Refractive-Index Profiling of Graded-Index Planar Waveguides from Effective Indexes Measured with Different External Refractive Indexes

Kin Seng Chiang, *Member, IEEE*, *Member, OSA*, Chi Lai Wong, Sin Yip Cheng, and Hau Ping Chan, *Member, IEEE*

***Abstract*—We extend the well-known inverse Wentzel–Kramer–Brillouin (WKB) method for refractive-index profiling of graded-index planar waveguides. We demonstrate, with numerical examples and experimental results, that the refractive-index profile of a graded-index planar waveguide can be calculated from effective indexes measured with different external refractive indexes. With this technique, single-mode and two-mode waveguides can be profiled easily to a good accuracy.**

***Index Terms*—Gradient index optics, optical planar waveguides, optical waveguides, refractive index measurement, Wentzel–Kramer–Brillouin (WKB) method.**

I. Introduction

IT IS of fundamental importance to determine the refractive-index profile of a graded-index planar waveguide, as the profile can give useful information about the waveguide fabrication process and the transmission properties of the waveguide. The most popular method for refractive-index profiling of planar waveguides appears to be the inverse WKB method [1], [2], as the method is easy to implement and does not assume any knowledge about the profile to be determined.

In the WKB approximation, the refractive-index profile of a waveguide is defined uniquely by the relationship between the effective index and the mode order, i.e., the effective-index function [2], [3]. The idea of the inverse Wentzel–Kramer–Brillouin (WKB) method [2] is to construct the effective-index function with measured effective indexes, and calculate the corresponding profile from the function. As a profile is characterized by its peak index, depth, and shape, to obtain a meaningful approximation of the effective-index function, at least three effective indexes are required. Therefore, the early version of the method [2], which relies on measurements for one mode type (either TE or TM modes), can only be applied to waveguides that support at least three modes of the same type [4].

To extend the inverse WKB method to single-mode and two-mode waveguides, we have proposed techniques to combine measurements for both mode types, and at different wavelengths [5]. However, the technique of combining measurements for both mode types cannot be applied to waveguides that do not support both mode types or contain unknown modal birefringence. The technique of combining measurements at different wavelengths is more general, but requires several laser sources and an accurate knowledge of the dispersion properties of the waveguide material. In this paper, we propose a technique of combining effective indexes measured with different external refractive indexes. This technique is simple to implement and allows accurate profiling of single-mode and two-mode waveguides at a fixed wavelength.

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II. Theory

We consider a graded-index profile $n(x)$, which is a monotonically decreasing function of x for $x \geq 0$ with a peak value n_0 at $x = 0$. The substrate index at $x = +\infty$ and the external index for $x < 0$ are denoted as n_s and n_e, respectively. The WKB eigenvalue equation for the guided modes of the waveguide can be written as [2]

$$k \int_0^{x_t(m)} [n^2(x) - N^2(m)]^{1/2}\, dx = (m + 0.25)\pi + \Phi(N, n_e) \quad (1)$$

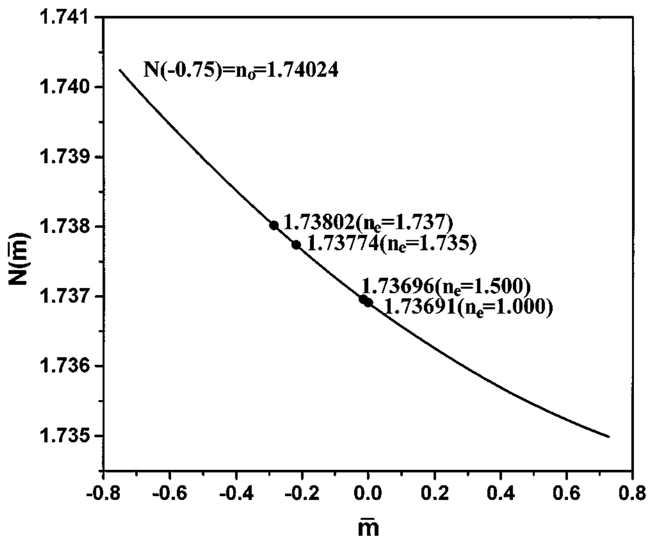
with the phase change Φ given by

$$\Phi(N, n_e) = \arctan\left[r_e \left(\frac{N^2 - n_e^2}{n_0^2 - N^2} \right)^{1/2} \right] \quad (2)$$

