Efficient FD Formulation for Lossy Waveguide Analysis Based on Quasi-Static Field Characteristics

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***Abstract*—This letter treats the finite-difference (FD) analysis of transmission lines including metallic loss. Computational costs are reduced drastically by incorporating correction factors into the FD equations in frequency domain. These factors are obtained from the quasistatic field behavior in the vicinity and within the metallic conductors. Thus, FD analysis can be performed using a coarse grid without loss of accuracy. The new method is verified for a typical monolithic microwave integrated circuit (MMIC) coplanar waveguide.**

***Index Terms*— Coplanar waveguide, finite difference, MMIC, quasi-statics, transmission-line analysis.**

I. Introduction

THE finite-difference (FD) method is one of the most powerful electromagnetic simulation tools today and its time-domain formulation (FDTD) is in widespread use. Nevertheless, its capabilities still fall short off the needs in monolithic microwave integrated circuit (MMIC) design. There is the necessity to include housing effects, and finally one would like to analyze entire chips. However, the smallest dimensions on the chip are in the range of micrometers, such as metallization thickness in coplanar MMIC's, whereas the largest ones are in the millimeter range. In the conventional FD method, this huge difference in length leads to an excessively large mesh size. If conductor losses are considered situation gets even worse since skin-depth further reduces the smallest cell size required.

Since the first formulation of the FDTD method by Yee, a lot of work has been done to improve efficiency with regard to this problem. In the beginning of the 1990's an enhanced FDTD method was proposed incorporating the asymptotic field behavior around microstrip discontinuities [1]. Other contributions followed [2]–[4] using *a-priori* knowledge from analytical or numerical solutions of static fields. Until now, however, the treatments are confined to the lossless case.

A common method to include conductor loss is the surface impedance [5] approach, which, however, relies on a simplified one-dimensional (1-D) field dependence within the conductors. This assumption is questionable for structures with metal thicknesses comparable to skin depth and inherently two-dimensional (2-D) field distributions, as is the case for MMIC coplanar lines, for instance.

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The approach presented here does not involve such restrictions but applies only the quasi-static assumption. The principal idea is that geometrical details and skin-depth are treated *a priori* by a quasi-static approach. Since in most MMIC structures wavelength is much larger than these dimensions, our approach enhances efficiency without sacrificing accuracy. The method is similar to [3], which however was restricted to the lossless case. In this letter, ohmic losses are considered in such a numerical approach for the first time. Doing this adds complexity since now the full magnetostatic formulation has to be treated including the frequency-dependent eddy-current problem. Because skin depth usually is the smallest length quantity, however, the savings in computational efforts due to the new approach are higher than in the lossless case of [3]. To demonstrate validity and benefits of the improved algorithm, it is applied to a typical CPW structure and compared to the conventional FD frequency-domain (FDFD) method.

II. The Improved FD Formulation

According to [6] we derive the finite-difference frequency-domain (FDFD) approach starting from the integral forms of Ampère's law (1) and Faraday's law (2) for each elementary cell

$$\oint_{C(A)} \frac{\vec{B}}{\mu} \cdot d\vec{s} = \iint_A j\omega\vec{D} \cdot d\vec{A} \quad (1)$$

$$\oint_{C(A)} \vec{E} \cdot d\vec{s} = -j\omega \iint_A \vec{B} \cdot d\vec{A}. \quad (2)$$

Matrix equations are set up for each of (1) and (2) which possess the same properties (*div rot* $\vec{F} = 0$, etc.) as their analytical counterparts. These equations are then combined to give a matrix wave-equation for the unknown electric field components in the FD grid.

The task of determining the propagation characteristics of a longitudinally homogeneous waveguide is reduced to an eigenvalue problem for the transverse electric field by imposing the condition $div\varepsilon\vec{E} = 0$ in the FD grid. This eigenvalue problem is then solved by an implicitly restarted Arnoldi method.

