The RPS Method Applied to the Numerical Solution of Multimode Slab Waveguides with Complex Indexes

Jianjun Li, Changhua Chen, and Guangdi Shen

***Abstract*—In order to obtain the numerical solution of multimode slab waveguides with complex indexes, we propose a method named RPS, which integrates real index numerical method, perturbation method and shooting method reasonably. This RPS method avoids searching the solution in the whole complex plane, meanwhile, it is simple, fast and precise. The application shows that the RPS method is suitable for both the transverse electric (TE) and the transverse magnetic (TM) mode.**

***Index Terms*—Optical propagation, optical waveguides, optical waveguide theory, simulation.**

I. Introduction

GENERALLY, the actual waveguides contain gain and/or loss regions, both the gain and the loss have a relationship with the imaginary part of the complex refractive index. So in order to analyze strictly the mode characteristics for a waveguide, the complex propagation constant should be calculated. We often only consider the real refractive index because the imaginary part is too small compared with the real part. But in most case, such as for the lateral mode analysis of semiconductor lasers and the mode analysis of semiconductor laser amplifiers, the complex eigenvalue, namely, the complex propagation constant must be solved for these devices.

Traditionally, the mode gain is obtained by the product of the optical confinement factor and the gain of the source material [1]. This method may provide an approximation for transverse electric (TE) mode, but for transverse magnetic (TM) mode, it is not applicable [2]. Up to now, several methods had been developed to calculate the complex propagation constant. The perturbation method [3], [4] is simple and fast, but there is no means of assessing its accuracy. The complex shooting method [5] can get the solution accurately, but it needs an initial value, and for the case of multimode waveguide, some eigenvalue may be lost. Although the scattering matrix approach [6] may find all complex mode propagation constants, it must search the root within a range of the complex plane, so more CPU time will be consumed. The embedding method [7] is suitable for waveguides with large imaginary parts of indexes, but it needs more iterative time.

In order to get all complex propagation constants for multimode complex waveguides, we developed a method named RPS, which integrates Real index numerical method, Perturbation method, and Shooting method reasonably. This method takes advantages of the three methods, avoids the root search in the entire complex plane, and is able to obtain all complex constants. An application of RPS shows that this method is simple, fast and accurate.

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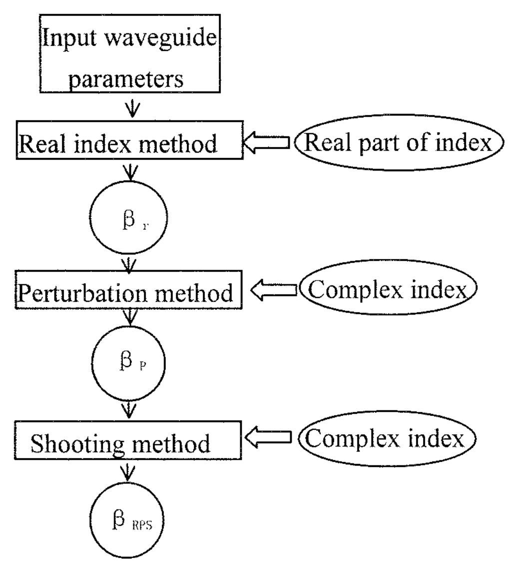

Fig. 1. The main idea of RPS.

II. Simulation Technique

The main idea of RPS is shown in Fig. 1. After inputting the waveguide parameters, such as the layer's thickness and indexes, first, we only consider the real parts of refractive indexes, by using the real numerical method, all real mode solutions β_r will be obtained. Then considering the imaginary part of the refractive index, we may get all approximate mode complex propagation constants β_p through the perturbation method. Last, when each β_p is treated as the initial value of the complex shooting method, all accurate complex mode propagation constants $\beta_{\rm RPS}$ will be gotten by the shooting method.

The main point of RPS is that the total mode number is first determined through the real solution, then it utilizes the fact that the imaginary part of the index is far less than its real part, all approximate complex solutions are obtained by the perturbation method, and because the perturbation solution is treated as the initial value of the shooting method, there is no need to search the root in the whole complex plane. For the virtue of the shooting method, the solution precision is only determined by the iterative error.