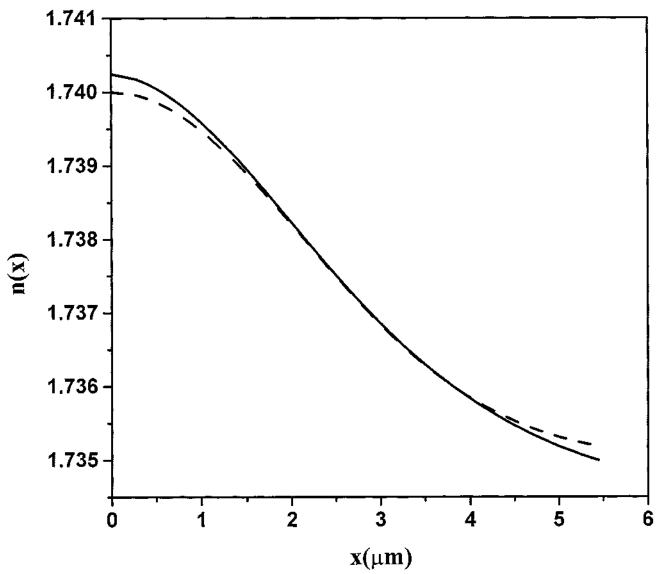
where

$k = 2\pi/\lambda$	free-space wavenumber with λ the wavelength;
m	mode order;
$N(m)$	effective index (propagation constant divided by k);
x_t	turning point at which $n(x_t) = N(m)$;
$r_e = 1$	TE modes;
$r_e = (n_0/n_e)^2$	TM modes.

The method reported in [2] is based on treating m as a real number, so that $N(m)$, which is regarded as a continuous function of m (the effective-index function), can be constructed by curve-fitting of the effective indexes $N(1), N(2), N(3), \ldots$. The refractive-index profile $n(x)$ can be recovered from $N(m)$ with an efficient algorithm [2]. For the construction of the effective-index function, at least three effective indexes must be available [4]. Therefore, if measurements for only one mode

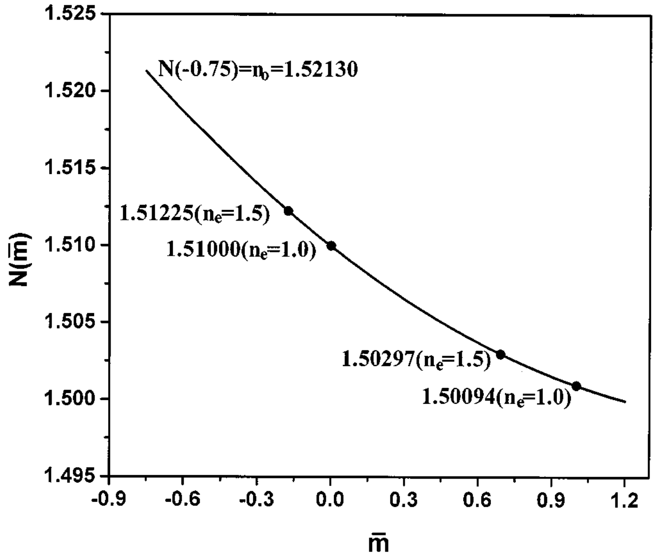


(a)



(b)

Fig. 1. Recovery of a single-mode Gaussian profile by using effective indexes obtained with four external refractive indexes: (a) Eeffective indexes and the best-fit effective-index function $N(\bar{m})$; (b) Profile calculated from the effective-index function in (a) (solid curve) and the exact profile (dashed curve).



