An Approach to the Analysis of Arbitrarily Shaped Helical Groove Waveguides

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***Abstract*—A simple but accurate approach to the analysis of the arbitrarily shaped helical groove structures is presented in this letter. The unified dispersion equation is obtained by means of an approximate field-theory analysis, in which the profile of the groove is approximately replaced by a serious of steps and the field continuity at the interface of two neighboring steps together with the matching conditions at the interface between the groove region and central region are employed. The derived transcendental equation is resolved numerically. A half-circular helical groove structure was manufactured and the cold measurement was made. The experimental data are in good agreement with the numerical calculation results.**

***Index Terms*—Gyro-TWT, helical groove waveguide, millimeter-wave traveling wave tube, slow-wave structure.**

I. Introduction

RECENTLY, during the development of high-power broad-band traveling wave tube (TWT), the helical groove waveguide [1]–[4] has attracted the scholar's interest due to some of its peculiarities: large size, the transverse dimensions comparable to those of coupled cavity structures; high precession of manufacturing and assembling, the pitch can be machined to close tolerance; and superthermal conductivity and low loss, which make it worth consideration, especially at millimeter frequencies. And there is an increasing interest in variable-shape helical groove structure for use as a slow-wave structure (SWS) in a TWT [1]–[4]; meanwhile, the use such kinds of structures as operating systems for a gyrotron TWT allows significant widening of its bandwidth and an increase in efficiency [5]. Thus, the helical groove waveguide will play a pivotal role in the future traveling wave tube designs.

The investigation on the helical groove waveguide with rectangular groove shows that this structure is more dispersive, which is suitable for use in narrow-band millimeter-wave communication TWT's. The introduction of capacitive ridge-loading tends to reduce the dispersion of this circuit [1], [3], however the instantaneous bandwidth of the Raytheon's ridged helical TWT is only 1.3% [1]. The analysis of the helical half-circular-shaped groove waveguide indicates that this circuit has relatively broader band than that of the helical rectangular-shaped one [2]. For further broad-banding this structure, the authors recently have developed the circuit by means of increasing the arc-angle. So far, all the researchers have only considered the regular shape.

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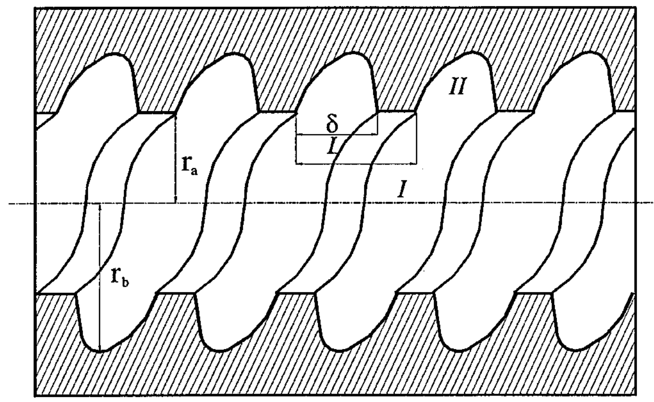

Fig. 1. Arbitrarily shaped helical groove waveguide.

For both analyzing such practical circuit such as the deformed structure, etc., and obtaining a lower dispersion circuit, it is necessary to consider the influence of the groove form, such as V-type, cosine-type, or sector-shape, etc. However, a unified approach to analyzing this kind of helical groove structure, even a simple one, is not presented. In the following text, a simple but accurate approach of analyzing helical arbitrarily shaped groove structures, as shown in Fig. 1, is presented by means of an approximate field-theory analysis.

II. Theory

In Fig. 1, δ is the width of the groove, L is the pitch, ψ is the pitch angle, and r_a and r_b are the radii of the mouth and bottom of the groove, respectively. Note that the boundary of the groove is arbitrarily whether it is smooth or not. In the analysis of the helical rectangular-shaped groove structure, the approximate expressions of the field components may be obtained by employing a helical coordinate system [4]. However, when the shape of the groove is nonrectangular, the theoretical solution of the wave equation cannot be obtained. Here, some approximations and assumption are made: 1) the profile of the groove is replaced by a series of rectangular steps which are as close to the profile as possible; 2) the discontinuity capacitance at the interface of two consecutive steps is neglected; and 3) each step supports the fundamental TE model which propagates along the helical direction, as carried out by a rectangular groove [4]. Then, the staircase representation of an arbitrarily groove is shown in Fig. 2, where the groove space is divided into M regions and