In order to improve efficiency, the FD equations are modified as in [1] and [3] using *a-priori* knowledge of the quasi-static field behavior. This information is introduced by means of so-called correction factors, which are multiplied to the integrals over the electric and magnetic field. In contrast to the lossless case, however, the correction factors for the magnetic field quantities are frequency dependent in the lossy case.

To illustrate the incorporation of correction factors into the FD formulation, one may consider the line and surface integral approximations given in (3) and (4), respectively. F' denotes the discretized field value. In the conventional FD method, the line integral is approximated by $F'\Delta x$. In the new method, a correction factor CF_x is introduced, which is determined from the ratio between line integral and centered value of the quasi-static case. The surface integral in (4) is treated correspondingly

$$\int F \cdot dx \approx F' \cdot \Delta x \cdot CF_x \quad \text{where}$$

$$CF_x = \frac{\int F \cdot dx}{F' \cdot \Delta x} \quad \text{in the quasi-static case.} \tag{3}$$

$$\iint F \cdot dx\, dy \approx F' \cdot \Delta x \cdot \Delta y \cdot CF_A \quad \text{where}$$

$$CF_A = \frac{\int\int F \cdot dx\, dy}{F' \cdot \Delta x \cdot \Delta y} \quad \text{in the quasi-static case.} \tag{4}$$

The correction factors are obtained from a quasi-static FD analysis, which is performed before its dynamic counterpart. Note that a quasi-static approach is always possible if the characteristic dimensions l of a structure are small compared to wavelength λ.

The quasi-static approach consists of an electrostatic and a quasi-static magnetic part yielding the correction factors for the corresponding electric and magnetic field integrals, respectively. In the electrostatic case, a FD approximation of the Poisson equation is solved for the electrostatic potential. The solution of the corresponding symmetric linear system of equations can be obtained using a conjugate-gradient-type method. For the quasi-static magnetic part, a more involved treatment is required since in the presence of losses the fields become frequency dependent and cannot be reduced to a scalar potential. For the quasistatic magnetic problem the first Maxwell equation reduces to

$$\oint_{C(A)} \frac{\vec{B}}{\mu} \cdot d\vec{s} - \iint_A \sigma\vec{E} \cdot d\vec{A} = \iint_A \sigma\vec{E}_0 \cdot d\vec{A}. \tag{5}$$

Equation (5) contains the electric field source E_0, for which (without loss of generality) a homogeneous distribution over a conductor cross section is assumed. In the case of the CPW in Fig. 1, for instance, we assumed a constant electrical field in z-direction $E_0 = +1$ V/m over the cross section of the outer conductor and $E_0 = -1$ V/m over the cross section of the inner conductor, respectively. Equation (5) is discretized in the same way as (1). The FD matrix equations of (5) and (2) are then combined such that the magnetic flux density is eliminated. The resulting system of equations for the electric grid field is solved by means of Krylov Subspace methods. For transmission-line analysis, the electric field of the quasistatic *magnetic* part has only the longitudinal component z.[1] This means one has a scalar problem. Finally, the transverse magnetic flux density is obtained by using (2).

[1] Note that the transverse components are covered by the *electrostatic* part.

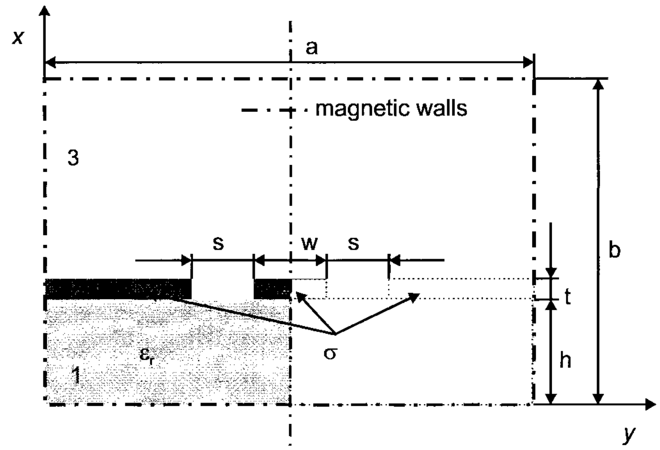

Fig. 1. Coplanar cross section under investigation (dimensions are $w = 20$ μm, $s = 15$ μm, $t = 3$ μm, $h = 500$ μm, $a = 450$ μm, $b = 1503$ μm, $\varepsilon_r = 12.9$, $\sigma = 3 \cdot 10^7$ S/m).

