Analysis of Three-Dimensional Embedded Transmission Lines (ETL's)

Ali Darwish, Amin Ezzeddine, Ho C. Huang, and Misoon Mah

***Abstract*— Three-dimensional (3-D) circuits promise a significant reduction in circuit size and cost. In 3-D circuits, a few transmission line configurations are encountered including the offset stripline and the embedded transmission line (ETL), a stripline-like topology with two dielectrics. The ETL may have either two ground planes (similar to an offset stripline) or one ground plane (similar to an inverted microstrip). There is a need for an accurate solution to predict the effective dielectric constant and characteristic impedance of these transmission line structures. This paper provides an accurate (1% maximum error) closed-form empirical formula for the effective dielectric constant and compares it with full-wave simulations. In addition, this letter provides an empirical formula for the characteristic impedance and compares it with full-wave simulations of the structure. Close agreement between the two approaches is observed over a wide range of parameters.**

***Index Terms*— MMIC, multilayer, nonhomogeneous media, polyimide films, stripline, three-dimensional, transmission line theory.**

I. Introduction

The recent boom in wireless communications underscores the need for providing inexpensive microwave circuits and higher levels of on-chip integration. A promising approach is to build three-dimensional (3-D) microwave circuits by laminating multiple microwave circuit layers on top of each other while keeping all active devices on the semiconductor layer. This approach lowers cost by saving valuable real estate space [1]. Recently, several multilevel monolithic microwave integrated circuits (MMIC) have been reported [2]–[4]. To fully benefit from this technology, we must develop simple mathematical models to accurately predict the behavior of various 3-D components. The current approach of full-wave simulation of these structures is time consuming and challenging from a design point of view. This paper addresses two fundamental aspects of transmission line analysis: the effective dielectric constant and the characteristic impedance. The more complicated issue of circuit loss calculations shall be deferred to another publication. In an earlier work, the authors developed a closed-form expression for the effective dielectric constant and characteristic impedance [5]. This letter takes a different approach to the problem. The formula presented here for the effective dielectric constant is, in general, more accurate than that in the earlier work.

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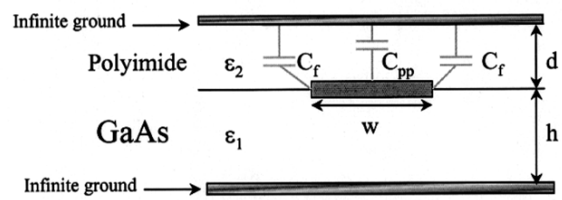

Fig. 1. An ETL with two different dielectric constants.

In a typical 3-D integrated circuit (3DIC), active devices, resistors, and capacitors are implemented on a GaAs (or Si) wafer. Next, a thin dielectric film (e.g., BCB or polyimide) is spun on top of the wafer and a metal layer is deposited on the dielectric to serve as a ground plane. One may add more dielectric and metal layers. The different ground planes are electrically connected using via holes. The first layer of transmission lines patterned (or fabricated) on top of the GaAs (or Si) wafer and is covered by another dielectric and a ground plane on top (see Fig. 1). Thus we have an offset stripline with an inhomogeneous dielectric. Flip-chip MMIC's use the same configuration [6], [7]. This configuration has been termed an *embedded transmission line* (ETL) configuration [7].

II. Theoretical Analysis

The ETL presents a complex analytical problem since it has an inhomogeneous dielectric where a TEM mode of propagation is not supported. However, we may search for a quasi-TEM mode that is valid in the quasi-static case. In the microstrip case, an exact solution has been expressed in implicit form in terms of elliptic functions and a simpler approximate solution was developed through conformal mapping [8]. Also, a symmetric homogenous stripline has been analyzed rigorously by conformal mapping [9] to yield an exact expression for line capacitance in terms of elliptic functions. Another powerful approach for the calculation of the capacitance of a structure is the variational method [10], [11]. The variational method can handle a wide variety of structures including microstrip, stripline, and suspended substrate lines [12].

