An Artificially-Synthesized Absorbing Medium for the Truncation of FDTD Lattices

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***Abstract*—An artificially-synthesized absorbing material composed of a doubly-periodic array of lossy electric and magnetic media (i.e., an ε and μ checkerboard) is presented for the truncation of Finite-Difference Time-Domain (FDTD) lattices in waveguide simulations. It is shown numerically that this artificially-synthesized material exhibits excellent absorption properties when used in waveguide simulations. However, unlike the Perfectly Matched Layer (PML) absorbing medium, the artificially-synthesized medium presented in this letter does not require any modification of the standard FDTD formulation. Numerical examples demonstrate that the FDTD implementation of the artificially-synthesized absorbing medium is stable as well as computationally efficient.**

***Index Terms*—Absorbing boundary condition, FDTD, waveguide.**

I. Introduction

THE finite-difference time-domain (FDTD) method has been proven to be a highly efficient technique for numerical applications in electromagnetics [1]. One of the greatest challenges of applying the FDTD technique to open radiation problems has been the development of accurate and computationally efficient absorbing boundary conditions. The three most commonly used mesh truncation techniques for FDTD simulations are the Mur's [2], Liao's [3], and PML [4] absorbing boundary conditions. The PML boundary condition is a lossy material boundary layer that is perfectly matched to the solution space, and can be divided into two categories, *viz*., split and unsplit PML boundary conditions. The split PML technique requires field splitting [4], [5] or coordinate stretching [6], [7], and leads to a modified set of Maxwell's equations. The fields within the PML medium and governing equations are non-Maxwellian. Research on the improvement of the PML technique is focused on the reduction of memory requirements and computational complexities [8]. On the other hand, the unsplit PML technique [9]–[11] uses Maxwell's equations. However, the update for the electric and magnetic fields remains a complicated and time consuming procedure.

In this letter, a new absorbing medium composed of a doubly-periodic array of lossy electric and magnetic media in the form of a checkerboard is presented. It is demonstrated numerically that this material, if designed appropriately, matches well to free space. The basic concept behind the technique presented in this letter is that the reflections from the lossy electric and magnetic slabs in the checkerboard will cancel each other, provided the constitutive parameters are chosen properly. In particular, if the parameters of the electric and magnetic blocks satisfy the condition $\varepsilon'/\varepsilon'' = \mu'/\mu''$, then the checkerboard will be matched to free space. To validate the method proposed in this letter, we use a checkerboard to truncate a PEC parallel plate waveguide and a rectangular waveguide. Numerical examples show good agreement between the checkerboard and a PML medium of the same order. In order to avoid handling each interface, edge and corner in the checkerboard, we introduce an alternative version of the FDTD scheme, in which the parameters of the medium are defined at the center of each FDTD cell, as opposed to at the locations of the E and H fields.

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II. FDTD Method

A. An Alternative FDTD Scheme

An alternative FDTD scheme based on the concept of effective permittivity and conductivity is used to model the numerous interfaces, edges and corners of the checkerboard. In this letter, we introduce an alternative scheme that defines the parameters of the media at the centers of the FDTD cells instead of at the location of the fields. In this scheme, it is necessary to calculate the effective permittivities and conductivities (ε and σ) of the medium at the locations of the electric fields by averaging these parameters at the four points surrounding the electric field. The effective permeabilities and magnetic conductivities (μ and σ^*) at the locations of the magnetic fields are found by averaging these parameters at points above and below these locations. This alternative scheme yields universal formulas that are valid both in the regions where the medium is uniform and at interfaces, edges and corners.

