Phase Noise Improvement in a Loop Configuration of Oscillating Antennas

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***Abstract*—Phase noise in a four-element array of oscillating antennas is analyzed for a loop configuration and a unilateral coupling topology. It is demonstrated that this structure, basically designed for circular polarization (CP) applications, also yields a 75% reduction of the near-carrier phase noise compared to a single oscillator. Experimental validation is made at 4 GHz. For this structure, deviation of the coupling phase from its nominal value has no influence on the phase noise performance.**

***Index Terms*—Coupled oscillator array, injection-locking, phase noise analysis.**

I. Introduction

In a recent paper [1], a loop configuration of four oscillating antennas (Fig. 1) was proposed to generate circular polarization (CP) in a large beamwidth. The phase shift $\Delta\theta = \pm 90°$ between neighboring antennas was obtained with a proper choice of the phase ϕ introduced by the coupling circuits. It was shown that a phase value equal or "near" the nominal value $\phi = \pm 90°$ (118° in the experiment at 4 GHz) and the unidirectional coupling provided by amplifiers allowed the elimination of unwanted modes in the array. The study, based on the theory developed in [2], only concerned the existence of stable modes, but the phase noise analysis of the structure has not been performed yet and is proposed in this letter. With the help of the general analysis presented in [3] for globally coupled oscillator arrays, we show that the specific case of a loop configuration using unilaterally injection-locked oscillators allows a phase noise division by the number of oscillators. This property does not depend of the variation on the coupling phase ϕ, which makes the structure robust toward fabrication tolerance on the coupling lines and amplifiers. The theoretical demonstration is validated experimentally for the circularly polarized array.

II. Theoretical Developments

In the theoretical development, the following assumptions are made on the oscillators and the coupling circuits.

- Identical oscillators and antennas, i.e., identical steady-state amplitudes, free-running frequencies, and quality factors.
- Identical coupling circuits with unilaterally injection-locked oscillators in a loop configuration. Only the coupling between neighboring antennas is considered. Every oscillator is the master of the next oscillator and the slave of the previous one.

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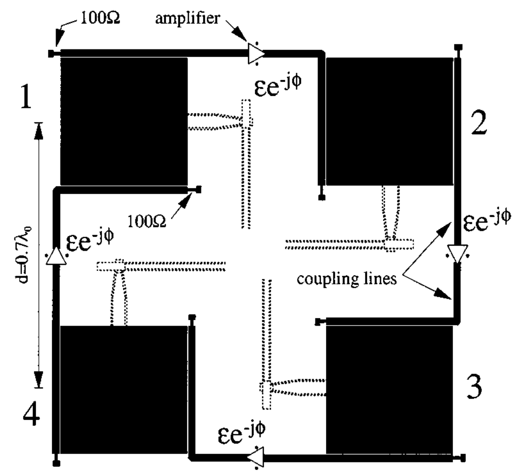

Fig. 1. A 2 × 2 microstrip array generating circular polarization with linearly polarized square patch antennas. Amplifiers are in a clockwise configuration. The active circuitry is etched in the ground plane and is described in [1].

In [3], it has been demonstrated that unilateral injection locking does not improve the total phase noise for a linear arrangement of the array. For linear arrays, bilateral coupling was necessary to allow a phase noise reduction near the carrier frequency in direct proportion to the number of oscillators. But [3] did not consider a loop configuration using unilateral injection-locking which improves the total phase noise as demonstrated below for four oscillators.

The phase dynamics of four coupled oscillators can be modeled by a differential equation system [2], [3] as

$$\frac{\partial\theta_i}{\partial t} = \omega_0 - \frac{\omega_0}{2Q}\sum_{j=1}^{4}\varepsilon_{ij}\sin(\theta_i - \theta_j + \phi) - \frac{\omega_0}{2Q}B_{ni}(t), \quad i = 1,2,3,4 \tag{1}$$

where θ_i, ω_0, and Q are the instantaneous phase, free-running frequency, and quality factor of the ith oscillator. B_{ni} is the equivalent noise susceptance of the ith oscillator (see [3]) supposed identical for all the oscillators ($B_{ni} = B_n$). ε_{ij} and $\phi_{ij} = \phi$ are the magnitude and phase of the coupling circuit between oscillators i and j with

$$\varepsilon_{ij} = \begin{cases} \varepsilon, & \text{for } i - j = 1 \quad \text{or} \quad (i = 1 \text{and } j = 4) \\ 0, & \text{otherwise} \end{cases} \tag{2}$$

if the amplifiers are oriented clockwise like in Fig. 1 or

$$\varepsilon_{ij} = \begin{cases} \varepsilon, & \text{for } j - i = 1 \quad \text{or} \quad (j = 1 \text{ and } i = 4) \\ 0, & \text{otherwise} \end{cases} \tag{3}$$

if the amplifiers are oriented counterclockwise. For noise analysis, (1) is perturbed by substituting $\theta_i \rightarrow \hat{\theta}_i + \delta\theta_i$ where $\hat{\theta}_i$ is the steady-state solution to (1) and $\delta\theta_i$ is the phase fluctuation of the ith oscillator. Assuming small fluctuations and neglecting the mutual transformation between the AM noise and PM noise, (1) can be linearized around $\hat{\theta}_i$

$$\frac{\partial \delta\theta_i}{\partial t} = -\frac{\omega_0}{2Q}\left[\sum_{j=1}^{4} \varepsilon_{ij}(\delta\theta_i - \delta\theta_j)\cos\left(\hat{\theta}_i - \hat{\theta}_j + \phi\right) + B_{ni}(t)\right], \qquad i = 1, 2, 3, 4. \tag{4}$$

