# Analysis and Design of an Inhomogeneous Transformer with Hard Wall Waveguide Sections

Mete Ozkar, *Student Member, IEEE,* and Amir Mortazawi, *Member, IEEE*

***Abstract*—A new inhomogeneous waveguide transformer with hard walls is presented. Mode matching technique along with an optimization routine is used to design the transformer. The generalized scattering matrix (GSM) of the whole block is calculated which can be used to predict the fields at the output given the incident excitations. An example of a three-section transformer, which replaces a tapered hard horn, is shown. The transformer has better performance in the bandwidth of interest compared to the tapered hard horn having twice the length of the transformer. This type of transformers could be useful for excitation of quasi-optical amplifiers and reflector feeds.**

***Index Terms*—Inhomogeneous transformers, overmoded waveguides.**

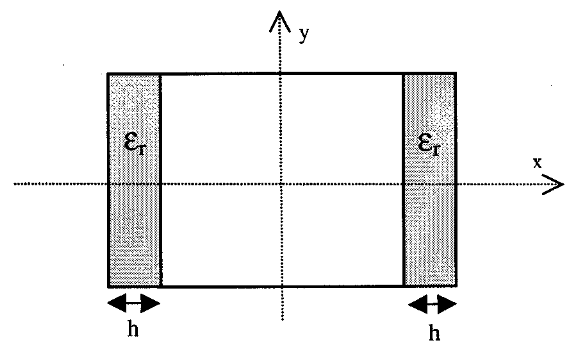


Fig. 1. A transverse plane view of a dielectric-loaded waveguide.

## I. Introduction

WAVEGUIDES and horn antennas with hard walls are known to support hybrid modes known as LSE and LSM modes [1]. Due to the uniform field distribution across their aperture, such hard horn antennas have high gain and find applications in cluster feeds and arrays [2]. Hard horn antennas have also been previously employed for uniform excitation of quasi-optical amplifier arrays [3], [4]. The uniform field distribution at the hard horn aperture is achieved by dielectric loading the walls of the regular horn antennas [4]. However, since the flare angle for a horn antenna should be kept at less than 15° for a smooth transition, the antenna length is considerably long compared to wavelength. In this paper, inhomogeneous hard wall waveguide transformers are proposed which can be notably shorter in length and can achieve a similar performance as the tapered hard horns.

In inhomogeneous transformers, the ratios of internal wavelengths and characteristic impedances at different positions along the direction of propagation may change with frequency. Even though the lengths of such transformers are shorter than the tapered transitions, they show similar behavior around the design frequency. In this paper, for the first time, the application of inhomogeneous transformers is extended to the design of dielectric loaded multisection waveguide transformers.

Design equations for conventional waveguide transformers based on quarter-wave transformer prototypes are given in [5] and [6]. In [7], inhomogeneous, nonquarterwave rectangular waveguide transformer designs through the use of computer optimization was achieved, but the frequency range was chosen such that the higher order modes of the larger waveguide would not be excited. Mode matching along with optimization has been used to design double plane transformers for empty waveguides in [8] and, unlike the previous designs, higher order modes (which affect the resonator lengths and step dimensions) are considered in the optimization process. In this paper, the optimization is done for the overmoded dielectric-loaded waveguide sections and the discontinuity is in both E and H planes.

Manuscript received October 18, 1999; revised January 20, 2000. This work was supported by an Army Research Office MURI Grant under Contract DAAG-55-97-132.

The authors are with the Electrical and Computer Engineering Department, North Carolina State University, Raleigh, NC 27695-7911 USA.

Publisher Item Identifier S 1051-8207(00)03148-2.

## II. Theory

In dielectric-loaded waveguides, TE and TM modes are coupled together to form an orthogonal set of hybrid modes known as LSM and LSE modes [9]. If the dielectric is on the E-planes of the waveguide, TE to $x$ (LSE$^x$) and TM to $x$ (LSM$^x$) modes are supported, where $x$ direction is as indicated in Fig. 1. It is possible to make use of this artificial hard surface at the sidewalls to achieve a uniform field distribution at the transverse plane of the waveguide. The approximate dielectric thickness necessary to obtain a uniform distribution is determined by [1]

$$h = \frac{\lambda_0}{4\sqrt{\varepsilon_r - 1}} \tag{1}$$

where

$\lambda_0$ free space wavelength at the design frequency;
$\varepsilon_r$ dielectric constant of the material;
$h$ thickness of the dielectric material as shown in Fig. 1.

The analysis of hard horns is based on mode-matching technique [1]. The tapered horn is approximated by a sequence of double step junctions. Each junction can be represented by a GSM. The GSM of the hard horn can then be obtained by cascading these individual GSM's. GSM of each junction described above can be found using the well-known mode matching technique.

