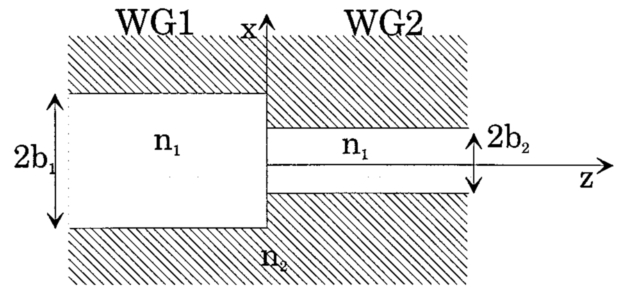

Fig. 1. Schematic of a waveguide step-transition.

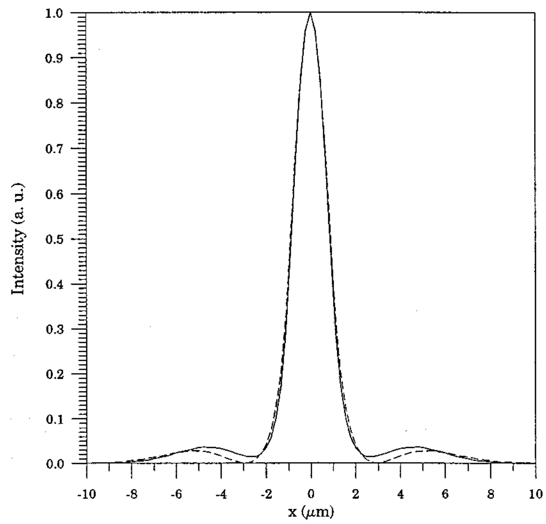

Fig. 2. Intensity profile of an optical field after propagating a distance of 50 μm from the waveguide step-transition: $n_1=3.32$, $n_2=3.3$, $\lambda_0=1.55$ μm, $2b_1=6$ μm, $2b_2=2$ μm. Solid line: HGCM ($M=99$, $w_0=1$ μm); dashed line: FSRM.

is basically a simplified version of the free space radiation mode (FSRM) method, [17]. In general the optical field is expanded in terms of the complete set of discrete and continuum modes of WG2. For the FSRM, however, the continuum spectrum is calculated approximately with the assumption of a homogeneous medium (of refractive index n_c), i.e., in the limit of the core width of WG2 tending to zero. The value for n_c has been chosen to be that of the modal refractive index of WG2.

Results are presented for the specific example of the input field, $F_0(x)$, corresponding to the fundamental mode of WG1, assuming that WG2 supports only one bound mode. The fields calculated by the HGCM and FSRM for this example are presented in Fig. 2 for a propagation distance of 50 μm into WG2. The required CPU times are 68 and 30 s for the HGCM and the FSRM, respectively (an HP712 machine with a 66 MHz RISC processor has been use to compute all the results presented in this paper). The difference between the two computing times can be explained by the fact that the first method is a (total field) propagation scheme, whereas the latter essentially involves just a Fourier transform. The two results are in good agreement, thus again supporting the claim that both guided and radiated fields can be quite accurately analyzed by the HGCM.

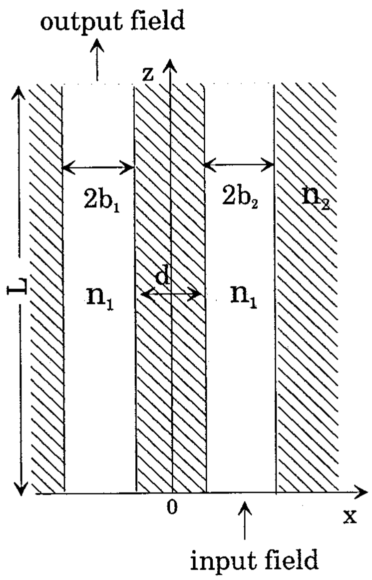

Fig. 3. Schematic of a coupled waveguide structure.

