Analysis of Microstrip Lines in Multilayer Structures of Arbitrarily Varying Thickness

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***Abstract*—A general approach to the full-wave analysis of microstrip lines in multilayer dielectrics of arbitrarily varying thickness is developed. It is based on the discrete mode matching technique (DMM) and uses a full-wave equivalent circuit for the stratified structure, which is simple to apply in a numerical procedure. As an example, the propagation constant of a microstrip line in the interface of two dielectrics as a function of different shape characteristics is computed.**

***Index Terms*—Arbitrarily shaped interface, conformal, discrete mode matching, DMM, microstrip lines, multilayer.**

I. INTRODUCTION

FUTURE mobile communications and navigation systems demand antennas and feed networks that can be integrated in the surface of vehicles, airplanes, and satellites. For this purpose, microstrip patch antennas are best suited due to their low weight and flexibility. The design of the network and other microwave circuit elements requires fast and exact procedures for the precise determination of their electrical characteristics. While proofed commercial software is already available for planar antennas and circuits, the analysis of conformal structures is often restricted to cylindrical bodies [1]–[3]. Procedures like finite differences (FDTD) or finite elements (FE), which can be used for arbitrarily shaped objects, require large storage and computation time, especially for thin layers. In [4], the advantages of discrete mode matching (DMM) for the analysis of multilayer planar structures have been shown, and in [5] this procedure was adapted to waveguides filled by nonplanar stratified dielectric without metallizations. In this letter, the DMM is extended to microstrips with layers of arbitrarily varying thickness. In DMM, fields and currents are represented by an orthogonal set of basis functions, which are the eigensolutions of Helmholtz' wave equation with suitable lateral boundary conditions. To match the fields at the interfaces of different layers, the basis functions are disretized, but this is necessary in only one dimension.

II. ANALYSIS

The fundamental geometry for the following analysis is depicted in Fig. 1. A wave propagation $\exp j(\omega t - k_z z)$ in z-direction is assumed. To obtain a full-wave solution, we start with

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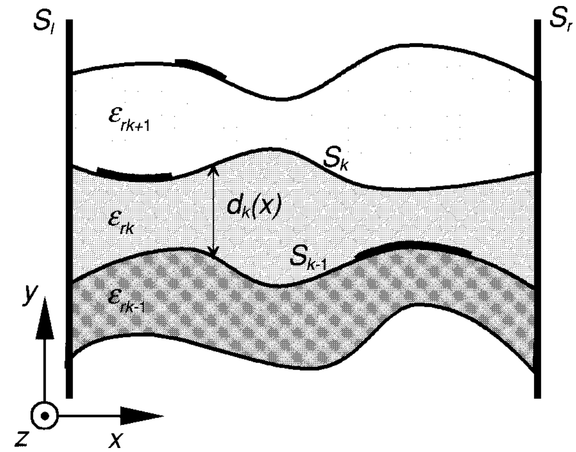

Fig. 1. Microstrip lines in the interfaces of stratified dielectrics with arbitrarily varying thickness.

Helmholtz' wave equation (normalized by k_0), which is given within an arbitrary layer by

$$\left(D_x^2 + D_y^2 + \varepsilon_d\right)\psi = 0,$$
$$\varepsilon_d = \varepsilon_r - k_z^2, \quad D_\mu = \frac{\partial}{\partial \mu}, \quad \mu = x, y \tag{1}$$

for the two independent field components $\psi = E_z, H_z$. We expand ψ into a Fourier series truncated after N terms

$$\psi(x,y) = \mathbf{t}^T \cdot \tilde{\psi} = \sum_i^N \tau(k_{x,i}\, x)\tilde{\psi}_i(y) \tag{2}$$

where τ represents sine (cosine) functions for E_z and H_z, satisfying the Dirichlet and Neumann boundary conditions, respectively, at the metallic walls S_l and S_r. Substituting (2) in (1) and solving the resulting ordinary differential equations at N sampling points x_j, we obtain

$$\tilde{\psi}_i(y) = A_i \cosh(k_{y,i} y) + B_i \sinh(k_{y,i} y)|_{x_j} \tag{3}$$

with $k_{y,i}^2 = k_{x,i}^2 - \varepsilon_d$. The constants A_i, B_i are different for each sampling point.

From (3) follows that the field components on both sides S_{k-1}, S_k of an arbitrary layer k are related by

$$\begin{bmatrix} D_y\tilde{\psi}_i|_{y=S_{k-1}} \\ D_y\tilde{\psi}_i|_{y=S_k} \end{bmatrix} = k_{y,i}\begin{bmatrix} -\gamma_k & \alpha_k \\ -\alpha_k & \gamma_k \end{bmatrix} \cdot \begin{bmatrix} \tilde{\psi}_i|_{y=S_{k-1}} \\ \tilde{\psi}_i|_{y=S_k} \end{bmatrix} \tag{4}$$

with

$$\gamma_k = \frac{1}{\tanh(k_{y,i} d_k(x))}, \quad \alpha_k = \frac{1}{\sinh(k_{y,i} d_k(x))}$$

to be taken at $x = x_j$.