In (4), focus is on the PM-to-PM noise conversion while AM-to-AM noise conversion is neglected which is justified close to the carrier frequency where noise is dominated by phase noise [3]. In the CP application, a constant phase progression along the loop is assumed to establish the circular polarization

$$\Delta\hat{\theta}_i = \hat{\theta}_i - \hat{\theta}_j = \Delta\hat{\theta} = \pm 90^\circ. \tag{5}$$

The sign of $\Delta\hat{\theta}$ both depends on the orientation of the amplifiers and the line lengths and gives the direction of rotation of the radiated wave ($\Delta\hat{\theta} = +90^\circ$ for right-hand circular polarization in Fig. 1). A proper choice of ϕ, resulting from a stability analysis [1], ensures the existence and stability of the solution. In a more general loop configuration with N oscillators, the constant phase shift between the oscillators would be $2k\pi/N$ where k is an integer and the determination of k would be fixed by the value of ϕ. Following the procedure described in [3], (4) is Fourier transformed into

$$j\left(\frac{\omega}{\omega_{3\,\text{dB}}}\right)\delta\tilde{\theta}_i = -\left[\sum_{j=1}^{4} \varepsilon_{ij}(\delta\tilde{\theta}_i - \delta\tilde{\theta}_j)\cos(\Delta\hat{\theta} + \phi) + \tilde{B}_{ni}\right], \qquad i = 1, 2, 3, 4 \tag{6}$$

where ω is the noise frequency measured relative to the carrier and $\omega_{3\,\text{dB}} = \omega_0/2Q$. Using (2), (6) can be rewritten in the following form when a clockwise orientation of the amplifiers is considered:

$$\begin{cases} \tilde{B}_{ni} = \varepsilon\cos(\Delta\hat{\theta} + \phi)[\delta\tilde{\theta}_{i-1} + \delta\tilde{\theta}_i(-1 - jx)], \text{ for } i = 2, 3, 4 \\ \tilde{B}_{n1} = \varepsilon\cos(\Delta\hat{\theta} + \phi)[\delta\tilde{\theta}_4 + \delta\tilde{\theta}_1(-1 - jx)] \end{cases} \tag{7}$$

where $x = (\omega/\omega_{3\,\text{dB}})/(\varepsilon\cos(\Delta\hat{\theta} + \phi))$. Using (3), similar equations can be obtained for a counterclock orientation of the amplifiers leading to similar developments. The next step consists in putting (7) into a matrix form

$$\bar{\bar{N}} \cdot \overline{\delta\tilde{\theta}} = \bar{\tilde{B}}_n \tag{8}$$

where

$$\overline{\delta\tilde{\theta}} = \begin{pmatrix} \delta\tilde{\theta}_1 \\ \delta\tilde{\theta}_2 \\ \delta\tilde{\theta}_3 \\ \delta\tilde{\theta}_4 \end{pmatrix} \qquad \bar{\tilde{B}}_n = \begin{pmatrix} \tilde{B}_1 \\ \tilde{B}_2 \\ \tilde{B}_3 \\ \tilde{B}_4 \end{pmatrix}$$

and

$$\bar{\bar{N}} = \varepsilon\cos(\Delta\hat{\theta} + \phi) \cdot \begin{bmatrix} -1 - jx & 0 & 0 & 1 \\ 1 & -1 - jx & 0 & 0 \\ 0 & 1 & -1 - jx & 0 \\ 0 & 0 & 1 & -1 - jx \end{bmatrix}. \tag{9}$$

The matrix $\bar{\bar{N}}$ characterizes the unilateral coupling between four oscillators in a loop configuration. The information on the total phase noise can be extracted from the knowledge of the inverse matrix $\bar{\bar{P}}$ verifying: $\bar{\bar{P}} \cdot \bar{\bar{N}} = \bar{\bar{N}} \cdot \bar{\bar{P}} = \bar{\bar{I}}$. One can write

$$\sum_{j=1}^{4} n_{ij} \cdot p_{jk} = \delta_{ik} \Rightarrow$$
$$\sum_{i=1}^{4}\sum_{j=1}^{4} n_{ij} \cdot p_{jk} = \sum_{j=1}^{4} p_{jk}\left(\sum_{i=1}^{4} n_{ij}\right) = 1. \tag{10}$$