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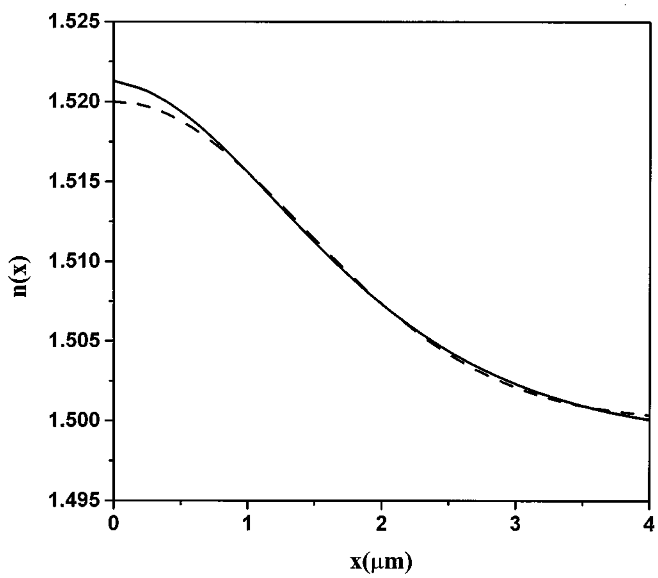

(b)

Fig. 2. Recovery of a two-mode Gaussian profile by using effective indexes obtained with two external refractive indexes: (a) Effective indexes and the best-fit effective-index function $N(\bar{m})$; (b) Profile calculated from the effective-index function in (a) (solid curve) and the exact profile (dashed curve).

type (TE or TM) are used, the waveguide must support at least three guided modes of the same type. When data measured for both mode types can be combined, two-mode waveguides can be profiled [5]. By combining data measured at different wavelengths, we can profile even single-mode waveguides [5]. Here we propose to combine effective indexes measured with different external indexes so that single-mode waveguides can be profiled at a fixed wavelength.

The key idea of the method is the introduction of an effective mode order $\bar{m}$:

$$\bar{m} = m - \frac{\Phi(N, n_a) - \Phi(N, n_e)}{\pi} \tag{3}$$

where n_a is a reference external index (air with $n_a = 1$ is normally used as the reference). Using the new variable $\bar{m}$, (1) can be written as

$$k \int_0^{x_t(\bar{m})} [n^2(x) - N^2(\bar{m})]^{1/2}\, dx = (\bar{m} + 0.25)\pi + \Phi(N, n_a) \tag{4}$$

where $N(\bar{m})$ is now regarded as a continuous function of $\bar{m}$, which is the effective-index function that characterizes the refractive-index profile. When evaluated at the values of $\bar{m}$ given by (3), $N(\bar{m})$ gives the effective indexes obtained with various external indexes. In particular, at $\bar{m} = m$, $N(\bar{m})$ is equal to the effective index obtained with the reference external index n_a. Clearly, with the introduction of the effective mode order, the effective indexes obtained with different external indexes become