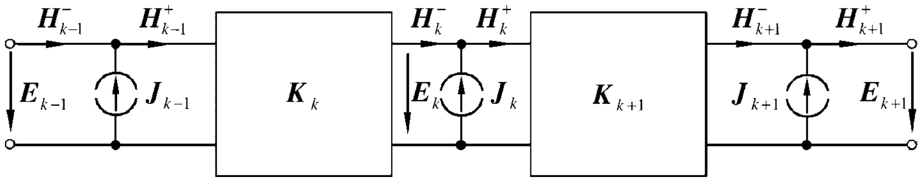

Fig. 2. Equivalent two-port representation of the stratified dielectric.

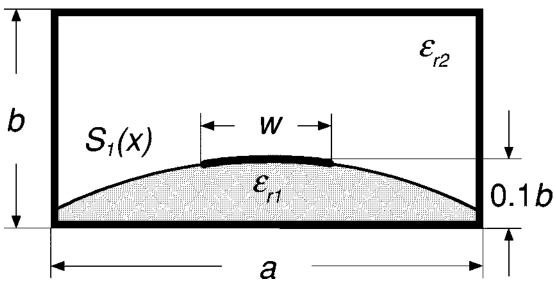

Fig. 3. Microstrip line on a dielectric layer with variable thickness. $\varepsilon_{r1} = 8.875, \varepsilon_{r2} = 1, b = 7.5$ mm, $a = 2b, w = 0.25a$.

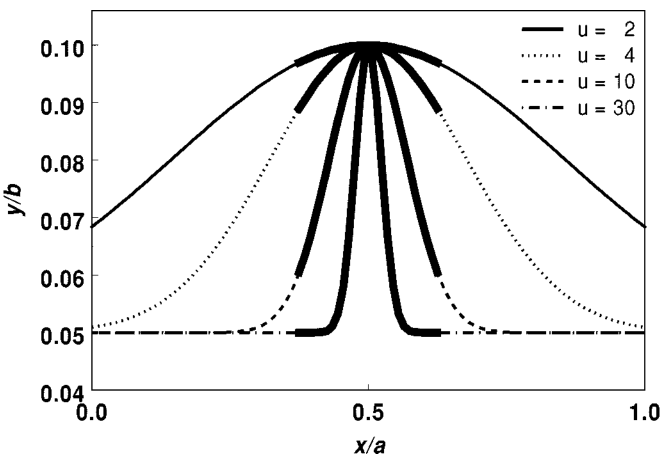

Fig. 4. Shape of the dielectric layer $y = d_1(x) = S_1(x)$ given by (9) for different model parameters u.

At the interfaces, the tangential field components E_z, H_z and E_t, H_t, which are given by

$$(E_t, H_t) = (E_x, H_x)\cos\delta(x_j) + (E_y, H_y)\sin\delta(x_j)$$
$$\delta(x_j) = \arctan\left(\left.\frac{dS_k(x)}{dx}\right|_{x_j}\right) \tag{5}$$

must be matched. For this purpose, E_x, H_x, E_y and H_y are expanded according to (2) with suitable boundary conditions at S_l and S_r. Using (4) and the relation between the field components given in [5] all discretized tangential field components at both interfaces of an arbitrary layer k can be represented by hybrid vectors and block matrices

$$\boldsymbol{F}_{k-1} = \boldsymbol{K}_k \cdot \boldsymbol{F}_k, \quad \boldsymbol{F}_k = \begin{bmatrix} \boldsymbol{E}_k \\ \boldsymbol{H}_k \end{bmatrix}$$
$$\boldsymbol{E}_k = \begin{bmatrix} \boldsymbol{E}_{z,k} \\ \boldsymbol{E}_{t,k} \end{bmatrix}, \quad \boldsymbol{H}_k = \eta_0 \begin{bmatrix} \boldsymbol{H}_{t,k} \\ \boldsymbol{H}_{z,k} \end{bmatrix}. \tag{6}$$

This leads to a simple equivalent two-port representation of the stratified dielectric (Fig. 2). If there are interfaces with ideal

