Accelerated Computation of the Propagation Constants of Multiconductor Planar Lines

Otman Aghzout and Francisco Medina, *Member, IEEE*

***Abstract*—This paper reports on an efficient and simple technique to speed up the full-wave analysis of strip/slot-like transmission lines embedded in a multilayered substrate. The method is primarily based on the use of interpolation techniques in the generation of the characteristic eigenvalue matrix. Moreover, the features of the basis functions are exploited so as to reduce the size of the working matrix by following the guidelines reported in [1]. It is demonstrated that important CPU time saving is achieved without sacrificing accuracy.**

***Index Terms*—Computer-aided design, planar lines, spectral domain analysis.**

I. Introduction

The propagation constants (β's) of strip/slot-like transmission lines on multilayered iso/anisotropic substrates (see Fig. 1) can be efficiently computed by means of the spectral domain analysis (SDA) [2] or the singular integral equation method [3]. Sophisticated analytical techniques have been added to the original formulation of those methods, leading to very fast computer codes (see [4] and references therein). Nevertheless, if the number of coupled strips/slots is large, the required numerical effort could be still excessive for computer-aided design (CAD) applications. Some strategies have been reported to improve the situation. Thus, Yang *et al.* [5] assume that the dynamic free surface current distribution on a multistrip system is close to the quasi-static one. They insert the fast quasi-static solution into the full-wave formulation, thus reducing the computational effort. Huynen *et al.*[6] make use of variational expressions for the propagation parameters together with quasi-static approximations for the field over the cross section of the structure under study (coplanar waveguide). Unfortunately, the true full-wave field/current distribution may be completely distinct from the quasi-static one (this is particularly true for suspended substrate structures). In addition, it is not easy to built approximated expressions for fields or currents for a general structure. Therefore, the approximations used in the mentioned methods may be rather crude or limited in some situations of practical interest. We propose here a completely different approach that does not rely on *a priori* approximations. Our proposal is based on a simple fact: although the characteristic function whose roots determine the desired eigenvalues (usually the determinant of a numerically generated characteristic matrix $[A]$) is a fast varying function of the unknown, β, the individual elements of the matrix $A_{ij}(\beta)$ are smooth functions of such variable, provided appropriate basis functions are used. Since the evaluation of $[A]$ is responsible for most of the computational effort, it is obvious that the application of an interpolation scheme to generate $[A]$ would yield a drastical reduction of CPU time (it is worth mentioning that the application of interpolation techniques with respect to the frequency parameter was proposed in [7] so as to speed up the generation of the moment method impedance matrix in wide-band computations). It is expected that the CPU time saving is more significant when the number of required evaluations of $[A]$ increases: multiple strip/slot or need for using many basis functions (closely spaced wide strips, for example). If the elements of $[A]$ can be accurately approximated with a low degree polynomial, the CPU time involved in their computation becomes negligible in comparison with the time devoted to compute the determinant of $[A]$. Fortunately, thanks to the features of the used basis functions, it is also possible to reduce the size of the matrix whose determinant has to be eventually computed [1].

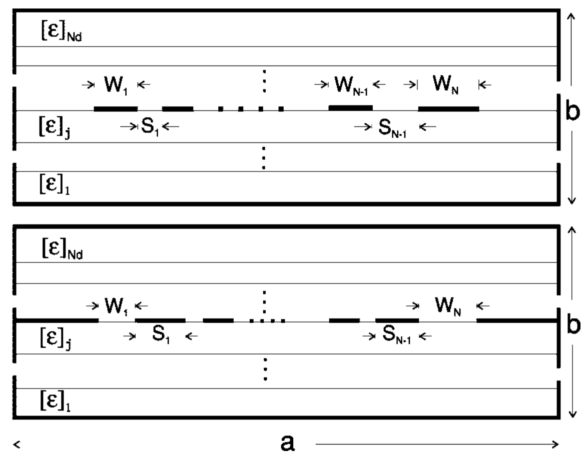

Fig. 1. Cross sections of (a) strip-like and (b) slot-like boxed planar structures.

Manuscript received November 23, 2000; revised January 2, 2000. This work was supported by the Comisión Interministerial de Ciencia y Tecnología, Spain, under Project TIC98-0630.