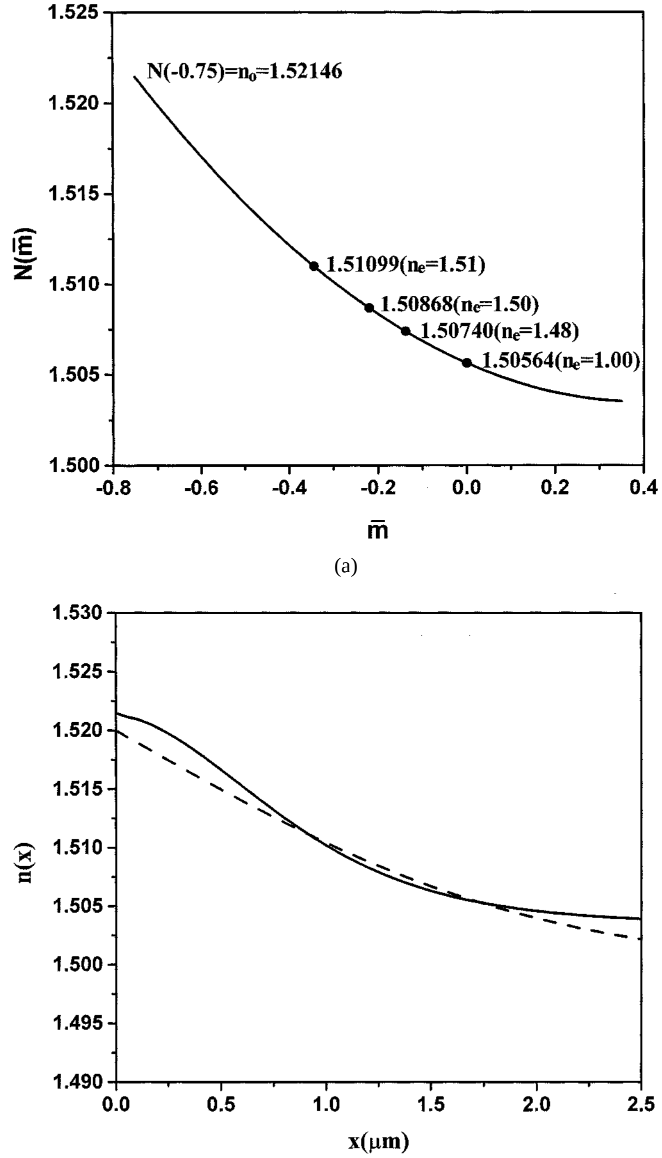

Fig. 3. Recovery of a single-mode complementary-error-function profile by using effective indexes obtained with four external refractive indexes: (a) Effective indexes and the best-fit effective-index function $N(\bar{m})$; (b) Profile calculated from the effective-index function in (a) (solid curve) and the exact profile (dashed curve).

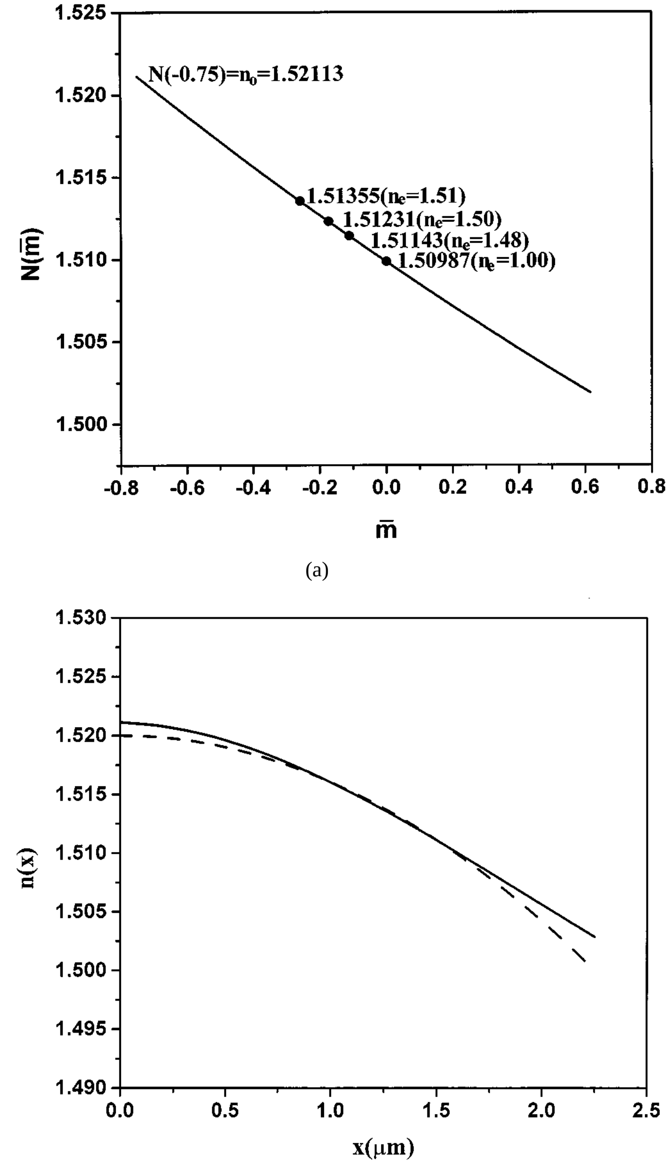

Fig. 4. Recovery of a single-mode truncated parabolic profile by using effective indexes obtained with four external refractive indexes: (a) Effective indexes and the best-fit effective-index function $N(\bar{m})$; (b) Profile calculated from the effective-index function in (a) (solid curve) and the exact profile (dashed curve).

specific points of the same effective-index function $N(\bar{m})$. This provides the basis of constructing the effective-index function from a set of measurements with different external indexes by the curve-fitting technique.

In practice, the effective-index function can be constructed by the following steps.

Step (i) Use an initial guess of n_0 $(n_0 > N(m))$ to evaluate the phases $\Phi(N, n_a)$ and $\Phi(N, n_e)$, and estimate the effective mode orders $\bar{m}$ from (3).