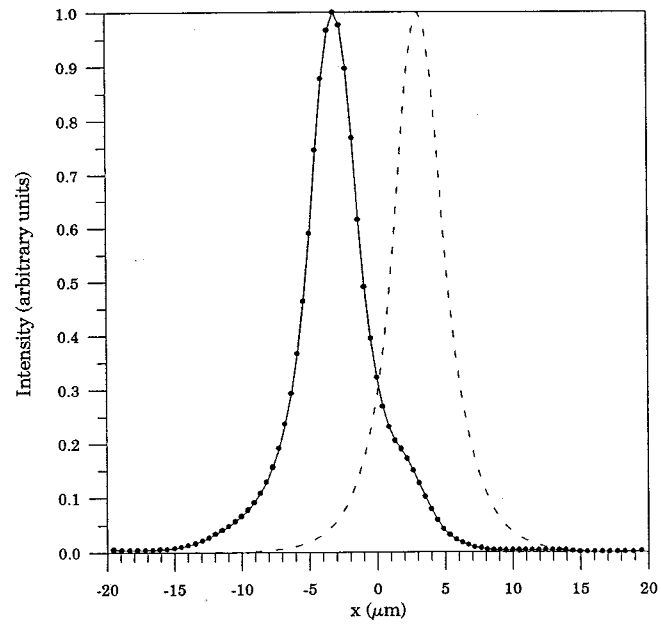

Fig. 4. Field intensity profile calculated with the HGCM (dots, $M = 99$, $w_0 = 2$ μm) and compared with the analytic solution (solid line) for a coupled waveguide structure: $n_1 = 3.284\,48$, $n_2 = 3.282\,41$, $\lambda_0 = 1.55$ μm, $2b_1 = 2b_2 = 3$ μm, $d = 3$ μm and $L = 1220$ μm. The dashed line represents the input field at $z = 0$.

B. Coupled Waveguides

The computation of propagation in coupled waveguides, Fig. 3, is considered as yet another test for the HGCM. The structure is clearly not longitudinally nonuniform but in this case multimodal propagation in the composite waveguide, in effect, produces the longitudinally varying lateral field profile. The HGCM computed results are compared against analytic solutions obtained from the use of only the bound modes of a five-layer slab waveguide, [3], defined by (13) at the bottom of the page.

The results presented in Fig. 4 have been computed for an input field specified as the fundamental mode of the (isolated) waveguide on the right hand side of Fig. 3. The propagation length, L, used in the example has been arbitrarily chosen to be half the coupling length. The agreement between the results computed by the two quite different methods is remarkably good, indicating that, for this case, the discrete modes are dominant while the radiation mode spectrum, implicitly included in the HGCM, contributes negligibly to the lateral field profile.

It may be useful to observe that the CPU time required by the HGCM depends on the number M of expansion terms used in (6), on the propagation step, Δz, and, although very moderately, on the HG width parameter w_0 which also determines the extent of the computational window. It is to be expected that the computing time increases with M (approximately as M^2), Fig. 5, because of the matrix formulation of the HGCM. On the other hand, the HGCM solutions seem to be almost insensitive to changes in $M, \Delta z$ and w_0, as illustrated by Fig. 6,

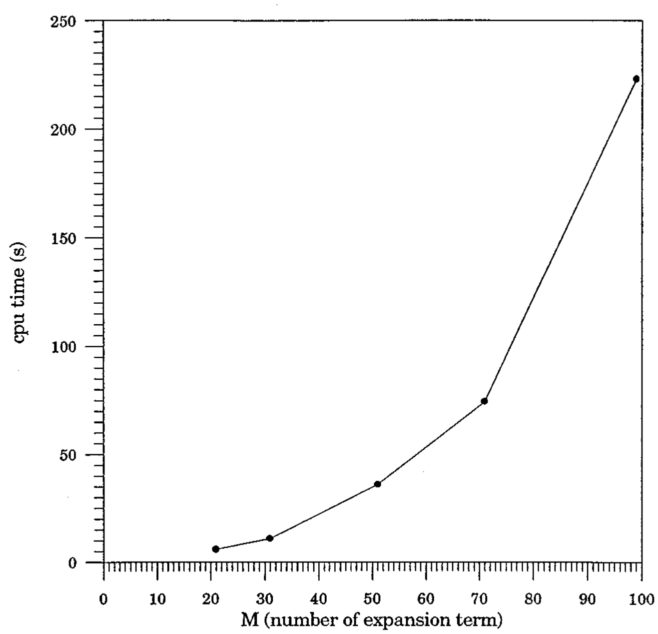

Fig. 5. CPU time as a function of the number M of expansion terms used in the HGCM solution of the coupled slab waveguides ($w_0 = 2$ μm).