As an example of RPS, we give the solution procedure for TE mode. For the waveguide shown in Fig. 2, the electrical field polarizes along the y axis and propagates along the z direction. For the outer sections of $x < x_s$ and $x > x_e$, the refractive index are complex constants n_s and n_e, respectively, while starting from $x_s$and ending up at x_e, the index has an arbitrary distribution $n(x)$. Next, we will give the method of real solution, perturbation solution and shooting solution, respectively.

A. For the Real Solution

If we only consider the real part n_r of indexes, the wave equation for TE mode should be:

$$\frac{\partial^2 E_{yr}}{\partial x^2} + k_0^2[n_r^2(x) - (\beta_r/k_0)^2]E_{yr} = 0 \tag{1}$$

where β_r and E_{yr} are the propagation constant and electric field when only the real part of indexes are considered, $k_0 = 2\pi/\lambda_0$ is the free space number.

We fractionalize the solution domain into a multilayer structure with constant refractive index in each layer, with $x < x_s$ the first layer and $x > x_e$ the mth layer, then the electric field in the jth layer is ($1 \leq j \leq m$):

$$\begin{aligned} E_{yr}^j(x) &= a_{jr}\ \exp[\alpha_{jr}(x - x_j)] \\ &\quad + b_{jr}\ \exp[-\alpha_{jr}(x - x_j)] \\ \alpha_{jr} &= \sqrt{\beta_r^2 - (k_0 n_{jr})^2} \quad (\beta_r > k_0 n_{jr}) \end{aligned} \tag{2a}$$

$$\begin{aligned} E_{yr}^j(x) &= a_{jr}\ \cos[\alpha_{jr}(x - x_j)] \\ &\quad + b_{jr}\ \sin[-\alpha_{jr}(x - x_j)] \\ \alpha_{jr} &= \sqrt{(k_0 n_{jr})^2 - \beta_r^2} \quad (\beta_r < k_0 n_{jr}) \end{aligned} \tag{2b}$$

where

- a_{jr}, b_{jr} coefficients to be determined;
- n_{jr} real part of the index;
- x_j bottom boundary position of the jth layer, respectively.

According to the continuity of the electric field and its derivative at the boundaries, by using the transfer method, we can obtain the relationship of coefficients between the first layer and the mth layer

$$\begin{pmatrix} a_{mr} \\ b_{mr} \end{pmatrix} = \begin{pmatrix} tr_{11}(\beta_r) & tr_{12}(\beta_r) \\ tr_{21}(\beta_r) & tr_{22}(\beta_r) \end{pmatrix} \begin{pmatrix} a_{1r} \\ b_{1r} \end{pmatrix}. \tag{3}$$

For the guided mode, the field should be evanescent for $x < x_s$ and $x > x_e$, so we have $b_{1r} = 0$ and $a_{mr} = 0$, then the problem of solving β_r is converted to the root search of $\mathrm{tr}_{11}(\beta_r) = 0$. By selecting the searching step properly, through the bisection method, we may obtain the propagation constant β_r for all modes.

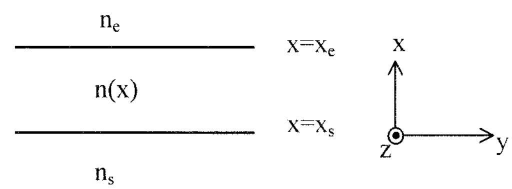

Fig. 2. Waveguide structure.

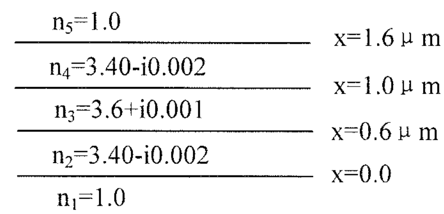

Fig. 3. Five-layer waveguide.