In the mode matching technique, all modes are considered at a double step plane discontinuity and the total field is obtained as the sum of all the excited modes. The total power on each side of the junction is matched and the GSM for each junction is obtained. The details of mode matching applied to the specific case of dielectric-loaded waveguides are described in [4]. The same GSM approach was used for analyzing transformers with different waveguide sections. Conventional transformer design rules are not valid in this case, because of the double plane discontinuities, dielectric-loaded walls, and the existence of higher order modes. Therefore, an optimization procedure had to be integrated with the mode matching approach to obtain the final design parameters. The approach used here is similar to the one in [8]. An error function is formed for the optimization process as follows:

$$g(a_i, b_i, l_i) = \sum_{j=f_1}^{f_2} \frac{\left|S_{11}^{\mathrm{TE}_{10}}\right|^2}{\left|S_{21}^{\mathrm{TE}_{10}-\mathrm{LSE}_{10}}\right|^2} \qquad (2)$$

where $a_i$, $b_i$, and $l_i$ are the waveguide width, the waveguide height, and length of each section, respectively, $f_1$ is the start frequency, and $f_2$ is the final frequency in the frequency region of interest. $S_{21}$ is the transmission coefficient from the input mode to the dominant output mode. The conversion between the $\mathrm{TE}_{10}$ mode at the input and the $\mathrm{LSE}_{10}$ mode at the output waveguide is maximized through this optimization whereas the reflections seen at the input are minimized.

## III. Design and Simulations for a Hard Wall Transformer and Verification Using Commercial FEM Software

A three-section double plane transformer with dielectric walls as shown in Fig. 2 was designed using the above optimization method and was incorporated into the mode matching algorithm. The waveguide dimensions are given in Table I, where $a$ and $b$ are the widths and heights, and $l$ is the length of each section. The input waveguide is a regular empty waveguide whereas the output waveguide and the sections are dielectric-loaded. Same dielectric material with $\varepsilon_r = 2.2$ was used in both the horn and the transformer. The dielectric thickness was chosen to achieve uniform field distribution around 15 GHz. After analyzing the tapered horn and the inhomogeneous hard wall transformer using the mode-matching program, they were also analyzed using a commercial three dimensional (3-D) FEM simulator (HFSS). A good agreement between the two techniques is obtained. The input reflection coefficient for the dominant mode is compared with a tapered waveguide transition of length 50 mm in Fig. 3. The reflections seen at the input waveguide are reduced in the bandwidth of interest by using the double plane (inhomogeneous) transformer. The total length of the three-section transformer is 23.31 mm (about half of the hard horn length).

Cutoff frequencies shown as vertical lines in Fig. 3 are calculated from the characteristic equations that result from even- and odd-mode analysis [9]. Even modes are the modes that have even symmetry with respect to the $y$-axis and odd modes are the modes that have odd symmetry with respect to the $y$-axis. It is clear that the output waveguide is overmoded in the frequency range of interest.

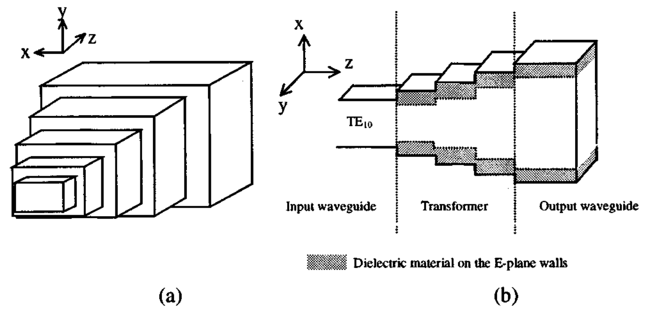


Fig. 2. A three-section double plane waveguide transformer. (a) 3-D view. (b) Top cross section.

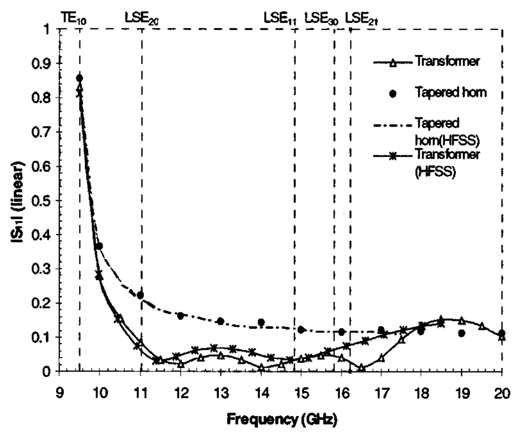


Fig. 3. Magnitude of $S_{11}$ of the $\mathrm{TE}_{10}$ mode (of the input waveguide) as a function of frequency. Cutoff frequencies for different LSE modes at the output waveguide and for $\mathrm{TE}_{10}$ at the input are shown in vertical lines.