O. Aghzout was with the Departamento de Electrónica y Electromagnetismo, Facultad de Física, Universidad de Sevilla, Sevilla 41012 Spain and the Electronics and Microwaves Group, Faculty of Sciences, Université Abdemalek Essaadi, 93000 Morocco. He is now with the Departamento de Electrónica y Automática, Universidad Las Palmas de Gran Canaria, Las Palmas 35017 Spain.

F. Medina is with the Departamento de Electrónica y Electromagnetismo, Facultad de Física, Universidad de Sevilla, Sevilla 41012 Spain (e-mail: medina@cica.es).

Publisher Item Identifier S 1051-8207(00)04881-9.

II. Method of Analysis

SDA, the most widely used method for the analysis of planar lines, yields a characteristic matrix $[A]$ whose determinant vanishes for the desired eigenvalues (β's). Obtaining the proper eigenvalues requires one to build up $[A(\beta)]$ for many different values of β (this number is relatively large for multiple strip/slot lines). Even though very fast methods have been developed to perform such computations [4], this is still the most time-consuming step in the solution process. The overall computational cost would be drastically reduced if the numerical/analytical evaluation of the entries of $[A]$ could be avoided. At a first glance, this could be done using an interpolated version of the determinant of $[A]$ (as a function of β) instead of the original function. However, that determinant is a fast varying function of β, and its correct approximation would require a huge number of sample points. Nothing then would be gained that way. However, if appropriate basis functions are used for the strip currents/slot fields, the individual elements of $[A]$ are very smooth functions of β. In such a case, a low degree polynomial could be used to generate $[A]$ from a few sample points over the range of interest. This is the case if first and second kind Chebyshev polynomials weighted by the Meixner edge condition are used to approximate the unknown functions [4]. This property is closely related to the nature of the integral equation kernel. It can be theoretically understood—and it has been numerically verified by the authors—that the reaction integrals defining those elements of $[A]$ involving transverse currents or longitudinal electric fields are almost independent of β or, at most, can be approximated with a linear function of β. On the other hand, the dominant longitudinal currents (or transverse electric fields) are approximated by a zero-order function, giving the total current on the strips or total voltage drop through the slots, plus a set of high-order contributions. Those reaction integrals involving high-order functions can be accurately approximated by means of linear or quadratic functions of β. Finally, the zero-order fundamental functions give place to reaction integrals requiring a slightly higher degree polynomial (it should be mentioned that these functions account for most of the relevant information on current/field distribution). Therefore, the whole $(N \times M) \times (N \times M)$ matrix $[A]$— where N is the number of strips/slots and M the number of basis functions per strip/slot—has to be numerically computed just a few times. But, in such a case, the CPU time devoted to the computation of the determinant becomes dominant. Fortunately, since most of the information about the solutions is in the $(N \times N)$ submatrix involving the dominant or fundamental contributions, the remaining being a perturbation, it is possible to apply a block triangular decomposition of $[A]$ so as to produce an equivalent $(N \times N)$ matrix problem $[A]'$ having those eigenvalues of $[A]$ we are looking for [1]. Indeed, the matrix $[A]$ can be written as

$$[A] = \begin{bmatrix} [A]_{\rm ff} & [A]_{\rm fh} \\ [A]_{\rm hf} & [A]_{\rm hh} \end{bmatrix} \tag{1}$$

where $[A]_{\rm fh} = [A]_{\rm hf}^t$ is a $N \times [(N \times M) - N]$ rectangular matrix and $[A]_{hh}$ is an $[(N \times M) - N] \times [(N \times M) - N]$ square matrix. It is simple to show that

$$\det[A] = \det[A]' \cdot \det[A]_{\rm hh} \tag{2}$$

where $[A]' = [A]_{\rm ff} - [A]_{\rm fh}[A]_{\rm hh}^{-1}[A]_{\rm hf}$ is an $N \times N$ square matrix. For the basis functions in this paper, the eigenvalues corresponding to the fundamental (and first higher order modes) are all associated to $\det[A]'$. The roots of $\det[A]_{\rm hh}$ are associated with very high-order modes outside our region of interest. Therefore, what we actually need is to compute the determinant of a $(N \times N)$ matrix $[A]'$. Notice that the elements of $[A]_{\rm fh}$ and $[A]_{\rm hh}^{-1}$ can be generated by means of very low-order polynomial interpolation (thus, matrix inversion is carried out no more than two or three times in practice). In order to keep very high accuracy, a slightly higher order is advisable for the elements of $[A]_{\rm ff}$. The combination of interpolation and block diagonal decomposition has allowed us to develop a very fast code for the full-wave analysis of multiple coupled planar lines without sacrificing the accuracy of the original formulation. Accuracy is not diminished because the correct current (or field) expansion is used at any frequency or for any structure without *a priori* assumptions, which can fail for some geometries or operating frequencies.