B. For the Perturbation Solution

After getting β_r, we consider the imaginary part of indexes and obtain the approximate complex propagation constant for each mode according to the perturbation method [3], [4],

$$\beta_p^2 = \beta_r^2 + \frac{\int_{-\infty}^{+\infty} (n^2 - n_r^2)E_{yr}^2\, dx}{\int_{-\infty}^{+\infty} E_{yr}^2\, dx} \tag{4}$$

where n is the complex index, β_r and E_{yr} have been determined from Section II-A. Here, the approximate propagation constant is obtained by treating the imaginary part of $n(x)$ as a perturbation to the real part n_r. Through the perturbation method, we can get the propagation constants quickly, but its accuracy can not be assessed.

C. For the Shooting Solution

When the imaginary part of $n(x)$ is considered, the wave equation has a same form as (1), the only difference is that β_r, n_r and E_{yr} are replaced by the complex β,n and E_y, respectively. The electric field in the jth layer is

$$\begin{aligned} E_y^j(x) &= a_j\ \exp[\alpha_j(x - x_j)] + b_j\ \exp[-\alpha_j(x - x_j)] \\ \alpha_j &= \sqrt{\beta^2 - (k_0 n_j)^2}. \end{aligned} \tag{5}$$

Again, the transfer matrix method is adopted according to the continuity of E_y and dE_y/dx at the boundaries, the coefficient

TABLE I
THE CALCULATED MODE INDEXES FOR VARIOUS METHOD

mode	neff$_r$	neff$_p$	neff$_{RPS}$	neff$_{SMA}$
TE$_0$	3.5035135	3.5035010+i7.1018397E-3	3.5034433+i7.1029987E-3	3.5034433+i7.1030010E-3
TE$_1$	3.3373277	3.3373251-i2.2912350E-4	3.3372869-i2.2949241E-4	3.3372869-i2.2949110E-4
TE$_2$	3.2516435	3.2516412-i5.3161401E-4	3.2516853-i5.3051366E-4	3.2516852-i5.3051478E-4
TE$_3$	3.1042640	3.1042597+i1.3475890E-3	3.1042515+i1.3379858E-3	3.1042514+i1.3379863E-3
TE$_4$	2.8786258	2.8786227-i1.7587958E-4	2.8786369-i1.7372970E-4	2.8786368-i1.7372989E-4
TE$_5$	2.6281238	2.6281189+i1.5834542E-3	2.6281394+i1.5486446E-3	2.6281393+i1.5486443E-3
TE$_6$	2.2439623	2.2439574+i7.3441047E-4	2.2439515+i7.0837744E-4	2.2439514+i7.0837796E-4
TE$_7$	1.7681716	1.7681655+i1.4428402E-3	1.7681912+i1.3532174E-3	1.7681910+i1.3532172E-3
TE$_8$	1.0742819	1.0742761+i3.0570532E-3	1.0742623+i2.4578920E-3	1.0742620+i2.4578915E-3
TM$_0$	3.4967439	3.4967400+i6.6407201E-3	3.4966838+i6.5439798E-3	3.4966838+i6.5439817E-3
TM$_1$	3.3307270	3.3307245-i2.7371771E-4	3.3306972+i3.5185421E-5	3.3306971+i3.5186422E-5

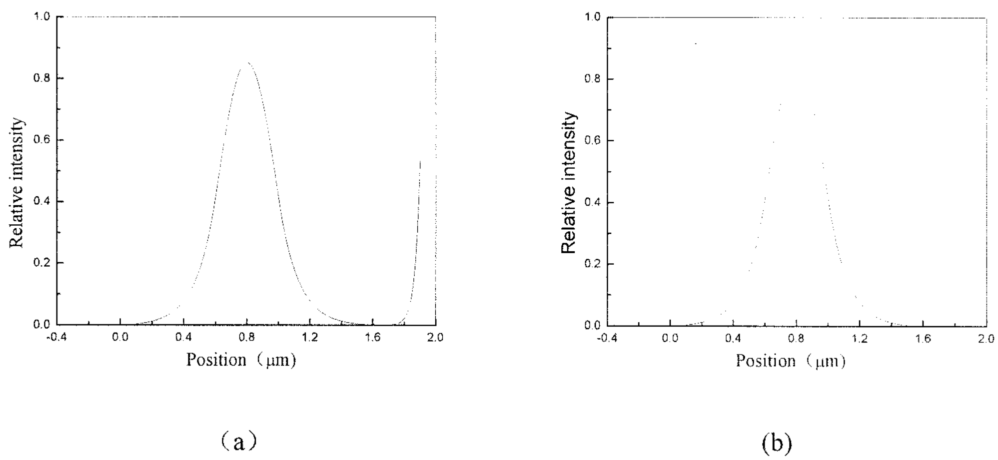

Fig. 4. Near-field distribution of TE$_0$ mode for (a) perturbation method and (b) RPS method