The closed-form empirical expressions we derive in this paper are based on the variational method used in [12]. Applying the variational method to the specific case at hand, we arrive at the following expression for the capacitance [12, eq. (17)]):

$$\frac{1}{C} = \frac{1}{\pi Q^2 \varepsilon_0} \int f(x)^2 g(x)\, dx \tag{1}$$

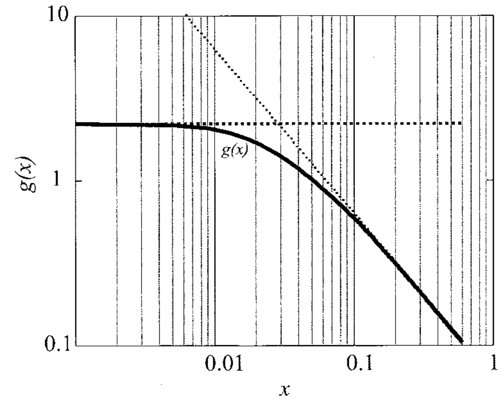

Fig. 2. Piecewise approximation (dashed line) of $g(x)$ (solid line).

where

$$\frac{f(x)}{Q} = \frac{8}{5}\,\frac{\sin(x\cdot w/2)}{x\cdot w/2} + \frac{12}{5\cdot(x\cdot w/2)^2} \cdot \left\{\cos(x\cdot w/2) - \frac{2\sin(x\cdot w/2)}{x\cdot w/2} + \frac{\sin^2(x\cdot w/4)}{(x\cdot w/4)^2}\right\} \tag{2}$$

$$g(x) = \frac{1}{|x|\cdot(\varepsilon_1\cdot\coth(|x|\cdot h) + \varepsilon_2\cdot\coth(|x|\cdot d))} \tag{3}$$

where $\varepsilon_1, \varepsilon_2, w, d$, and h, are as shown in Fig. 1, Q is the total line charge per unit length, $f(x)$ is related to the Fourier transform of the charge distribution on the strip, and $g(x)$ is a function related to the Fourier transform of the potential distribution. As it stands, the integral in (1) cannot be solved analytically. However, as shown graphically in Fig. 2 we can approximate $g(x)$ as

$$g(x) = \begin{cases} \dfrac{1}{(\varepsilon_1/h + \varepsilon_2/d)} & x \le \dfrac{(\varepsilon_1/h + \varepsilon_2/d)}{(\varepsilon_1+\varepsilon_2)} \\ \dfrac{1}{|x|\cdot(\varepsilon_1+\varepsilon_2)} & x \ge \dfrac{(\varepsilon_1/h + \varepsilon_2/d)}{(\varepsilon_1+\varepsilon_2)}. \end{cases} \tag{4}$$

Then through series expansion we may write $[f(x)/Q]^2 g(x)$ as

$$\left(\frac{f(x)}{Q}\right)^2 g(x) \approx \begin{cases} \dfrac{1}{\left(\dfrac{\varepsilon_1}{h}+\dfrac{\varepsilon_2}{d}\right)}\left(1 - \dfrac{2}{5}y^2 + \dfrac{37}{600}y^4 + \cdots\right), \\ \qquad x \le \dfrac{(\varepsilon_1/h + \varepsilon_2/d)}{(\varepsilon_1+\varepsilon_2)} \\ \dfrac{1}{\left(\dfrac{\varepsilon_1}{h}+\dfrac{\varepsilon_2}{d}\right)}\left(\dfrac{1}{y} - \dfrac{2}{5}y + \dfrac{37}{600}y^3 + \cdots\right), \\ \qquad x \ge \dfrac{(\varepsilon_1/h + \varepsilon_2/d)}{(\varepsilon_1+\varepsilon_2)} \end{cases} \tag{5}$$

where $y = xw/2$. After the series expansion we can easily carry out the integration of $1/C$ above and calculate the effective dielectric constant by computing the ratio of C/C_{air}, where C_{air} is the capacitance of the structure with all dielectrics replaced by air. Thus, we obtain the following expression for ε_{eff}:

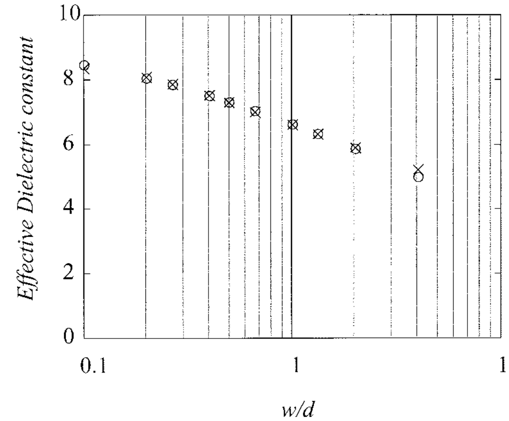

Fig. 3. Effective dielectric constant of ETL line on 100-μm GaAs substrate and covered with polyimide of thickness d and $w = 20\ \mu$m. We compare full wave simulation "×" with and our empirical formula "o."

$$\varepsilon_{\text{eff}} = \left(\frac{\varepsilon_1+\varepsilon_2}{2}\right)\cdot\frac{U(K(\varepsilon_1,\varepsilon_2))}{U(K(1,1))} \tag{6}$$

where

$$U(x) = 1.15 - \ln(x) + 0.08\cdot x^2 \tag{7}$$

$$K(\varepsilon_1,\varepsilon_2) = \frac{1}{\varepsilon_1+\varepsilon_2}\cdot\left[\frac{\varepsilon_1}{\left(\dfrac{2h}{w}\right)} + \frac{\varepsilon_2}{\left(\dfrac{2d}{w}\right)}\right]. \tag{8}$$

Comparing the values obtained from the previous expression for ε_{eff} with those calculated by full-wave EM simulation [13], for a wide variety of practical line widths and polyimide thicknesses, we observe an error less than 1% for $\text{K}(\varepsilon_1,\varepsilon_2) < 1$ (in typical cases, this will include the range of $0.1 < w/h < 2$ and $0.1 < w/d < 2$). And when $\text{K}(\varepsilon_1,\varepsilon_2) \ll 1$, the accuracy is much better than 1%. Fig. 3 shows a comparison of the effective dielectric constant calculated with the above formula and full-wave simulation for an ETL with $w = 20\ \mu$m, $h = 100\ \mu$m, $\varepsilon_1 = 3.2$ (polyimide), and $\varepsilon_2 = 12.9$ (GaAs). Finally, we plot the percentage error observed for a wide variety of ETL cases with randomly selected parameters (see Fig. 4). The agreement between our result (based on variational method) and full-wave EM simulation (based on a Method of Moments code) is significant since we arrived at each answer using a different route.

Given the above expression for the effective dielectric constant we may proceed to calculate the characteristic impedance using

$$Z_o = \frac{\sqrt{\mu_0\varepsilon_0\varepsilon_{\text{eff}}}}{C} = \frac{\sqrt{\mu_0\varepsilon_0}}{\sqrt{\varepsilon_{\text{eff}}}C_{\text{air}}}. \tag{9}$$

The capacitance C_{air} can be calculated based on (1). Alternatively, we can calculate C_{air} more accurately based on classical expressions [14]–[17]. Based on classical expressions (assuming negligible conductor thickness) C_{air} equals

$$C_{air} = \varepsilon_0(C_{pp} + 2\cdot C_f)_{x=h} + \varepsilon_0(C_{pp} + 2\cdot C_f)_{x=d} \tag{10}$$