TABLE I
EFFECTIVE DIELECTRIC CONSTANTS FOR THE SIX QUASI-TEM TYPE FUNDAMENTAL MODES OF A SIX-STRIP STRUCTURE (DATA IN TEXT)

freq (GHz)	ref. [4] ϵ_{ef}	This method $N_h = 1, N_f = 2$ ϵ_{ef}	t_e/t_i	This method $N_h = 2, N_f = 3$ ϵ_{ef}	t_e/t_i
1.0	6.4492	6.4504	12.64	6.4492	10.32
	6.5486	6.5518		6.5486	
	6.7751	6.7791		6.7751	
	7.2417	7.2437		7.2417	
	8.1310	8.1300		8.1310	
	9.6108	9.6083		9.6108	
5.0	6.4869	6.4866	17.17	6.4869	14.23
	6.6075	6.6086		6.6075	
	6.8867	6.8881		6.8867	
	7.4786	7.4790		7.4785	
	8.6380	8.6378		8.6380	
	10.3819	10.3826		10.3819	
15.0	6.7983	6.7989	17.80	6.7983	14.97
	7.0779	7.0777		7.0779	
	7.6847	7.6837		7.6848	
	8.7223	8.7227		8.7223	
	9.9919	9.9912		9.9919	
	11.0100	11.0134		11.0103	

III. NUMERICAL RESULTS

The technique outlined in the previous section has been applied to the computation of the propagation constants of multiple strip/slot transmission lines composed by an arbitrary number of conducting strips or slots and dielectric layers. The results obtained after applying interpolation and reduction of the matrix size have been systematically checked against the ones computed by straightforward application of the optimized SDA reported in [4]. As an example, Table I shows the values of $\epsilon_{\rm ef}$ (effective dielectric constant) for the six quasi-TEM type fundamental modes of six evenly spaced ($S_i = 0.5$ mm) equal width ($W_i = 1$ mm) coupled strips printed on a single isotropic substrate ($\epsilon_r = 11.7$, thickness, $h = 1$ mm) inside a metallic box ($a = 20$ mm, $b = 11$ mm) at three different frequencies. The number of basis functions of the kind used in [4] was chosen so as to ensure that all the figures in the first column are correct (four functions for the longitudinal surface current density and three functions for the transverse one). The first column stands for data computed using [4]. The

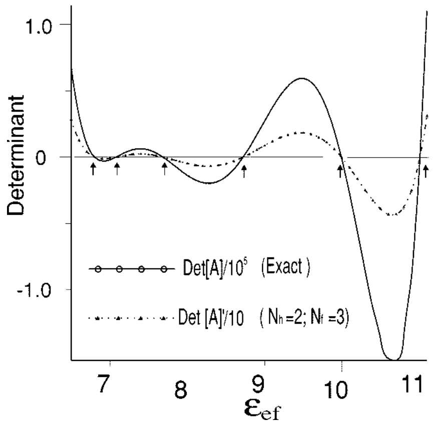

Fig. 2. Determinants of $[A]$ and $[A]'$ for the example structure in Table I. Frequency: 15 GHz.

TABLE II
EFFECTIVE DIELECTRIC CONSTANTS FOR THE FIVE QUASI-TEM TYPE FUNDAMENTAL MODES OF A SIX-SLOT COPLANAR WAVEGUIDE (DATA IN TEXT)

	ref. [4]	This method $N_h = 1, N_f = 2$		This method $N_h = 2, N_f = 3$	
freq (GHz)	ϵ_{ef}	ϵ_{ef}	t_e/t_i	ϵ_{ef}	t_e/t_i
5.0	2.5077	2.5081	18.23	2.5078	15.35
	3.0390	3.0370		3.0394	
	3.4520	3.4500		3.4520	
	3.7466	3.7432		3.7460	
	3.9260	3.9243		3.9260	