$$\varepsilon(x_j, z) = \varepsilon(x_j) = \begin{cases} n_1^2 & -\frac{d}{2} - 2b_1 \le x \le -\frac{d}{2}; \quad \frac{d}{2} \le x \le \frac{d}{2} + 2b_2 \\ n_2^2 & x \le -\frac{d}{2} - 2b_1; \quad -\frac{d}{2} \le x \le \frac{d}{2}; \quad x \ge \frac{d}{2} + 2b_2 \end{cases} \tag{13}$$

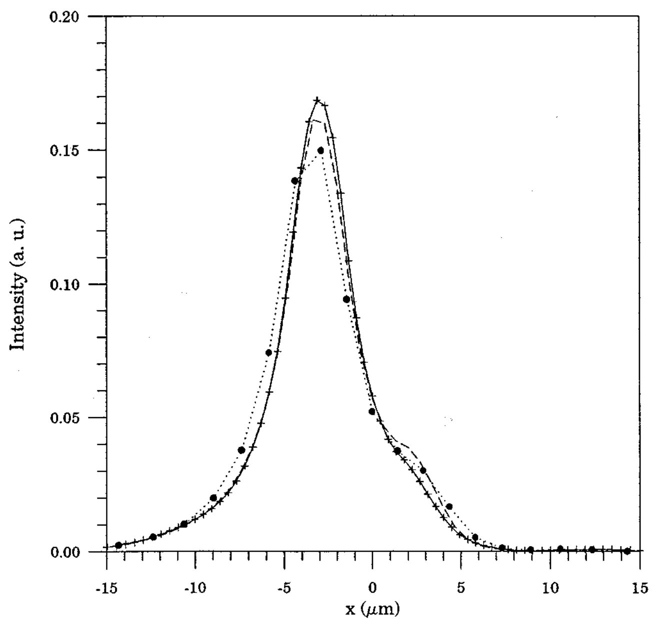

Fig. 6. Coupled slab waveguides: the HGCM computed solution as a function of the parameters $M, \Delta z$ and w_0; continuous line: $M = 99, \Delta z = 112$ μm, $w_0 = 2$ μm; crosses: $M = 99, \Delta z = 1$ μm, $w_0 = 2$ μm; dashed line: $M = 99, \Delta z = 20$ μm, $w_0 = 3$ μm; dots: $M = 31, \Delta z = 20$ μm, $w_0 = 3$ μm.

where the coupled waveguide problem has been solved by the HGCM using different values of the above mentioned parameters, which gives an indication of the convergence and stability of the method.

C. Dielectric Tapers

Tapered geometry structures have been extensively used for metal waveguide horn antennas at microwave frequencies, [18], while tapered dielectric elements have been applied to design efficient optical mode converters, [19]. Relatively recently, tapered geometries have been used for the development of high-power semiconductor optical sources, [20]–[22]. Indeed, the parabolic taper geometry, Fig. 7, in particular, has been found to be very attractive for a number of guided wave passive, [23], and active, [24], optical devices. Evidence presented here demonstrates convincingly that the HGCM can be efficiently used to solve for propagation in tapered dielectric structures.

As an example the HGCM is used to solve for propagation in a parabolic taper waveguide, Fig. 7, having a core of half width $b(z) = \sqrt{b_0^2 + t \cdot z}$ and refractive index n_1 surrounded by a medium of refractive index $n_2 < n_1$. In using the HGCM the lateral discretization derives from the application of the collocation method as described in Section III. For the longitudinal discretization an appropriate propagation step needs to be chosen according to the tapered structure of interest. Importantly note that the combination of the two discretization schemes may produce significant numerical errors because the representation of the longitudinally varying width, $b(z)$, of the core layer depends critically on the spacing of the collocation points. To minimize computational errors it has been found to be most convenient to represent the abrupt lateral refractive index step by a continuously varying Super-Gaussian (SG) profile, Appendix A.

