FDTD Modeling of an Artificially Synthesized Absorbing Medium

Wenhua Yu, Raj Mittra, and Douglas H. Werner

***Abstract*— In this letter we investigate an artificially synthesized absorbing medium by using the finite-difference time-domain (FDTD) and waveguide simulation techniques. The artificial medium comprises a doubly periodic array of lossy electrical and magnetic media (i.e., an ε-only and μ-only checkerboard) and we compute its reflection coefficient for normal and oblique angles of incidence. It is demonstrated that, if properly designed, the reflection characteristics of the checkerboard are far superior to those of a uniform material of the same thickness with $\varepsilon = \mu$.**

***Index Terms*—Electromagnetic absorbing material, FDTD, periodic structures.**

I. Introduction

PERIODIC materials and the phenomenon of wave propagation in such media have received considerable attention in recent years [1]–[5]. This is primarily due to the possibility of using these periodic materials to achieve low reflection over a broad frequency band. In this letter, we discuss a novel periodic structure comprised of lossy dielectric and magnetic materials in a checkerboard-type configuration. The search for an artificial structure that exhibits a good match to free space was motivated by the fact that, to date, it has not been possible to find a real material which satisfies the criterion $\varepsilon_r' = \mu_r'$ and $\varepsilon_r'' = \mu_r''$—or similar ones that present a perfect match to free space—as does the well-known perfectly matched layer (PML) medium widely used for finite-difference time-domain (FDTD) and finite-element (FE) mesh truncation. For purposes of this letter, it is convenient to adopt the convention commonly used in the PML literature, where ε'' and μ'' are expressed in terms of an electric and magnetic conductivity σ and σ^* respectively. Therefore we let $\varepsilon_r = \varepsilon_r' - j\varepsilon_r''$ and $\mu_r = \mu_r' - \mu_r''$ where

$$\varepsilon_r'' = \frac{\sigma}{\omega\varepsilon_0} \tag{1}$$

$$\mu_r'' = \frac{\sigma^*}{\omega\mu_0} \tag{2}$$

which suggests that if $\varepsilon'' = \mu''$ then $\sigma^* = \eta_0^2\sigma$.

In this letter, the FDTD technique [6] is applied in conjunction with a periodic boundary condition to calculate the scattered fields from the periodic checkerboard structure mentioned above, for both normal and oblique incidence cases. Because the computation is carried out in the time domain, only one FDTD simulation is required to calculate the reflection coefficients in the frequency range of interest by using Prony's method.

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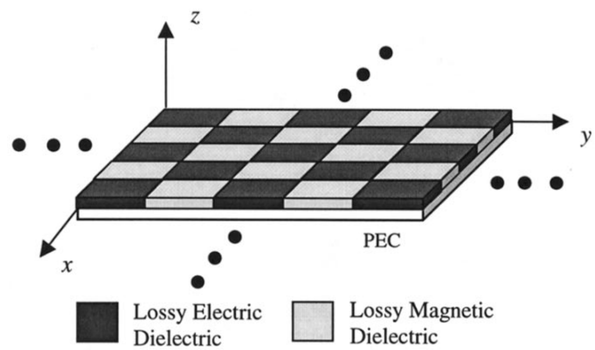

Fig. 1. Periodic structure backed by a PEC ground plane.