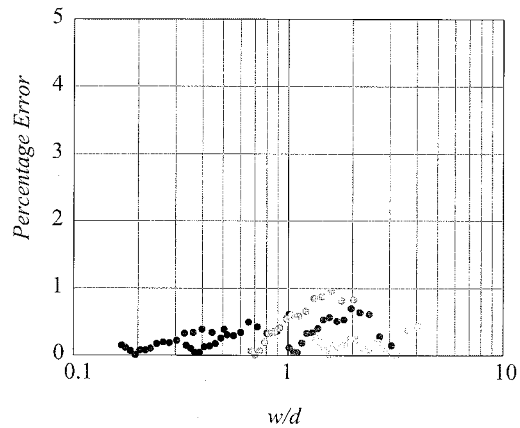

Fig. 4. Percentage error observed for a wide variety of ETL lines with randomly selected parameters.

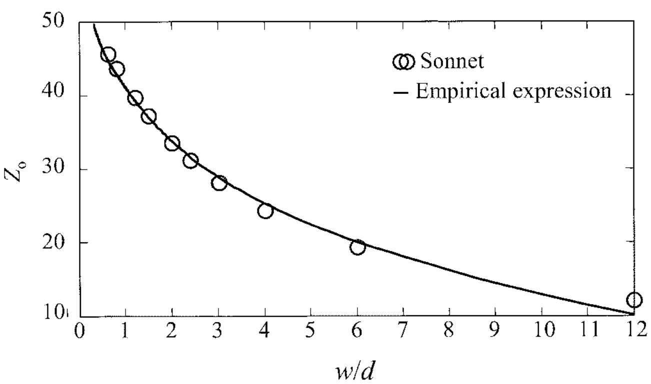

Fig. 5. Characteristic impedance of ETL line on 100-μm GaAs substrate and covered with polyimide of thickness $d = 25$ μm, where w is varied. We compare full-wave simulation with and our empirical formula.

where

$$C_{pp} = \frac{w}{x} + \begin{cases} 0, & \frac{w}{2x} \geq 0.35 \\ -2(0.35 - w/2x)^2, & \frac{w}{2x} \leq 0.35 \end{cases} \quad (11)$$

$$C_f = \frac{\ln(4)}{\pi}. \quad (12)$$

In (10), C_{air} is defined as the sum of parallel plate capacitance (C_{pp}) and fringing capacitance (C_f) between the line and upper and lower grounds (see Fig. 1). Now, we can substitute (6) and (10) into (9) to obtain the characteristic impedance Z_o. We compare Z_o computed using (9) with full-wave EM simulation (see Fig. 5) for an ETL with $d = 25$ μm, $h = 100$ μm, $\varepsilon_1 = 3.2$, and $\varepsilon_2 = 12.9$ where w is varied. In general, we observe that the error is less than 5% provided that: 1) we maintain $\text{K}(\varepsilon_1, \varepsilon_2) < 1$ (for the series expansion in (5) to converge) and 2) keep $0.1 < d/h < 10$. Most of the error in characteristic impedance Z_o comes from the expression for C_{air} rather than the expression for ε_{eff}. In the preceding analysis, the metal thickness t was assumed to be negligible compared with the line width w and the thickness of the dielectrics d and h ($t/w \ll 1, t/d \ll 1$, and $t/h \ll 1$). In practice, the metal can be as thick as 5 μm and the polyimide thickness can be as thin as 6 μm. In that case, the capacitance expressions (11) and (12) should be modified to reflect the increased capacitance (refer to [14]–[17]).

III. Conclusion

Based on variational principles, we developed a simple closed-form formula for the calculation of the effective dielectric constant of the stripline form of ETL with small error (less than 1%). The formula's range of validity covers the majority of cases encountered in practice. In addition, we developed a formula for the characteristic impedance using a standard expression for the capacitance of an offset stripline. Further analysis of 3-D integrated circuit components and effects are needed to bring the 3-D technology to the market.

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