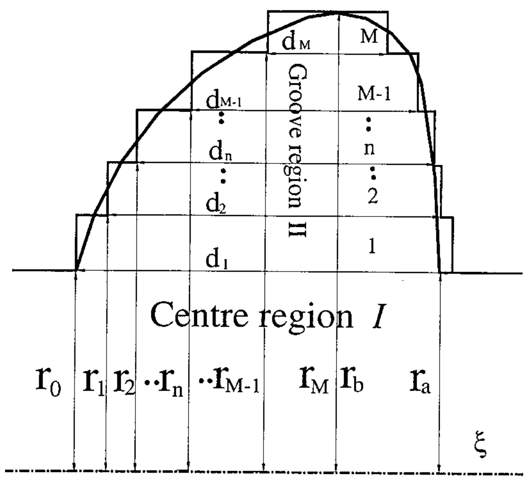

Fig. 2. The staircase representation of an arbitrary groove.

d_n is the width of the nth region. In each area the helical coordinate system $(r,\ \theta,\ \xi)$ is employed [4] to solve the wave equation. Thus, the field expressions in the nth region may be written as

$$\left.\begin{aligned} E^{\rm II}_{\xi(n)} &= [A_n J_v(kr) + B_n J_{-v}(kr)]e^{-jv\theta}\\ H^{\rm II}_{\theta(n)} &= -jY_0[A_n J'_v(kr) + B_n J'_{-v}(kr)]e^{-jv\theta}\\ H^{\rm II}_{r(n)} &= \frac{v}{r\omega\mu_0}\,[A_n J_v(kr) + B_n J_{-v}(kr)]e^{-jv\theta}\\ E^{\rm II}_{\theta(n)} &= E^{\rm II}_{r(n)} = H^{\rm II}_{r(n)} = 0 \end{aligned}\right\} \tag{1}$$

where

$A^{\rm II}_n$ and $B^{\rm II}_n$	field amplitude factors;
k	wavenumber in the free space;
Y_0	wave admittance in the free space;
v	angular propagation coefficient in the groove region and the relationship between it and β_0. The axial phase propagation constant in the center region can be determined by the fact that the phase change per pitch in the center region should be equal to the phase change per turn in the groove, and this leads to $v = \beta_0 L/2\pi$ [3];
$J_{\pm v}$	normal Bessel Function of the first kind and order $\pm v$, which indicates the fundamental mode excited here is oscillating in the r-direction [4];
$J'_{\pm v}(kr)$	first differential of $J_{\pm v}(kr)$with respect to kr.

The normalized transverse admittance of the $(n-1)$th region at radius $r = r_{n-2}$ is defined by

$$\begin{aligned} Y_{n-2,\,n-1} &= \frac{1}{-jY_0}\,\frac{H^{\rm II}_{\theta(n-1)}(kr_{n-2})}{E^{\rm II}_{\xi(n-1)}(kr_{n-2})}\\ &= \frac{J'_v(kr_{n-2}) + C_{n-1}J'_{-v}(kr_{n-2})}{J_v(kr_{n-2}) + C_{n-1}J_{-v}(kr_{n-2})} \end{aligned} \tag{2}$$

where $C_{n-1} = B_{n-1}/A_{n-1}$. Using the admittance matching conditions at the interface of two successive steps leads to the following recurrence relationship of Y:

$$\begin{aligned} Y_{n-2,\,n-1} &= Q_v(kr_{n-1},\ kr_{n-2})\\ &\quad\cdot \left[\frac{d_{n-1}Y_{n-1,\,n} - d_n T_v(kr_{n-2},\ kr_{n-1})}{d_{n-1}Y_{n-1,\,n} - d_n Q_v(kr_{n-2},\ kr_{n-1})}\right] \end{aligned} \tag{3}$$

where the functions T_v and Q_v are defined as

$$\begin{aligned} T_v(x,\ y) &= \frac{J'_v(x)J'_{-v}(y) - J'_{-v}(x)J'_v(y)}{J'_v(x)J_{-v}(y) - J'_{-v}(x)J_v(y)}\\ Q_v(x,\ y) &= \frac{J_v(x)J'_{-v}(y) - J_{-v}(x)J'_v(y)}{J_v(x)J_{-v}(y) - J_{-v}(x)J_v(y)}. \end{aligned}$$

When $n = M$, that is $r_M = r_b$, there exists $E^{\rm II}_\xi = A_M J_v(kr_b) + B_M J_{-v}(kr_b) = 0$, thus

$$C_M = -J_v(kr_b)/J_{-v}(kr_b)$$

and

$$Y_{M-1,\,M} = \frac{J'_v(kr_{M-1})J_{-v}(kr_c) - J_v(kr_c)J'_{-v}(kr_{M-1})}{J_v(kr_{M-1})J_{-v}(kr_c) - J_v(kr_c)J'_{-v}(kr_{M-1})}.$$

The expression of $Y_{0,\,1}$ can be obtained by using recurrence relationship given in (3) and the initial values of C_M and $Y_{M-1,\,M}$.