In order to exploit the advantages of the hybrid method, two different meshes are to be used for the quasi-static and the dynamic analysis. A fine mesh is required for the quasi-static part, which, however, can be confined to regions of high-field gradients, i.e., inside and in the vicinity of the conductors, whereas the dynamic part is based on a coarse grid covering the entire volume. This leads to a significant reduction of computational costs because in most cases the extra processing time required for the quasi-static computations is considerably smaller than the time needed for a full-wave analysis.

III. Results

In order to verify the new hybrid FD method a typical MMIC CPW structure is studied. Fig. 1 shows the geometry and provides the parameters. Due to symmetry, only one half of the structure is considered introducing a magnetic wall at $a/2$.

Three different meshes are used which are referred to as the coarse, the intermediate, and the fine one in the following. A detailed view of the intermediate mesh in the vicinity and within the outer conductor is shown in Fig. 2. The dynamic FD analysis is performed on the coarse mesh, whereas the intermediate or the fine mesh is used to compute the correction factors. Additionally, to demonstrate efficiency improvement of the hybrid approach and to establish a reference the conventional FD method is applied on the fine mesh, too. All meshes are graded. Starting with the smallest cells, the cell size is increased successively by a constant factor q. For the coarse mesh, the smallest cell size, which is located at the corners of the inner conductor, is equal to $\Delta x_{\min} = t = 3$ μm and the largest one is $\lambda_{\min}/10 = 83$ μm at the surrounding magnetic wall ($\lambda_{\min}$ denotes wavelength for a permittivity ε_r at 100 GHz). The slot and the center conductor, respectively, are described with only one cell in y-direction. For the fine and the intermediate mesh, the skin-depth δ of the metal at the highest frequency (100 GHz) determines minimum cell size. In our case, it is chosen to be about $\delta/3 \cong 0.09$ μm and $\delta/20 \cong 0.0135$ μm, respectively, whereas the largest cell size is about $\lambda_{\min}/18 \cong 45.273$ μm for both meshes. The factor q is about 1.5 in the coarse mesh and 1.2 in the two other ones, respec-

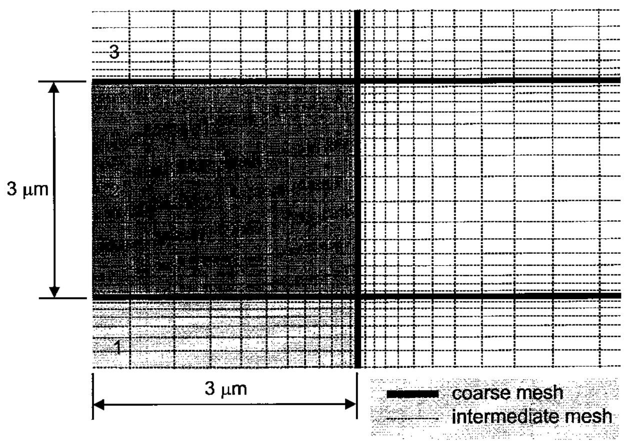

Fig. 2. Detailed view of the coarse and the intermediate mesh with $\Delta x_{\min} \approx \delta/3$ in the vicinity and within the right-hand edge of the outer conductor (see Fig. 1).