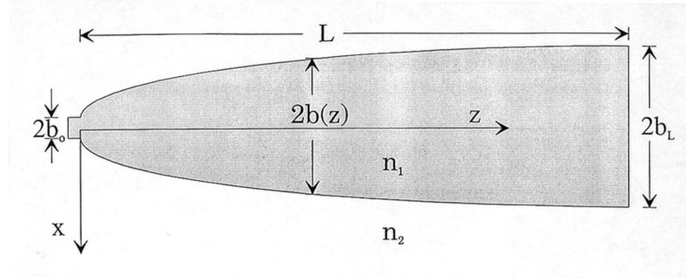

Fig. 7. Schematic of a parabolic taper.

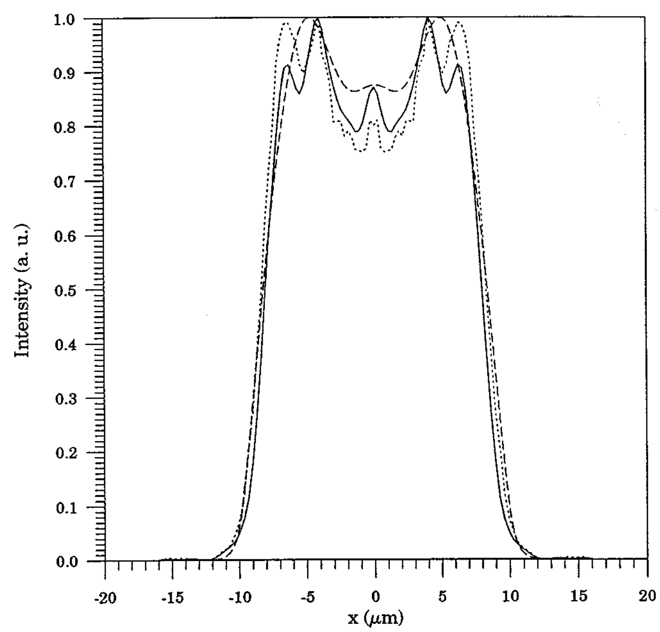

Fig. 8. Field intensity profile at the wider end of a parabolic taper ($2b_0 = 3$ μm, $2b_L = 20$ μm, $L = 500$ μm, $n_1 = 3.33, n_2 = 3.32, \lambda_0 = 0.86$ μm): solid line: HGCM ($M = 99, w_0 = 1.2$ μm) with SG refractive index distribution; dotted line: HGCM ($M = 99, w_0 = 1.2$ μm) with ST refractive index distribution; dashed line: BME. The input field is specified as the fundamental local mode at $z = 0$.

This important modification has remarkably improved the performance of the HGCM when applied to longitudinally nonuniform geometries.

The results from the HGCM are here compared with those obtained from the approximate version of the LME which makes use of only the (local) bound modes, referred to here as the bound mode expansion (BME), [25]. In fact, the BME is adequately accurate especially for parabolic dielectric structures since, following the excitation of the local fundamental mode, the field propagates by coupling predominantly to the local fundamental mode all along the length, [19].

With reference to Fig. 7, the input field is specified as the fundamental local mode at the narrow end of the parabolic taper. The results from the two different methods are plotted in Fig. 8. Note that there are two sets of HGCM computed results in Fig. 8—one is obtained using the step refractive index while the other uses the SG representation. The agreement between the HGCM-SG and the BME is particularly good and this underlines the importance of using a continuous (SG) represen-

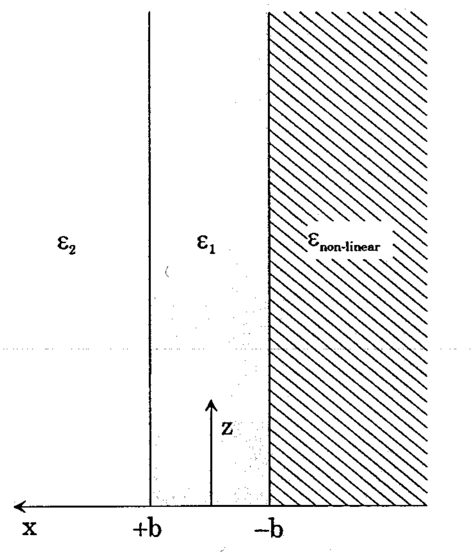

Fig. 9. Schematic of a slab waveguide with nonlinear substrate: $2b$ is the guiding layer width.