Step (ii) Use the values of $N(\bar{m})$ measured with different external indexes n_e and the effective mode orders calculated from the previous step to obtain a best-fit effective-index function and extrapolate it to $\bar{m} = -0.75$ to give an update value of n_0, i.e., $n_0 = N(-0.75)$ [2].

Step (iii) Use the results from Step (ii) to calculate a new set of effective mode orders from (3).

Step (iv) Repeat Step (ii) and (iii) till the value of n_0 (as well as the effective mode orders) converges. The refractive-index profile of the waveguide is then calculated from the final effective-index function by using the algorithm given in [2]. For completeness, the algorithm is given also in Appendix.

The external refractive indexes, required in the method, can be provided conveniently by index-matching liquids, which are commercially available and come with a wide range of refractive indexes. In the measurement of the effective indexes with the prism-coupling method [6], only a small amount of index-matching liquid is needed to wet the contact area

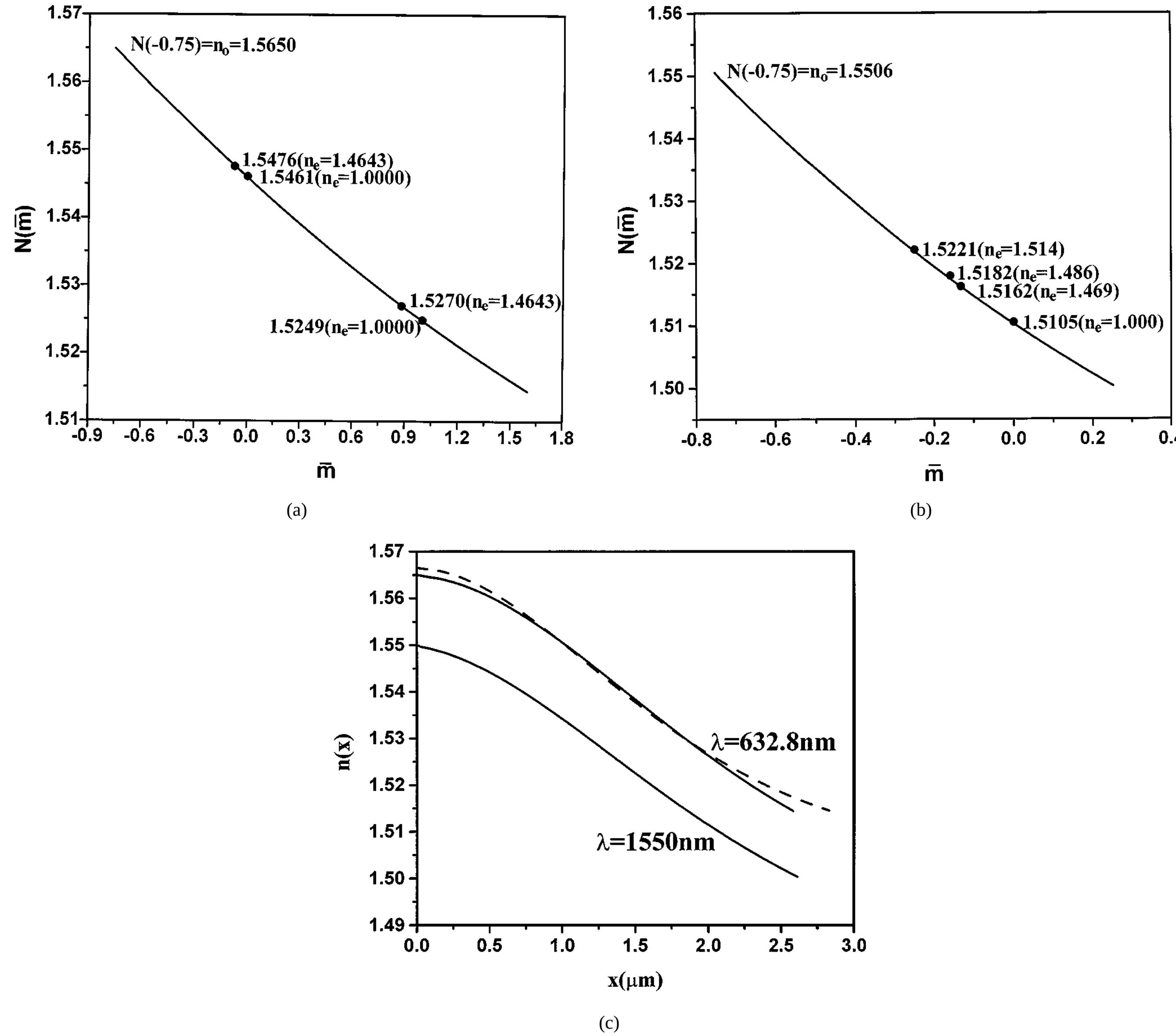

Fig. 5. Results of profiling a Ag-ion exchanged BK7-glass waveguide at the wavelengths 632.8 nm and 1550 nm: (a) Effective-index function at the wavelength 632.8 nm constructed from effective indexes measured with two external indexes; (b) Effective-index function at the wavelength 1550 nm constructed from effective indexes measured with four external indexes; (c) Profiles calculated with the present method (solid curves) and the method that combines the measurements for both the TE and TM modes (dashed curve—for 632.8 nm only) [5].

between the waveguide and the prism. In the case of profiling a single-mode waveguide, three measurements (using air and two different index-matching liquids) is the minimum requirement. If the waveguide supports two modes of the same type, two measurements (one of which employs index-matching liquid) will be sufficient. Of course, the reliability of the result improves as more measurement data are available. The present technique, if necessary, can be combined with the techniques based on measurements for both mode types and at different wavelengths [5].

III. Numerical Results