II. FDTD Modeling

The checkerboard type of periodic structure, described above, was numerically investigated by using the FDTD and its frequency response was computed. Fig. 1 shows the structure with a two-dimensional (2-D) periodicity, comprised of alternating dielectric and magnetic material blocks, backed by a perfect electric conductor (PEC) ground plane. The dark and light color blocks represent lossy dielectric and magnetic materials, respectively. A normally incident plane wave with E_y polarization is assumed to be propagating in the z-direction, and perfect magnetic conductor (PMC) and PEC boundary conditions are employed to truncate the periodic structure in the x- and y-directions, respectively. A PML type of absorbing boundary condition is imposed at $z = z_{\max}$, whereas a PEC boundary is inserted at $z = 0$. The unit cell for the periodic structure analyzed is shown in Fig. 2. The FDTD computational domain consists of $20 \times 20 \times 46$ cells, that includes two elements of the periodic structure in each direction, and utilizes a cell size of $\Delta x = \Delta y = \Delta z = 0.000\,25$ m. Time domain voltages are measured at four locations, which are 20, 22, 24, and 26 cells away from the interface of the periodic structure. They are then used to calculate the reflection coefficient of the checkerboard through Prony's method. The excitation source is a Gaussian pulse modulated by a sine function and is located 31 cells from the interface. The modeling of obliquely incident plane waves on a periodic structure is quite difficult to handle by conventional FDTD methods, and hence we employ a waveguide simulation technique to circumvent this problem. A TE_{10} mode, propagating along the z-direction, is

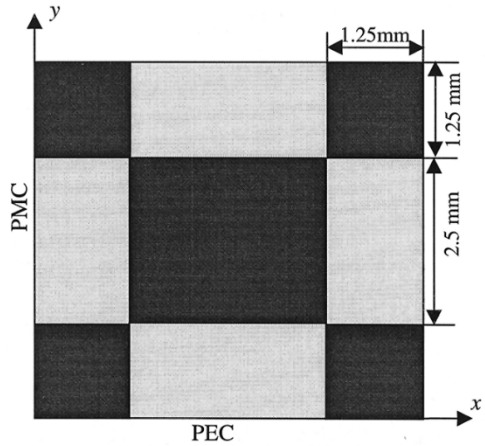

Fig. 2. Periodic structure truncated by a periodic boundary.

excited in the waveguide. The only difference from the normal incidence case is that the waveguide simulator utilizes three cells of the periodic structure in the x-direction, as opposed to only two for the normally incident case (see Fig. 5). The FDTD computational domain in the waveguide simulator consists of $30 \times 20 \times 46$ cells.

III. Numerical Results

In this section, we present some results for a few representative structures analyzed. The first of these is a periodic structure for which the relative permittivity and conductivity of the lossy dielectric are given by $\varepsilon_r = 16.0$ and $\sigma = 20$ S/m, respectively, whereas the relative permeability and conductivity of the lossy magnetic material are $\mu_r = 16.0$ and $\sigma^* = 20\ \mu_0/\varepsilon_0$ (see Table I). The material slabs are assumed to be square shaped, with a length of 0.0025 m on a side. The thickness of the periodic structure is 0.0025 m. Fig. 3 shows the time domain signatures at an observation point which is located at a distance of ten cells in front of the dielectric interface. The reflection from both the PEC plane and the dielectric interface can be observed in this signature. Furthermore, we can see that the reflection originating from the PEC plane vanishes as the dielectric becomes lossier. The FDTD results for the reflection properties of a uniform material with both electric and magnetic losses ($\mu = \varepsilon$ material), as well as that from the checkerboard structure, are plotted in Fig. 4. It is evident from this figure that the checkerboard structure is a much better absorber than the $\mu = \varepsilon$ material.

Modeling the oblique incidence case using a waveguide simulator is not as straightforward as it is for normal incidence, because a change in the frequency of excitation in a waveguide is accompanied by a change in the effective incidence angle (see Table II). One approach to handling this problem is to vary the transverse (x) dimension of the waveguide with a change in the frequency so as to hold the incident angle fixed. The geometry of the oblique incidence simulation is shown in Fig. 5. Once again, a TE_{10} mode is used to excite the waveguide and to simulate an effective angle of incidence, and the cutoff frequency of the waveguide is chosen to be 30 GHz. A ten-layer PML is used to truncate the waveguide at the input side and a checkerboard, backed by a PEC plane, is used to terminate the guide in the positive z direction.