The field components in the center region [3] can be obtained by using the Floquet's theorem in (4), shown at the top of the next page, where C_m and D_m are the field amplitude factors of the mth space harmonic. γ_m is the radial propagation constant, and defined as $\gamma_m^2 = |\beta_m^2 - k^2|$. If $\gamma_m^2 = \beta_m^2 - k^2 > 0$, $F_m(\gamma_m r) = I_m(\gamma_m r)$ and the upper of the signal "$\pm$" or "$\mp$" is selected, while $\gamma_m^2 = k^2 - \beta_m^2 > 0$, $F_m(\gamma_m r) = J_m(\gamma_m r)$, the lower case of the signal $\pm$ or $\mp$ is chosen. $I_m(\gamma_m r)$ and $J_m(\gamma_m r)$ represent the modified and ordinary Bessel functions of m order, respectively; $\beta_m$is the axial propagation constant of mth space harmonic, and can be expressed as $\beta_m = \beta_0 + 2m\pi/L$.

With the aid of the following field matching conditions at the interface between the groove space and central space [3]

$$\begin{aligned} E^I_Z &= E^{\rm II}_\xi \cos\psi,\ E^I_\phi\\ &= -E^{\rm II}_\xi \sin\psi \int_{-\delta/2}^{+\delta/2} (H^I_\phi \cos\psi + H^I_z \sin\psi)\,dz\\ &= \int_{-\delta/2}^{+\delta/2} H^{\rm II}_\theta\,dz \end{aligned} \tag{5}$$

the unified dispersion equation is derived

$$\begin{aligned} Y_{0,\,1} \pm \frac{\delta}{L}\sum_{m=-\infty}^{\infty}\frac{k}{\gamma_m}\left[\frac{F'_m(\gamma_m r_a)}{F_m(\gamma_m r_a)} - \frac{1}{k^2\gamma_m^2}\,\frac{F'_m(\gamma_m r_a)}{F_m(\gamma_m r_a)}\right.\\ \left.\cdot\left(\frac{m\beta_m}{r_a} \mp \gamma_m^2 \tan\psi\right)^2\right]\left(\frac{\sin\beta_m\delta/2}{\beta_m\delta/2}\right)^2 = 0 \end{aligned} \tag{6}$$

where $Y_{0,\,1}$ is the surface admittance of the groove at $r = r_a$.

Then, the properties of the power flow and the interaction impedance in this structure can be also obtained by making use of the solution of (6).

$$\left.\begin{aligned} E_z^I &= \sum_{m=-\infty}^{\infty} C_m \gamma_m^2 F_m(\gamma_m r) e^{j(m\phi - \beta_m z)} \\ H_z^I &= j \sum_{m=-\infty}^{\infty} D_m \gamma_m^2 F_m(\gamma_m r) e^{j(m\phi - \beta_m z)} \\ E_\phi^I &= \mp \sum_{m=-\infty}^{\infty} [C_m (m\beta_m / r) F_m(\gamma_m r) - D_m \omega \mu_0 \gamma_m F_m'(\gamma_m r)] e^{j(m\phi - \beta_m z)} \\ H_\phi^I &= \pm j \sum_{m=-\infty}^{\infty} [C_m (\gamma_m k^2/\omega\mu_0) F_m'(\gamma_m r) - D_m (m\beta / r) F_m(\gamma_m r)] e^{j(m\phi - \beta_m z)} \end{aligned}\right\} \tag{4}$$

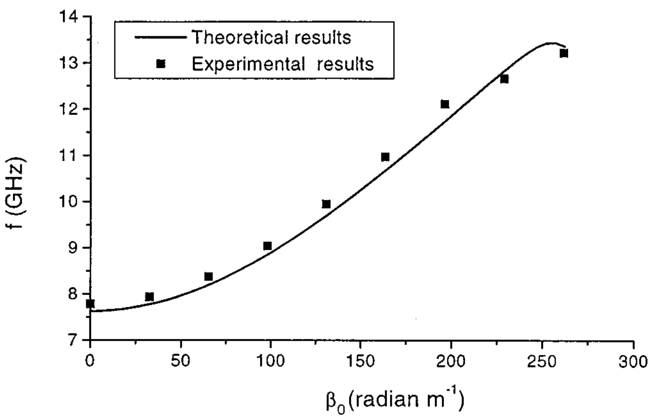

Fig. 3. The dispersion curve of a half-circular helical groove structure with the physical dimensions (unit: mm) $r_a = 14.5$, $r_b = 16.5$, $\delta = 4.0$, $L = 12.0$.