By inspection of (9), one can check out that the term in parentheses in (10) which is the sum of the jth column of $\bar{\bar{N}}$ is simply $-j(\omega/\omega_{3\,\text{dB}})$. Therefore, (10) gives

$$\sum_{j=1}^{4} p_{jk} = j(\omega_{3\,\text{dB}}/\omega) \tag{11}$$

which is the sum of the kth column of the matrix $\bar{\bar{P}}$. On the other hand, it has been shown in [3] that the total phase noise of an array of N oscillators is given by

$$|\delta\tilde{\theta}_{\text{total}}|^2 = \frac{|\tilde{B}_n|^2}{N^2} \sum_{k=1}^{N} \left|\sum_{j=1}^{N} p_{jk}\right|^2. \tag{12}$$

Equation (12) indicates that the total phase noise is found by summing the columns of the matrix $\bar{\bar{P}}$. As the phase noise of a single free-running oscillator is given by

$$|\delta\tilde{\theta}_i|^2_{\text{uncoupled}} = \frac{|\tilde{B}_n|^2}{\left(\frac{\omega}{\omega_{3\,\text{dB}}}\right)^2} \tag{13}$$

we finally conclude by combining (11)–(13) that the phase noise improvement due to the loop configuration in the CP application is given by

$$\frac{|\delta\tilde{\theta}_{\text{total}}|^2}{|\delta\tilde{\theta}_i|^2_{\text{uncoupled}}} = 1/4. \tag{14}$$

Equation (14) shows that a phase noise reduction is obtained in direct proportion to the number of oscillators ($N = 4$ in the CP application). This result can be extended to a larger number of antennas in a loop configuration, for instance in a circular array. The same $1/N$ reduction was obtained in [3] for reciprocally coupled arrays. In the loop configuration, an important difference results from the independence of the total noise toward ϕ in (11) and (12) due to the unilateral coupling. It means that the deviation of the coupling phase from its nominal values $\phi = \pm 90°$ does not have any influence on the phase noise performance which makes the structure robust toward fabrication tolerance on the coupling lines and amplifiers.

III. Experimental Validation

The oscillators active device was an HP-ATF10136 low-noise GaAs FET. The noise measurements were performed with a direct spectrum method based on the Hewlett Packard phase noise utility HP85671A [5], a down-loadable program used with the HP8560 Series spectrum analyzers. This method has been successfully applied in [4] to stress the phase noise improvement due to cavity backing in oscillating antennas. A circularly polarized horn antenna placed 3 m away from the oscillating array was used to detect the radiated power and then fed into a spectrum analyzer (HP-8563E). The experiment was placed in an anechoic chamber. The carrier frequency of the locked oscillating array was 4.0 GHz. The measurements were performed from 50 kHz to 10 MHz from the carrier (Fig. 2). According to HP characteristics, the noise floor is about -133 dBm for measurements at offsets of 1 MHz. This value is lower than the measured side-band noise and hence does not alter the results. Fig. 2 shows the averaged phase noise of the individual array elements when free-running and the averaged phase noise of the total array under synchronized conditions. The total output phase noise is clearly reduced as compared to those of the free-running oscillators with an averaged improvement around 6 dB. The dotted line shows the array theoretical phase noise calculated as the average free-running phase noise divided by four. Close agreement between theory and experiment is found for offset frequency ranging from 100 kHz to 10 MHz. The oscillators were not stable enough to allow accurate results below 100 kHz with our measurement setup. The difference between oscillators results from the dispersion in transistor characteristics and can explain the slight discrepancy between theoretical and experimental results.

IV. Conclusion

This paper has demonstrated that a loop configuration of four oscillators coupled through unidirectional amplifiers allows a division by four of the phase noise near the carrier compared to a single oscillator. The phase noise improvement is observed in the specific structure providing circular polarization and is not sensitive to tolerances on the coupling phase. This property can be extended to N oscillators in a loop configuration with a $1/N$ reduction of phase noise, in circular arrays of oscillating antennas for instance and for any type of polarization. It could also be extended to sequential arrays of the subarray presented in this letter. In this case, a further improved phase noise is expected through unilateral couplings between subarrays while increasing the CP purity.

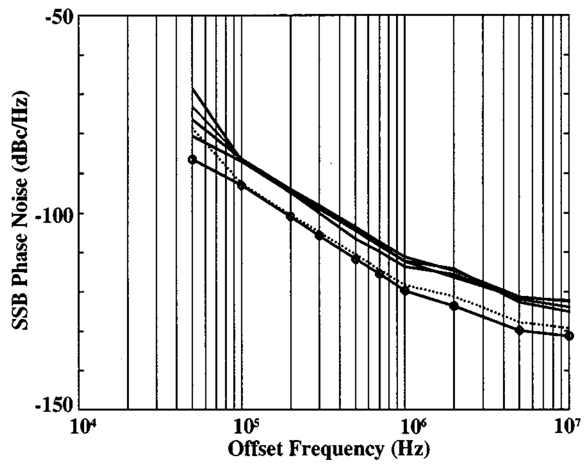

Fig. 2. Comparison of the free-running phase noise for each of the four oscillators in the experimental array with the total phase noise measured in the far field. The theoretical noise reduction is shown for comparison, which is the average free-running phase noise divided by four. —— free-running oscillators, ······ array (theory), and —•— array (experiment).

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