Fig. 6 shows the reflection characteristics of two commonly used absorbing boundary conditions viz., the first-order Mur and a ten-layer PML, and compares them to that of a one-layer checkerboard whose thickness is only 2.5 mm. The response of a two-layer PML is also shown for comparison purposes. We see that reflection of the one-layer checkerboard is comparable to that of Mur's boundary condition over a broad frequency band. We also note here that the overall reflection from a one-layer PML would be higher than that produced by either the first order Mur or the one-layer checkerboard. It is evident that the performance of the ten-layer PML is considerably superior to that of the one-layer checkerboard

TABLE I
Material Parameters of the Checkerboard

Frequency (GHz)	Permittivity of Electric Squares		Permeability of Magnetic Squares	
	ε'	$\varepsilon''_r = \dfrac{\sigma}{\omega\varepsilon_0}$	μ'_r	$\mu''_r = \dfrac{\sigma^*}{\omega\mu_0}$
3.0	16.0	119.8366	16.0	119.8366
5.0	16.0	71.9019	16.0	71.9019
7.0	16.0	44.9387	16.0	44.9387
9.0	16.0	39.9455	16.0	39.9455
11.0	16.0	32.6827	16.0	32.6827
13.0	16.0	27.6546	16.0	27.6546
15.0	16.0	23.9673	16.0	23.9673
17.0	16.0	21.1476	16.0	21.1476
19.0	16.0	18.9216	16.0	18.9216
21.0	16.0	17.1195	16.0	17.1195
23.0	16.0	15.6309	16.0	15.6309
25.0	16.0	14.3804	16.0	14.3804
27.0	16.0	13.3152	16.0	13.3152
29.0	16.0	12.3969	16.0	12.3969
31.0	16.0	11.5971	16.0	11.5971
33.0	16.0	10.8942	16.0	10.8942
35.0	16.0	10.2717	16.0	10.2717
37.0	16.0	9.7165	16.0	9.7165
39.0	16.0	9.2182	16.0	9.2182
40.0	16.0	8.9877	16.0	8.9877

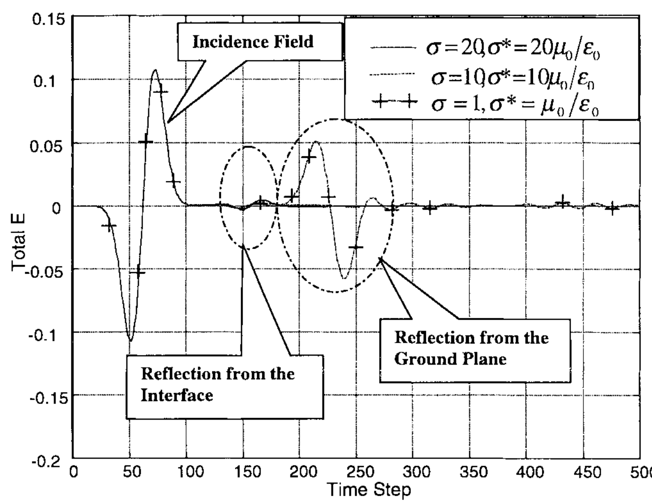

Fig. 3. Total field variation in the time domain calculated in front of the checkerboard for different values of material loss.

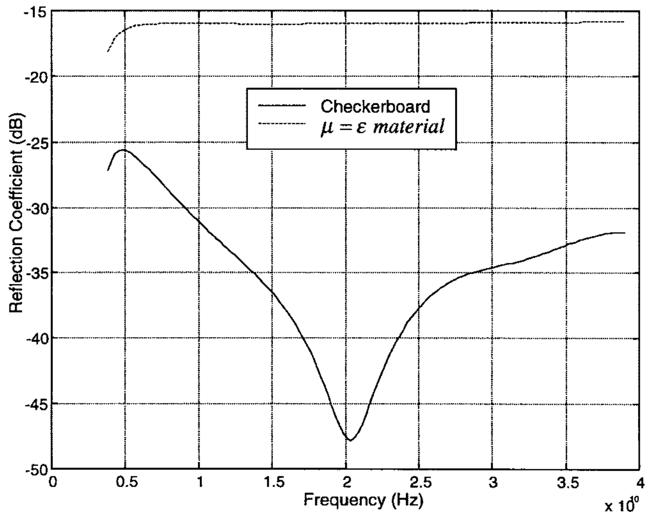

Fig. 4. Comparison between the reflection coefficient of a checkerboard and a lossy uniform material slab with $\mu = \varepsilon$ for normal incidence.