other two columns correspond to the method in this paper. The results in the second column were obtained using quadratic interpolation $(N_f = 2)$ for the elements of $[A]_{\mathrm{ff}}$ and linear interpolation $(N_h = 1)$ for the elements of $[A]_{\mathrm{fh}}$ and $[A]_{\mathrm{hh}}^{-1}$. Cubic $(N_f = 3)$ and quadratic $(N_h = 2)$ interpolations were used to generate the results in the third column. The interpolation interval was chosen wide enough to ensure that all the solutions were captured (ϵ_{ef} ranges from 4 to 11.7 in this example). The columns labeled t_e/t_i stand for the CPU time saving factor. It is clear that the results are very accurate even for very low order of interpolation, being virtually exact for $N_f = 3, N_h = 2$. The CPU time saving is, at least, of one order of magnitude. In Fig. 2, we have plotted the determinant of the whole characteristic matrix $[A(\epsilon_{\mathrm{ef}})]$ as a function of the tentative effective dielectric constant for the structure analyzed in Table I (frequency 15 GHz). The determinant of the reduced matrix $[A(\epsilon_{\mathrm{ef}})]'$ is also plotted. It is clear that the roots of both determinants coincide within the interval shown in the figure. New roots of the determinant of $[A]$, which are not roots of the determinant of $[A]'$, appear for imaginary or complex values of ϵ_{ef}, but they are obviously outside of the region of interest. As a final example, Table II shows a similar analysis for the five quasi-TEM type modes of a coplanar waveguide structure with six equal width slots ($W_i = 1$ mm) defined by five equal width ($S = 1$ mm) strips printed on a thin ($h = 0.2$ mm) AsGa substrate ($\epsilon_r = 12.9$). The whole is suspended and centered inside a metallic enclosure ($a = 20$ mm; $b = 10.2$ mm). Data correspond to 5 GHz. Conclusions are the same as for the microstrip structure. This type of numerical experiment has been performed for a large variety of geometries, leading to similar results. Thus, we can conclude that the results provided by the computer code incorporating interpolation and reduction of the size of the working matrix are extremely accurate, while CPU time is meaningfully reduced. Therefore, the method is very useful to accelerate SDA computations of planar lines.

IV. CONCLUSION

A simple procedure to accelerate the SDA computation of the frequency-dependent effective dielectric constants of multiconductor planar transmission lines has been described. The method is mainly based on the use of low degree polynomial interpolation to obtain very accurate approximations for the elements of the Galerkin matrix as functions of β. This is possible thanks to the smooth dependence of these elements on the β variable, in contrast with the strong dependence of the determinant of the Galerkin matrix. In addition, the working matrix size is established by the number of strips/slots and not by the number of basis functions, which can be much larger. Important reduction of CPU time is achieved with this method, whereas the generality and accuracy of the original formulation is not affected, in contrast with other fast methods reported in the literature.

REFERENCES

[1] R. H. Jansen, "Some notes on hybrid-mode versus quasistatic characterization of high frequency multistrip interconnects," in *Proc. 23rd Eur. Microwave Conf.*, Madrid, Spain, Sept. 1993, pp. 220–222.

[2] F. Medina, M. Horno, and H. Baudrand, "Generalized spectral analysis of planar lines on layered media including uniaxial and biaxial dielectric substrates," *IEEE Trans. Microwave Theory Tech.*, vol. 37, pp. 504–511, Mar. 1989.

[3] Y.-S. Xu and A. S. Omar, "Rigorous solution of mode spectra for shielded multilayer microstrip lines," *IEEE Trans. Microwave Theory Tech.*, vol. 42, pp. 1213–1222, July 1994.

[4] G. Cano, F. Medina, and M. Horno, "On the efficient implementation of SDA for boxed strip-like and slot-like structures," *IEEE Trans. Microwave Theory Tech.*, vol. 46, pp. 1801–1807, Nov. 1998.

[5] J. J. Yang, G. E. Howard, and Y. L. Chow, "A simple technique for calculating the propagation dispersion of multiconductor transmission lines in multilayer dielectric media," *IEEE Trans. Microwave Theory Tech.*, vol. 40, pp. 622–627, Apr. 1992.

[6] L. Huynen, D. Vanhoenacker-Janvier, and A. Vander Vorst, "Spectral domain form of new variational expression for very fast calculation of multilayered lossy planar line parameters," *IEEE Trans. Microwave Theory Tech.*, vol. 42, pp. 2099–2106, Nov. 1994.

[7] E. H. Newman, "Generation of wide-band data from the method of moments by interpolating the impedance matrix," *IEEE Trans. Antennas Propagat.*, vol. 36, pp. 1820–1824, Dec. 1988.