tation of the refractive index profile in further improving the accuracy of the HGCM computed results. For this example the CPU times required by the two methods are comparable being 244 s for the HGCM and 208 s for the BME.

V. Nonlinear Dielectric Media

Another class of problems that has been solved by the HGCM is that of optical propagation in nonlinear media. In particular, consider a Kerr-type nonlinearity where the dielectric distribution, $\varepsilon(x, z)$, is defined as, [26]

$$\varepsilon(x, z) = \varepsilon_{\rm lin}(x) + \kappa(x)|F(x, z)|^2 \tag{14}$$

where

$\varepsilon_{\rm lin}(x)$ (linear) built-in dielectric distribution;
$\kappa(x)$ nonlinearity coefficient;
$F(x, z)$ propagating field in the structure.

With the dielectric distribution as specified in (14), the paraxial wave equation (4) becomes nonlinear. To enable the use of the HG expansion (4) is quasilinearised by means of the iteration scheme used for the computation. Thus, (14) is written as

$$\varepsilon_{(v)}(x, z) = \varepsilon_{\rm lin}(x) + \kappa(x)\left|F_{(v-1)}(x, z)\right|^2 \tag{15}$$

indicating that in the vth iteration the dielectric distribution, $\varepsilon_{(v)}(x, z)$, is related to the (known) field distribution, $F_{(v-1)}(x, z)$, corresponding to the previous iteration. The field distribution for the first iteration is typically specified in terms of the eigenmodes of the built-in linear structure, $\varepsilon_{\rm lin}(x)$. At each iteration loop, v, the HGCM is used to solve the now linear paraxial wave equation

$$-2ip\frac{\partial f_{(v)}}{\partial z} + \frac{\partial^2 f_{(v)}}{\partial x^2} + \left\{k_0^2 \varepsilon_{(v)}(x, z) - p^2\right\} f_{(v)} = 0 \tag{16}$$