To evaluate the accuracy of the method, we first consider the Gaussian profile $n(x) = 1.735 + 0.005\exp[-(x/3)^2]$ for $x \geq 0$ (in micrometer), which is single-moded with $n_e = n_a = 1$ at the wavelength 632.8 nm. The exact effective indexes calculated for the TE_0 mode with $n_e = 1, 1.5, 1.735$, and 1.737 are 1.736 91, 1.736 96, 1.737 74, and 1.738 02, respectively. Using these four effective indexes, we can construct the effective-index function in the form of a best-fit second-order polynomial by following the steps described in the previous section and calculate the corresponding refractive-index profile with the algorithm in [2]. The effective-index function and the reconstructed profile are shown in Fig. 1(a) and (b), respectively, where the peak index n_0 is given by $N(-0.75) = 1.740\,24$. It can be seen that the reconstructed profile agrees very well with the original profile. The error in n_0 is only 5% with respect to the index difference $n_0 - n_s$, which is similar to that achieved with the multiwavelength technique [5].

It can be seen from Fig. 1(a) that the effective index for $n_e = 1.5$ (and hence the effective mode order) differs very little from

$$x_i = \frac{(\bar{m}_i + 0.25)\pi + \Phi(N_i, n_a) - k\sum_{j=1}^{i-1}\left\{x_j\left[\left(\bar{N}_j^2 - N_i^2\right)^{1/2} - \left(\bar{N}_{j+1}^2 - N_i^2\right)^{1/2}\right]\right\}}{k\left(\bar{N}_i^2 - N_i^2\right)^{1/2}}$$

that for $n_e = 1$ (air). It makes little difference whether this particular effective index is used or not in the construction of the effective-index function. Clearly, to obtain an accurate effective-index function, we should use well-separated effective indexes. This suggests that, in practice, the index of the liquid used should roughly match that of the waveguide material.

In the next example, we consider another Gaussian profile $n(x) = 1.50 + 0.02\exp[-(x/2)^2]$ for $x \geq 0$ (in micrometer). This waveguide, when placed in air, supports two TE modes at the wavelength 632.8 nm. As this waveguide supports two modes of the same type, we can construct the profile with effective indexes obtained with only two different external refractive indexes. The exact effective indexes calculated for the TE_0 and TE_1 modes are, respectively, 1.510 00 and 1.500 94 for $n_e = 1$, and 1.512 25 and 1.502 97 for $n_e = 1.5$. Using these four effective indexes, we construct the effective-index function in the form of a best-fit second-order polynomial, as shown in Fig. 2(a). The profile calculated from this function is shown in Fig. 2(b), which agrees well with the exact profile. The peak index n_0 is over-estimated by about 7% with respect to the index difference $n_0 - n_s$.