TABLE I
THE DIMENSIONS OF THE WAVEGUIDE TRANSFORMER IN mm

| | Ku-band waveguide | Section | | | X-band waveguide |
|---|---|---|---|---|---|
| | | 1 | 2 | 3 | |
| a | 15.8 | 16.73 | 18.81 | 21.66 | 22.86 |
| b | 7.9 | 10.50 | 10.73 | 10.65 | 10.16 |
| l | | 8.90 | 8.24 | 6.17 | |

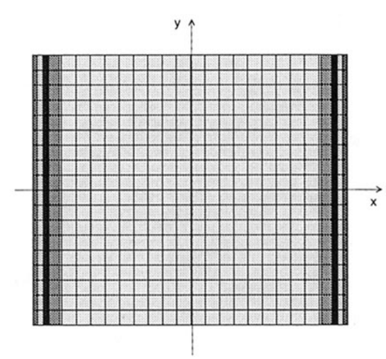


Fig. 4. The field distribution across the aperture (almost exactly the same for both the transformer and the horn aperture) at 15 GHz. Each shade of gray represents a $\pm 1$ dB power variation.

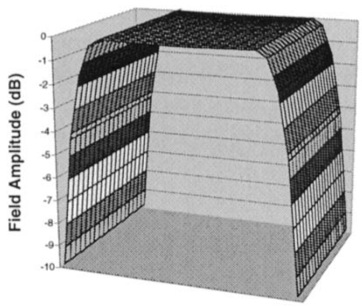


Fig. 5. 3-D field plot at the aperture of the three-section waveguide transformer at 15 GHz. 3-D field plot at the aperture of the horn at 15 GHz is the same as above.

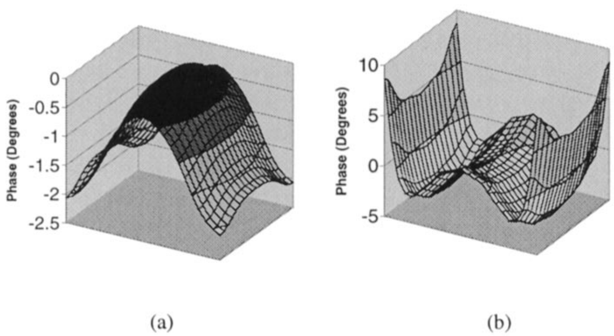


Fig. 6. Phase plot (a) at the aperture of the tapered horn at 15 GHz and (b) at the aperture of the three-section waveguide transformer at 15 GHz.

Fig. 4 shows the predicted $\pm$1-dB field distribution resulting across the aperture of the output waveguides for both the smooth transition and the three-section transformer cases.

Fig. 5 shows the 3-D field plot in magnitude at the aperture of the output waveguide for the three-section design and for the tapered design. Since both results were similar, only one figure is shown. The performance of the three-section design is comparable to that of the tapered transition's. Fig. 6 compares the phase distribution of the field at the apertures of both designs. Although it was not done here, the phase could also be included as a parameter in the error function of the optimization.

## IV. Conclusion

An inhomogeneous waveguide transformer with dielectric-loaded waveguide sections was presented. The analysis and design procedure of such transformers was verified in the case of the transition between an empty $Ku$-band waveguide and a dielectric loaded $X$-band waveguide. It was found that the transformer had similar performance compared to that of a hard horn in the bandwidth of interest. The simulation results agree well with the results obtained from a commercial FEM software. Some of the applications include high aperture efficiency reflector feeds and quasi-optical amplifier arrays feeds.

## References

[1] P. S. Kildal, "Definition of artificially soft and hard surfaces for electromagnetic waves," *Electron. Lett.*, vol. 24, pp. 168–170, 1988.
[2] P. S. Kildal and E. Lier, "Hard horns improve cluster feeds of satellite antennas," *Electron. Lett.*, vol. 24, pp. 491–492, April 1988.
[3] T. Ivanov and A. Mortazawi, "A two-stage spatial amplifiers with hard horn feeds," *IEEE Microwave Guided Lett.*, vol. 6, pp. 88–90, Feb. 1996.
[4] M. A. Ali, S. C. Ortiz, T. Ivanov, and A. Mortazawi, "Analysis and measurement of hard-horn feeds for the excitation of quasioptical amplifiers," *IEEE Trans. Microwave Theory Tech.*, vol. 47, pp. 479–487, Apr. 1999.
[5] G. L. Matthaei, L. Young, and E. M. T. Jones, *Microwave Filters, Impedance Matching Networks, and Coupling Structures*. New York: McGraw-Hill, 1964.
[6] G. C. Southworth, *Principles and Applications of Waveguide Transmission*. New York: Van Nostrand, 1950.
[7] J. W. Bandler, "Computer optimization of inhomogeneous waveguide transformers," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-17, pp. 563–571, 1969.
[8] H. Patzelt and F. Arndt, "Double-plane steps in rectangular waveguides and their application for transformers, irises, and filters," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-30, pp. 771–776, May 1982.
[9] C. A. Balanis, *Advanced Engineering Electromagnetics*. New York: Wiley, 1989.