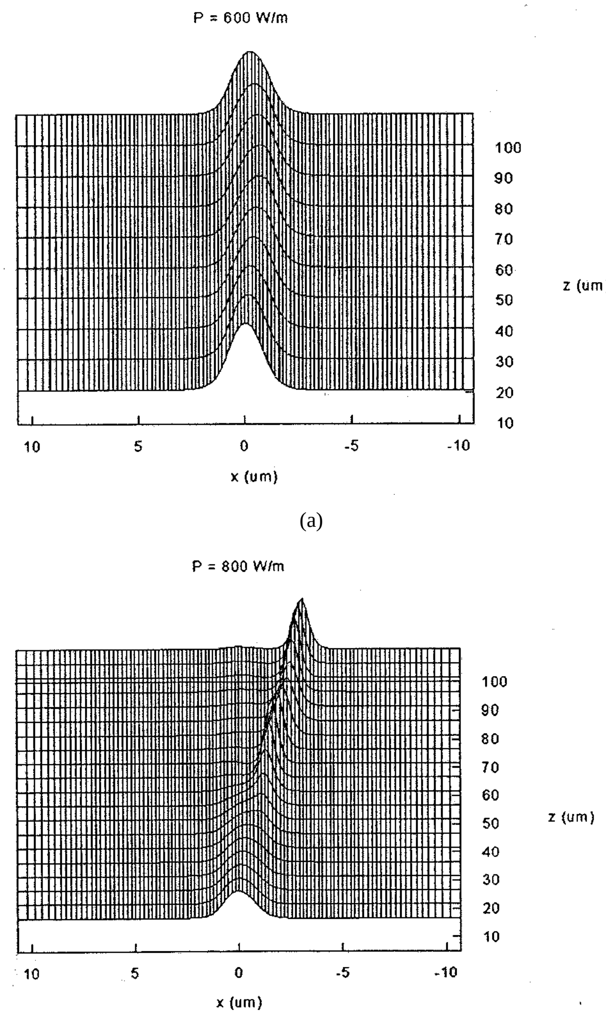

Fig. 10. Amplitude of a field propagating in a three-layer slab waveguide with nonlinear substrate, calculated with the HGCM ($M = 99, w_0 = 0.8$ μm) for two different input powers per unit length: (a) $P = 600$ W/m, (b) $P = 800$ W/m; $2b = 2$ μm, $\varepsilon_1 = (1.57)^2, \varepsilon_2 = (1.55)^2, \lambda_0 = 1$ μm, $\kappa_0 = 6.3771 \cdot 10^{-13}$ m^2/V^2, longitudinal step $= 0.05$ μm.

until convergence on the field profile is achieved.

Consider the case of paraxial propagation along a three-layer slab waveguide with a nonlinear substrate, Fig. 9, [26]–[29]. The built-in linear dielectric distribution is

$$\varepsilon_{\rm lin}(x) = \begin{cases} \varepsilon_1 & |x| \le b \\ \varepsilon_2 & |x| > b \end{cases} \tag{17}$$

where $\varepsilon_1 > \varepsilon_2$ are constants, $2b$ is the width of the core layer and the nonlinearity coefficient is

$$\kappa(x) = \begin{cases} \kappa_0 & x \le -b \\ 0 & x > -b \end{cases} \tag{18}$$

with κ_0 a constant parameter.

All relevant parameters, given in the caption of Fig. 10, are taken from [26] to allow for a useful comparison between the method proposed in that reference and the HGCM. Results obtained with the HGCM for propagation in this nonlinear waveguide are presented in Fig. 10 and are found to be in good agreement with those in [26]. The CPU time required by the HGCM strongly depends on the input power since large input powers imply reiterations to satisfy self-consistency. In this case the CPU time was 4800 s for 800 mW/m and 3624 s for 600 mW/m input power.

Hence, by quasilinearising the problem, the HGCM has been satisfactorily applied to solve also for nonlinear waveguide problems. Although the approximation made with the quasi-linearization needs to be carefully considered, particularly when the nonlinear terms become dominant, the advantages of a modest computational effort together with a straightforward implementation make the HGCM-iteration technique very attractive.

VI. Conclusion

The Hermite–Gauss collocation method (HGCM) has been extensively applied to a variety of field propagation problems which are encountered in typical present-day photonic device structures. Results computed by the HGCM have been shown to be in excellent agreement with those from other methods of solution.

In particular, under the approximation of a quasilinearization of the problem, the HGCM has been shown to be applicable for the computation of paraxial propagation also in explicitly nonlinear dielectric waveguides. This success provides further confidence in the use of the HGCM-iteration scheme to analyze active optical (semiconductor) devices, such as amplifiers and lasers, where the implicit nonlinearities are due to the interaction between the optical field and the charge carriers. Indeed, the versatility of the HGCM-iteration procedure for solving nonlinear differential equations has been further substantiated by solving the carrier diffusion equation that is typically encountered in the analysis of semiconductor optical devices [30].

Appendix A
Super-Gaussian Refractive Index Distribution

The use of a continuously varying refractive index distribution greatly improves the results obtained by the HGCM. Specifically, the structure to be analyzed can be conveniently described by a super-Gaussian (SG) refractive index profile of the type

$$n(x,z) = n_2 + (n_1 - n_2)\exp\left\{-\frac{1}{2}\left(\frac{x}{b(z)}\right)^s\right\} \tag{B1}$$

where s is the SG exponential and $b(z)$ the longitudinally varying width of the guiding layer. For example, for a linear taper

$$b(z) = b_0 + t \cdot z \tag{B2}$$

while for a parabolic taper

$$b(z) = \sqrt{b_0^2 + t \cdot z} \tag{B3}$$

where t is the tapering parameter and $b_0 = b(z=0)$.

The continuous SG refractive index distribution (B1) is very sharp for $s > 10$ so that it quite satisfactorily represents a step-index distribution.

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