relationship between the first layer and the mth layer can be expressed as

$$\begin{pmatrix} a_m \\ b_m \end{pmatrix} = \begin{pmatrix} t_{11}(\beta) & t_{12}(\beta) \\ t_{21}(\beta) & t_{22}(\beta) \end{pmatrix} \begin{pmatrix} a_1 \\ b_1 \end{pmatrix}. \quad (6)$$

For guided modes, E_y must vanish for $x < x_s$, so $b_1 = 0$, we have

$$\begin{pmatrix} a_m \\ b_m \end{pmatrix} = \begin{pmatrix} t_{11}(\beta) \\ t_{21}(\beta) \end{pmatrix} a_1. \quad (7)$$

For a test value $\tilde{\beta} = \tilde{\beta}_r + i\tilde{\beta}_i$, by setting $a_1 = 1$, we can get E_y^m through (7) and (5). For guided modes, E_y also must vanish for $x > x_e$, so if $\tilde{\beta}$ is the guided mode propagation constant, it should satisfy

$$\begin{aligned} f(\tilde{\beta}) &= f_r(\tilde{\beta}_r, \tilde{\beta}_i) + if_i(\tilde{\beta}_r, \tilde{\beta}_i) \\ &= \frac{dE_y^m(x_e)}{dx} + E_y^m(x_e)\sqrt{\tilde{\beta}^2 - (k_0 n_e)^2} = 0. \quad (8) \end{aligned}$$

Equation (8) can be solved by using the two-dimensional (2-D) Newton iterative method [5]. Here, we use β_p obtained from Section II-B as the initial test value of $\tilde{\beta}$, because β_p is very close to the true solution, not only the iterative time are much less, but also it can assure that the solution will not jump to other modes.

III. APPLICATION AND DISCUSSION

As an example, we select the five-layer slab waveguide [6] shown in Fig. 3 to verify the RPS method. The numerical results given in Table I tabulate all mode indexes $n_{\text{eff}}\,(=\beta/k_0)$ by using the real index method, the perturbation method and the RPS method, respectively, the corresponding wavelength is 1.3 μm, the results of the scattering matrix approach (SMA) [6] are also listed for comparing. It is obviously that the results of RPS method are consistent with SMA very well.

It should be noted that this method can also be used to TM mode. The basic equations and the solution procedures of TM mode are the same as that of TE mode, except that E_y is replaced with E_x and the continuity of $E_y$between two nearby layers is replaced with n^2E_x. Some results of TM mode are also listed in Table I.

In order to judge the accuracy of the result, a simple method is to see the near field distribution. For a correct solution, the field should vanish when $x < x_s$ and $x > x_e$. Fig. 4 shows the calculated near field distribution for TE_0 mode by using the perturbation method and the RPS method, the fields have been normalized. As shown in the figure, when $x > x_e$, the near field of the RPS method vanishes, but the near field of the perturbation method tends to infinity, so the perturbation solution can not be used as the solution.

IV. Conclusion

In conclusion, the RPS method, which combines the real index method, the perturbation method and the shooting method reasonably, can get all mode constants of multimode slab waveguides with complex indexes; the method is simple, fast and accurate. In addition, this method does not need to search the eigenvalue in the whole complex plane, and is an effective approach used to obtain the complex propagation constant.

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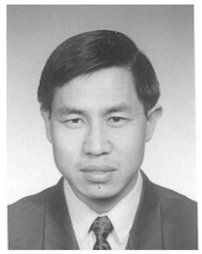

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Changhua Chen, photograph and biography not available at the time of publication.

Guangdi Shen, photograph and biography not available at the time of publication.