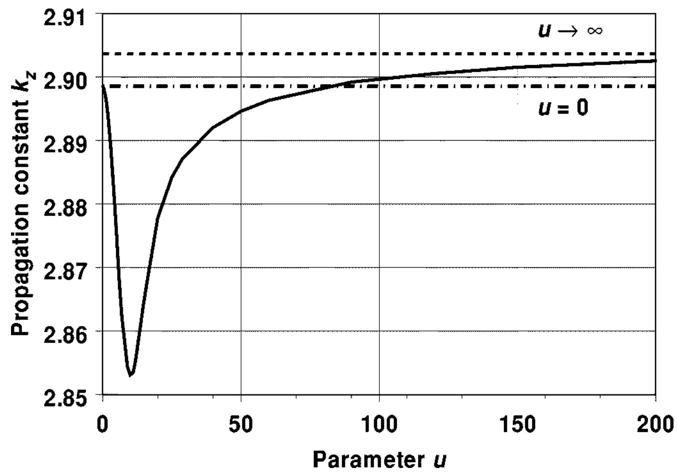

Fig. 5. Propagation constant k_z as a function of the shape of the dielectric layer determined by the model parameter u. Frequency $F = 30$ GHz, 32 sampling points on the metallization.

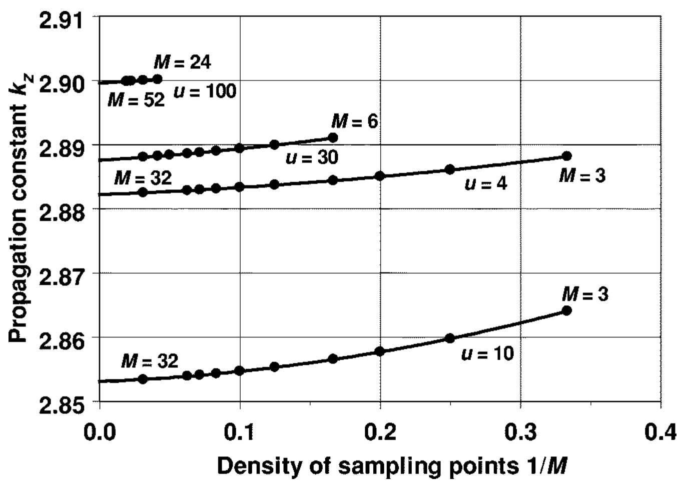

Fig. 6. Convergence of the propagation constant with increasing number of sampling points for four different shapes of the interface (model parameter $u = 4, 10, 30, 100$). M is the number of discretization points on the metallization of half the structure.

conducting lines, the appropriate continuity equations for the tangential field components and currents

$$\boldsymbol{E}_k^+ = \boldsymbol{E}_k^-, \quad \boldsymbol{H}_k^+ - \boldsymbol{H}_k^- = \boldsymbol{J}_k = \eta_0 \begin{bmatrix} -\boldsymbol{J}_{t,k} \\ \boldsymbol{J}_{z,k} \end{bmatrix} \tag{7}$$

must be taken into account by introducing a formal current source. Simple network analysis technique follows, also including the boundary conditions at the bottom and the top of the structure. From the condition, that the tangential electric field components on the metallizations and the currents outside must be zero, a system equation

$$\boldsymbol{Z} \cdot \boldsymbol{J} = \boldsymbol{0} \quad \text{or} \quad \boldsymbol{Y} \cdot \boldsymbol{E} = \boldsymbol{0} \tag{8}$$

is finally obtained. It can be solved for the propagation constants by means of $\det \boldsymbol{Z} = 0$ or $\det \boldsymbol{Y} = 0$.

III. Results

The propagation constant k_z of a microstrip line in the interface between two dielectric layers with varying thickness enclosed in a waveguide has been computed (Fig. 3). The shape of the interface is given by the function

$$S_1(x) = 0.05b\left(1 + \exp\left(-u^2\left(\frac{x}{a} - 0.5\right)^2\right)\right). \quad (9)$$

Different model parameters u have been used (Fig. 4). For $u = 0$, the structure is planar, where the thickness of the lower dielectric is given by $d = 0.1b$. If $u > 0$, the interface becomes convex, and for large u, its shape tends to a planar one with $d = 0.05b$, by reducing the convex segment to a thin peak in the middle. Since the structure is symmetric, a magnetic wall at $x = 0.5a$ reduces the computational effort and the calculation has been done with 32 sampling points on the metallization of half the structure (Fig. 5). To compare with, the propagation constants of planar microstrip lines with $d = 0.1b$ and $d = 0.05b$ are also shown in Fig. 5. The convex interface shape for u between these two limits results in a first decrease and final increase of the propagation constant. Unfortunately, no results for the given structure were found in the literature.

Fig. 6 shows the excellent convergence behavior of discrete mode matching. It permits an extrapolation to the exact solution by means of a simple quadratic curve-fitting algorithm. Each of the curves starts with a minimal number of sampling points suitable to approximate the shape of the interface.

IV. Conclusion

The discrete mode matching procedure has successfully been used to analyze microstrip lines on dielectric layers with arbitrarily varying thickness. It provides an easy and systematic CAD procedure with minimal discretization. It is also possible to include lateral absorbing boundary conditions for the investigation of radiation effects.

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