B. Artificially-Synthesized Absorbing Medium

The configuration of an artificially-synthesized checkerboard medium is shown in Fig. 1. It is composed of a doubly-periodic array of lossy electric and magnetic media in the x- and y-directions. The parameters of the lossy electric and magnetic material blocks satisfy the condition $\varepsilon'/\varepsilon'' = \mu'/\mu''$. It was found that better performance can be achieved, especially for oblique incidence, by configuring the lossy electric and magnetic blocks in such a way that they also alternate along the z direction. The electric conductivity σ and magnetic conductivity σ^* of the checkerboard are chosen to have spatial variation along the z-direction given by

$$\sigma = \sigma_{\max}\left(\frac{z - z_0 + 0.5\Delta z}{L}\right)^2 + \sigma_0 \qquad (1)$$

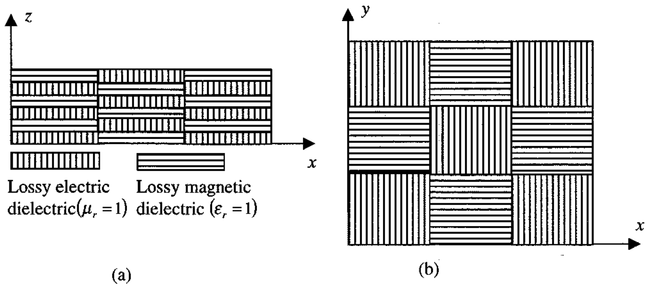

Fig. 1. Configuration of artificially-synthesized checkerboard material.

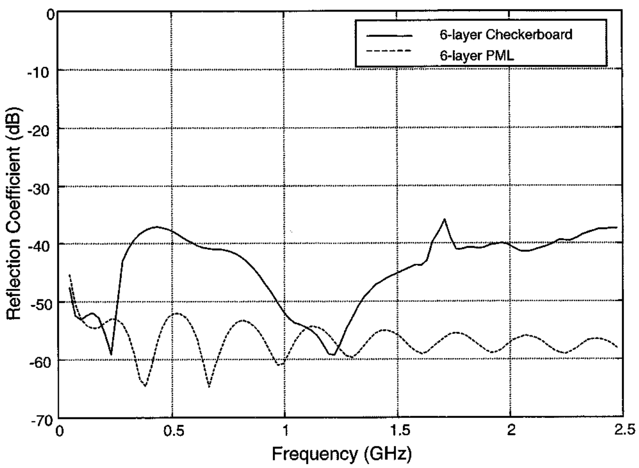

Fig. 2. Comparison of 6-layer artificially-synthesized checkerboard material and PML boundary condition for the normally incident case.

$$\sigma^* = \left[\sigma_{\max}\left(\frac{z - z_0 + 0.5\Delta z}{L}\right)^2 + \sigma_0\right]\frac{\mu_0}{\varepsilon_0} \tag{2}$$

where

z_0	location of the interface;
z	distance from the interface;
L	depth of the checkerboard;
Δz	cell size inside the checkerboard region;
$\sigma_{\max}$ and σ_0	constants.

III. Numerical Results

In this section, we present two results to validate the absorbing properties of the material proposed in this letter. The excitation source is a Gaussian pulse (3 dB cutoff frequency is 3 GHz) modulated by a sine function (1.5 GHz). The loss parameter distribution inside the checkerboard is calculated from (1) and (2), where $\sigma_{\max}$ and σ_0 were chosen to be 20 and 0.625, respectively. The $\sigma_{\max}$ and σ_0 values are chosen on the basis of the numerical experiments, and we did not observe any significant variation of reflection from the checkerboard when the value of $\sigma_{\max}$ was varied from 10 to 40. However, if the value of $\sigma_{\max}$ is chosen to be less than 10, we do observe the present of reflections originating from the PEC plane backing the absorber because the low loss material is not sufficiently absorptive. The relative permittivity and permeability of the checkerboard have the same value as free space and the element size of the checkerboard is 0.01×0.01 m^2. First, we use a 6-layer artificially-synthesized material to truncate a perfectly conducting parallel waveguide. The dimension of the entire computational domain is $0.02 \times 0.02 \times 0.056$ m^3 and includes $20 \times 20 \times 56$ cells. An excitation source with an E_y polarized plane wave is located at $z = 0.041$ m. The time voltage signature is sampled at $z = 0.025$ m. The reflection coefficient from the checkerboard is plotted in Fig. 2, along with the reflection coefficient of a 6-layer unsplit PML [10] included for comparison purposes. These results confirm the very low level of reflection produced by the checkerboard.