III. Test

Equation (6) is a unified determining equation, as $M = 2$, $r_2 = r_c$, $r_1 = r_b$, $r_0 = r_a$, $d_1 = \delta$, $d_2 = S$, it can be simplified to the dispersion equation of the ridge-loaded helical-groove structure, i.e.,

$$\begin{aligned} & Q_v(kr_b,\, kr_a) \left[\frac{Q_v(kr_b,\, kr_c) + (\delta/s) \cdot T_v(kr_a,\, kr_b)}{Q_v(kr_b,\, kr_c) + (\delta/s) \cdot Q_v(kr_a,\, kr_b)} \right] \\ & \quad \pm \frac{\delta}{L} \sum_{m=-\infty}^{\infty} \frac{k}{\gamma_m} \left[\frac{F_m'(\gamma_m r_a)}{F_m(\gamma_m r_a)} - \frac{1}{k^2 \gamma_m^2} \frac{F_m'(\gamma_m r_a)}{F_m(\gamma_m r_a)} \right. \\ & \quad \left. \cdot \left(\frac{m\beta_m}{r_a} \mp \gamma_m^2 \tan \psi \right)^2 \right] \cdot \left(\frac{\sin \beta_m \delta/2}{\beta_m \delta/2} \right)^2 = 0. \end{aligned} \tag{7}$$

This equation turns out to be basically the same as the results obtained in previous study [3] if neglecting the step discontinuity capacitance. It can also be reduced to the characteristic equation of the rectangular groove circuit when $r_M = r_b$, $r_{M-1} = r_0 = r_a$, $d_1 = \delta$, i.e.,

$$\begin{aligned} & \frac{J_v'(kr_a) J_{-v}(kr_b) - J_v(kr_b) J_{-v}'(kr_a)}{J_v(kr_a) J_{-v}(kr_b) - J_v(kr_b) J_{-v}(kr_a)} \pm \frac{\delta}{L} \sum_{m=-\infty}^{\infty} \frac{k}{\gamma_m} \\ & \quad \cdot \left[\frac{F_m'(\gamma_m r_a)}{F_m(\gamma_m r_a)} - \frac{1}{k^2 \gamma_m^2} \frac{F_m'(\gamma_m r_a)}{F_m(\gamma_m r_a)} \right. \\ & \quad \left. \cdot \left(\frac{m\beta_m}{r_a} \mp \gamma_m^2 \tan \psi \right)^2 \right] \left(\frac{\sin \beta_m \delta/2}{\beta_m \delta/2} \right)^2 = 0. \end{aligned} \tag{8}$$

This is identical to the result obtained in [4].

To solve the transcendental equation (6), the author has developed a general FORTRAN computer program, which permits the number of the steps M to be arbitrarily specified.

In order to study the accuracy of the presented method, we have manufactured a half-circular helical groove waveguide and made the cold test on dispersion characteristics by resonance method [6]. Fig. 3 shows the experimental results. It is very clear that experimental data are in good agreement with the numerical calculation data, which indicates this method is tenable and accurate.

IV. Conclusions

An approach to the analysis of the arbitrarily shaped helical groove waveguide has been described. It has been demonstrated that the proposed method is accurate on the predication of the dispersion property. The unified determining equation of the arbitrarily shaped helical groove structure (whether the boundary of the groove is smooth or not) is an important result obtained by this method, which can be used to analyze the properties of electromagnetic wave propagating through helical various shape groove structures including V-type, cosine-type, or sector-shape circuit, etc. The presented analysis is more general, and it is applicable to both the slow-wave and fast-wave regions.

References

[1] C. Liss, R. Harper, and M. P. Puri, "Helical waveguide millimeter wave TWT," in *IEDM Tech. Dig.*, 1988, pp. 374–377.

[2] K. J. Bunch and R. W. Grow, "The helically wrapped circular waveguide," *IEEE Trans. Electron Devices*, vol. 34, pp. 1873–1884, Aug. 1987.

[3] W. Wang, G. Yu, and Y. Wei, "Study of the ridge-loaded helical-groove slow-wave structure," *IEEE Trans. Microwave Theory Tech.*, vol. 45, pp. 1689–1695, Oct. 1997.

[4] K. W. H. Foulds and J. R. Mansell, "Propagation of an electromagnetic wave through a helical waveguide," *Proc. Inst. Elect. Eng.*, vol. 111, no. 11, pp. 1789–1798, 1964.

[5] G. G. Denisov, V. L. Bratman, A. D. R. Pheps, and S. V. Samsonov, "Gyro-TWT with a helical operating waveguide: New possibilities to enhance efficiency and frequency bandwidth," *IEEE Trans. Plasma Science*, vol. 26, pp. 508–517, June 1998.

[6] A. W. Horsley and A. Pearson, "Measurement of dispersion and interaction impedance characteristics of slow-wave structures by resonance method," *IEEE Trans. Electron Devices*, vol. 13, pp. 962–969, Dec. 1966.