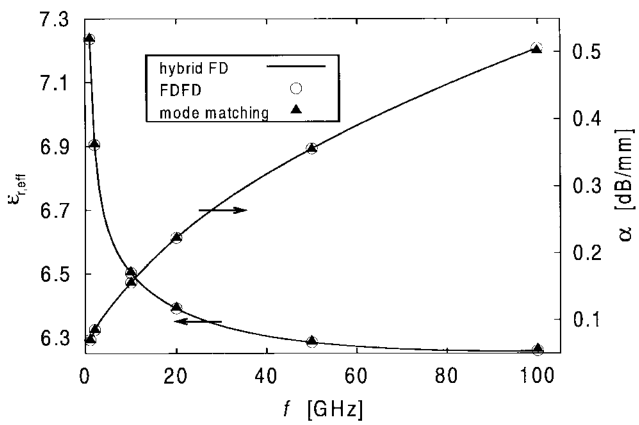

Fig. 3. Effective dielectric constant $\varepsilon_{r,\mathrm{eff}}$ and attenuation of the CPW in Fig. 1 against frequency: comparison between new hybrid method, conventional FD method (fine mesh with $\Delta x_{\min} \approx \delta/20$), and mode-matching results [7].

tively. In order to illustrate the differences in mesh resolution one may consider the slot: in the coarse mesh, it is discretized in y direction by a single cell, whereas in the intermediate and the fine mesh 32 cells and 52 cells, respectively, are used.

In Fig. 3, results of the new method are compared to the conventional FD method on the finest grid and mode-matching results obtained by the approach in [7]. Effective dielectric constant $\varepsilon_{r,\mathrm{eff}}$ and attenuation α are plotted as a function of frequency. The three curves cannot be distinguished within the plot scale thus verifying the new method. More precisely, the FD results agree with the mode-matching data with an error of less than 0.05% in β and 1% in α, respectively.

Fig. 4 provides a more detailed view presenting data on the relative errors of the propagation constant $k_z = \beta - j\alpha$. Reference for the errors is a conventional FDFD analysis on the fine grid. As can be seen the new hybrid approach describes the full-wave results with less than 1.25% error for frequencies up to 100 GHz if the intermediate mesh with minimum cell size of about $\delta/3$ is used. For the fine discretization, the deviations between hybrid and conventional approach decrease further.

Regarding the computational efforts, the eigenvalue problem on the coarse grid (i.e., the dynamic part of the approach) can

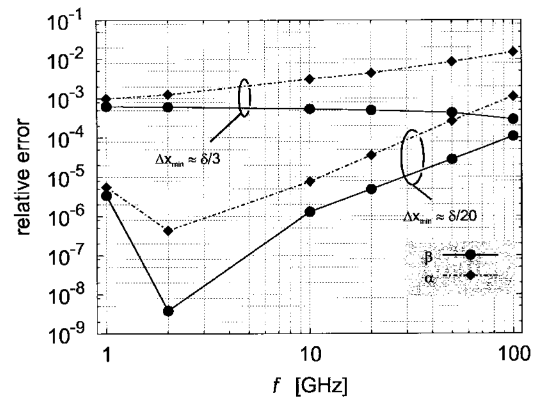

Fig. 4. Relative error of the complex propagation constant $k_z = \beta - j\alpha$ of the hybrid method for two different grids with $\Delta x_{\min} \approx \delta/3$ and $\Delta x_{\min} \approx \delta/20$ (reference: conventional FD method on fine grid with $\Delta x_{\min} \approx \delta/20$).

be solved about 2000 times faster than that on the fine grid. Of course, additional preprocessing time is required to perform the quasi-static analysis and to compute the correction coefficients. With our preliminary version we achieved for both the intermediate and the fine mesh an overall CPU time reduction of 75% (comparing the hybrid method to the conventional one). Using optimized algorithms for preprocessing, a further reduction to more than 90% should be possible. As the eigenvalue problem can be solved on a coarse grid and the quasi-static problems are only scalar the storage can be reduced considerably as well. This reduction is about 50%.

IV. Conclusions

The incorporation of correction factors into the FD formulation is shown to give a marked improvement in computational cost while maintaining accuracy compared to the standard FDFD method. This approach provides a particular advantage when including nonideal conductivity. Structural details and skin effect are considered on a fine grid by quasistatic computations only. This leads to drastic savings in CPU time and storage, because a given accuracy can be achieved using a much coarser grid. For transmission-line analysis of typical MMIC CPW structure, we find CPU time being reduced to less than 10% and storage to 1/2.

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