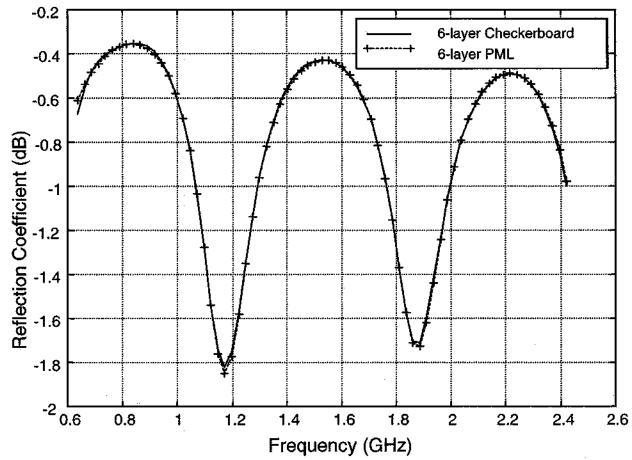

Fig. 3. Reflection coefficient comparison of a lossy dielectric slab using a 6-layer artificially-synthesized checkerboard material and the PML boundary condition in a waveguide simulation.

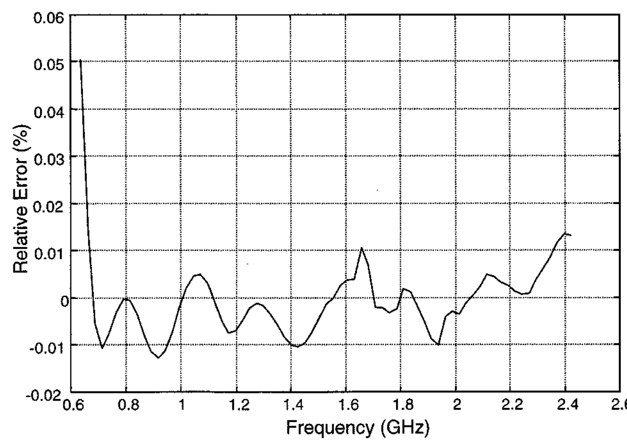

Fig. 4. Relative error of a 6-layer artificially-synthesized checkerboard material to the PML boundary condition for the reflection calculation of a lossy dielectric slab in a waveguide simulation.

Next, we use a 6-layer checkerboard to truncate one side ($z = 0$) of a waveguide. The dimension of the entire computational domain is $0.03 \times 0.02 \times 0.056$ m^3 and includes $30 \times 20 \times 56$ cells. The cut-off frequency corresponding to this geometry is 5.0 GHz. A lossy dielectric slab backed by a PEC plane is used to terminate the other side of the waveguide ($z = z_{\max}$). The thickness of this slab is 0.01 m, and its relative permittivity and conductivity are 4.0 and 0.1 S/m, respectively. A source that excites only the TE_{10} mode is used at $z = 0.025$ m and the time voltage signature is sampled at $z = 0.041$ m. The reflection coefficient of the lossy dielectric slab is plotted in Fig. 3. The results for a 6-layer unsplit PML absorbing material is again shown for comparison purposes. The relative error of the checkerboard compared to a 6-layer unsplit PML for the reflection coefficient of the lossy dielectric slab is plotted in Fig. 4. The two results are comparable, but the checkerboard neither splits the fields like the split PML technique nor employs a complicated updating procedure like the unsplit PML technique. All of these results have been obtained by using 18 000 time steps with a time step equal to 99.5% of the Courant limit. Additional runs, going up to 25 000 time steps, and no instability has ever been observed. In the numerical experiments, we did not see any noticeable changes in the results when the element size of the checkerboard from 0.007×0.007 m^2 to 0.014×0.014 m^2.

IV. Conclusions

A new artificially-synthesized absorbing material composed of lossy dielectric (only) and lossy magnetic (only) materials in a checkerboard configuration for the truncation of FDTD lattices has been presented. It was shown that by properly choosing the constitutive parameters, the absorbing medium can be perfectly matched to free space. Unlike the existing PML techniques, the absorbing medium presented in this letter is based on standard FDTD update equations. In the FDTD simulation, the absorbing material is handled as a general lossy object. Numerical examples demonstrate that the FDTD implementation of the new absorbing material in waveguide simulations is stable and efficient. The problem of two intersecting checkerboard absorbers will be investigated in the future.

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