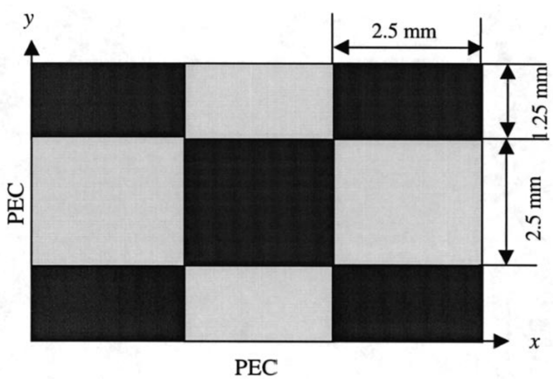

Fig. 5. The periodic structure used inside the waveguide.

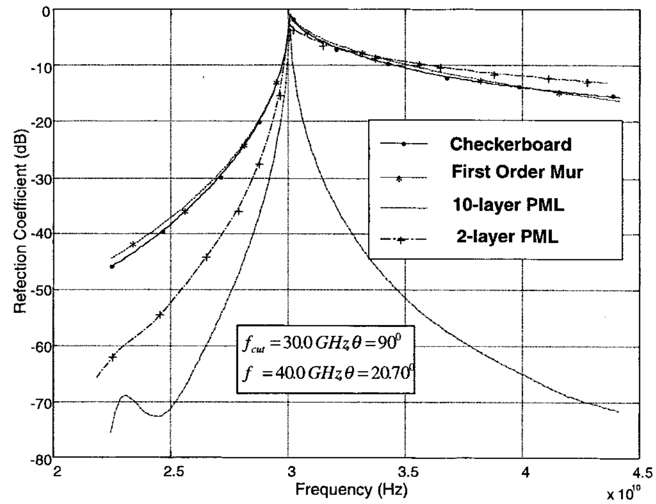

Fig. 6. Comparison of reflection coefficient for the checkerboard, first-order Mur, two-layer PML, and ten-layer PML absorbing boundary conditions used in waveguide simulation.

TABLE II
RELATIONSHIP BETWEEN FREQUENCY AND INCIDENT ANGLE

Frequency (GHz)	Angle (degrees)
30	90.00
31	75.41
32	69.64
33	65.38
34	61.93
35	59.00
36	56.44
37	54.18
38	52.14
39	50.28
40	48.59
41	47.03
42	45.58
43	44.24
44	42.99

and, not unexpectedly, the first-order Mur. However, the performance of the checkerboard can be improved by using multiple-layers as is typically done in a PML termination.

The simulations show that the reflection from the checkerboard is not very sensitive to the variation in the parameters of the dielectric and magnetic materials that make up the checkerboard. We also found that while the reflection characteristics of the checkerboard are somewhat dependent on the sizes of the ε and μ squares, we can compensate for this variation by adjusting the material parameters of these squares.

IV. CONCLUSIONS

We have shown that it is possible to synthesize an absorber by combining lossy dielectric (only) and magnetic (only) materials in a checkerboard configuration. The lossy dielectric slabs are readily available in practice. On the other hand, it is very difficult to find materials where the magnetic properties dominate at frequencies above about 1 GHz. The authors are currently exploring ways by which such magnetic media can be artificially synthesized by using appropriately shaped inclusions embedded in a background medium. The authors also plan to investigate the geometry and material parameters of the checkerboard that will provide optimal performance in terms of its absorption characteristics and to study the multilayer designs for the same.

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