We give two more examples in Figs. 3 and 4, one for the complementary-error-function profile $n(x) = 1.50 + 0.02\,\mathrm{erfc}(x/2.2)$ for $x \geq 0$ and the other for the truncated parabolic profile $n(x) = 1.50 + 0.02[1 - (x/2.25)^2]$ for $2.25 \geq x \geq 0$, where x is in micrometer. The wavelength is 632.8 nm in both examples. As shown by these results, similar performance is obtained for different profile shapes.

IV. Experimental Results

To verify the present method experimentally, we prepared a Ag-ion exchanged BK7-glass waveguide, which supported two modes at the wavelength 632.8 nm and one mode at the wavelength 1550 nm. The effective indexes of the TE modes were measured at both wavelengths with a commercial prism-coupler measurement system (Metricon, Model 2010). The external refractive index was varied by using different index-matching liquids with known refractive indexes. The measured effective indexes and the best-fit effective-index functions are shown in Fig. 5(a) and (b) for the wavelengths 632.8 nm and 1550 nm, respectively. The corresponding profiles are shown in Fig. 5(c). At 632.8 nm, the profile was also constructed with the method based on measurements for both mode types (since two TE and two TM modes were supported at this wavelength). As can be seen from Fig. 5(c), the profiles constructed with the two methods agree very well with each other. As the operating wavelength should have negligible effects on the shape of the profile, the profile at 1550 nm is expected to differ from that at 632.8 nm only by a constant offset due to chromatic dispersion. The results shown in Fig. 5(c) are in good agreement with our expectation.

It should be mentioned that the uncertainty in the measured effective indexes is typically $\sim 10^{-4}$, which can affect the accuracy of the method only for profiles with an exceedingly small index difference between the waveguide surface and the substrate. It can be shown from (3) that an uncertainty in the external index usually produces an uncertainty a few times less in the effective mode order. The temperature dependence of the refractive index of the index-matching liquid (approximately $-4 \times 10^{-4}/°\mathrm{C}$), which contributes mainly to the uncertainty in the external index, can, therefore, be ignored. In practice, errors arising from such uncertainties can be reduced by using more data points in the construction of the effective-index function.

V. Conclusion

We have demonstrated a novel technique to extend the inverse WKB method [2] for refractive-index profiling of graded-index planar waveguides. The present technique only relies on effective indexes measured with different external refractive indexes, and, from a practical point of view, is much simpler and less restrictive than the previous techniques that are based on measurements for both mode types and at different wavelengths [5]. With this technique, even single-mode waveguides can be profiled easily to a good accuracy. This technique will become an important tool for the characterization of graded-index planar waveguides.

Appendix
Calculation of $n(x)$ from $N(\bar{m})$

With reference to (4), at $x_t = 0$, we have $N(\bar{m}) = n_0$ and hence $\Phi(N, n_a) = 0.5\pi$. Therefore, by setting $x_t = 0$ in (4), we get $\bar{m} = -0.75$. In other words, by extrapolating $N(\bar{m})$ to $\bar{m} = -0.75$, the peak index at the waveguide surface n_0 can be obtained, i.e., $N(-0.75) = n_0$. To calculate the refractive-index profile $n(x)$, the effective-index function $N(\bar{m})$ is replaced by a large number of samples $N_i = N(\bar{m}_i)$ $(i = 0, 1, 2, \ldots)$ with $N_0 > N_1 > N_2 > \cdots$ and $N_0 = N(-0.75) = n_0$, each of which corresponds to a turning point x_i at which $n(x_i) = N_i$. The turning points can then be calculated from the following algorithm [2] (see the equation at the top of the page) for $i = 2, 3, \ldots$ with $x_0 = 0$ and $x_1 = [(\bar{m}_1 + 0.25)\pi + \Phi(N_1, n_a)]/[k(\bar{N}_1^2 - N_1^2)^{1/2}]$, and $\bar{N}_i = (N_i + N_{i-1})/2$. The refractive-index profile $n(x)$ can be constructed from the turning points by $n(x_i) = N_i$.

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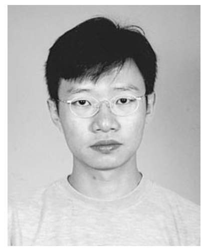

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