Improved Design Equations for Multilayer Microstrip Lines

Changhua Wan, *Senior Member, IEEE,* and Ahmad Hoorfar, *Senior Member, IEEE*

***Abstract*—This letter presents a physics-based improvement on the previous multilayer microstrip design equations that are found valid only for up to two layers. A three-layer example shows that the improvement leads to an excellent agreement between analytical calculations and moment method solutions. The improved equations are useful for designing any multilayer microstrip lines.**

***Index Terms*—Microstrip, multilayer microstrip.**

I. Introduction

INVERTED and suspended microstrips are useful two-layer microstrip lines in microwave applications. However, our interest here focuses on a multilayer microstrip line where the signal line resides on the first layer (substrate). Such a multilayer microstrip line can be used to feed stacked-patch antennas.

For a stacked-patch antenna structure with a multilayer microstrip feed, a mixed potential integral equation (MPIE) can be formulated in spatial domain and solved efficiently with the method of moments (MoM) [1]. Based on the MoM solution, one can extract the propagation constant of the feed and calculate the input impedance of the antenna using

$$Z = Z_f \frac{1+\Gamma}{1-\Gamma} \tag{1}$$

where Z_f is the characteristic impedance of the feed and Γ is the reflection coefficient of the load (antenna) presented to the feed [2].

The characteristic impedance Z_f of the feed can be obtained either from explicit design equations or from the effective permittivity concept with the extracted propagation constant and exact impedance of a corresponding free-space microstrip line or from other numerical methods. However, closed-form design equations, if available, are far more preferable in practice. Unfortunately, the existing equations for a multilayer microstrip line [3], [4] are found to be valid only for a structure with no more than two layers.

It is the purpose of this letter to improve the aforementioned design equations so that they become valid also for a multilayer microstrip structure with more than two layers. The key to the improvement is a study of a limiting case. Comparison with accurate MoM-based data will be made to verify the improved equations.

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C. Wan is with QuinStar Technology, Inc., Torrance, CA 90505 USA.

A. Hoorfar is with Department of Electrical and Computer Engineering, Villanova University, Villanova, PA 19085 USA.

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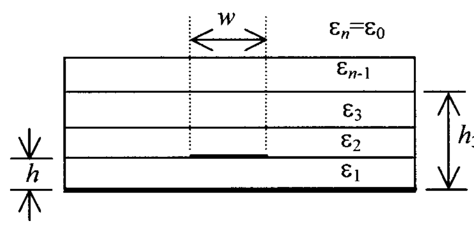

Fig. 1. Geometry of a multilayer microstrip line.

II. Improved Design Equations

To correlate mathematical expressions with physical dimensions, a cross-sectional view of a multilayer microstrip line is shown in Fig. 1. The width of the signal line, the cumulative thickness of the first j layers, and the dielectric constant of jth layer are respectively denoted by w, h_j, and ε_j while the thickness of the first layer (substrate) by h as in an one-layer microstrip line.

For the multilayer microstrip line in Fig. 1, Svačina [4] proposed the following analytical expressions for the filling factors of individual layers based on conformal mapping analysis.

For $w/h \geq 1$ (wide strip)

$$q_1 = 1 - \frac{1}{2\bar{w}_e}\ln(\bar{w}_e\pi - 1) \tag{2}$$

$$q_j = \frac{1}{2\bar{w}_e}\left\{\ln(\bar{w}_e\pi - 1) - (1-\bar{v}_j) \times \ln\left[\frac{2\bar{w}_e\cos(0.5\bar{v}_j\pi)}{2\bar{h}_j - 1 + \bar{v}_j} + \sin(0.5\bar{v}_j\pi)\right]\right\} \tag{3}$$

where

$$\bar{w}_e = \bar{w} + \frac{2}{\pi}\ln[17.08(0.5\bar{w} + 0.92)] \tag{4}$$

and

$$\bar{v}_j = \frac{2}{\pi}\tan^{-1}\left[\frac{2\pi}{\bar{w}_e\pi - 4}(\bar{h}_j - 1)\right]. \tag{5}$$

For $w/h < 1$ (narrow strip)

$$q_1 = 0.5 + \frac{0.9}{\pi\ln(0.125\bar{w})} \tag{6}$$

$$q_j = 0.5 - \frac{0.9 + 0.25\pi\ln b_j \cdot \cos^{-1}\left[\frac{\sqrt{b_j}}{\bar{h}_j}(\bar{h}_j - 1 + 0.125\bar{w})\right]}{\pi\ln(0.125\bar{w})} \tag{7}$$

where

$$b_j = \frac{\bar{h}_j + 1}{\bar{h}_j + 0.25\bar{w} - 1}. \tag{8}$$

From (2) to (8), all the bars over their corresponding letters stand for "normalized to h" and $j = 2, 3, \ldots, n-1$. Note that according to [3], other than $1 + \bar{v}_j$ in $1 - \bar{v}_j$ [4] is used in (3). Because of their physical interpretation, filling factors cannot be negative. However, calculations show that for a microstrip line with more than two layers, q_j in (3) or (7) sometimes becomes negative. To avoid this, consider a limiting case with h_j approaching infinity. In this case, the following equation should hold:

$$q_j = 1 - \sum_{i=1}^{j-1} q_i. \tag{9}$$

On the other hand, when $h_j \to \infty$, (3) and (7) become, respectively,

$$q_j = \frac{1}{2\bar{w}_e} \ln(\bar{w}_e \pi - 1) \tag{10}$$

and

$$q_j = 0.5 - \frac{0.9}{\pi \ln(0.125\bar{w})}. \tag{11}$$

Using (2) and (6), one immediately sees that only if $n = 3$ (two-layer case), (10) and (11) are equivalent to (9). As a result, (3) and (5) should be improved to become, respectively,

$$q_j = 1 - \sum_{i=1}^{j-1} q_i - \frac{1 - \bar{v}_j}{2\bar{w}_e} \times \ln\left[\frac{2\bar{w}_e \cos(0.5\bar{v}_j \pi)}{2\bar{h}_j - 1 + \bar{v}_j} + \sin(0.5\bar{v}_j \pi)\right] \tag{12}$$

and

$$q_j = 1 - \sum_{i=1}^{j-1} q_i - \frac{\ln b_j \cdot \cos^{-1}\left[\frac{\sqrt{b_j}}{\bar{h}_j}(\bar{h}_j - 1 + 0.125\bar{w})\right]}{4\ln(0.125\bar{w})}. \tag{13}$$

Other expressions in [4] remain unchanged.

III. Results

As an example, the effective permittivity and characteristic impedance of a three-layer ($n = 4$) microstrip where the signal line is on the first layer are calculated using previous and improved design equations and compared with the moment method based data in Table I.

TABLE I
EFFECTIVE PERMITTIVITY AND CHARACTERISTIC IMPEDANCE OF A THREE-LAYER MICROSTRIP LINE WITH $\varepsilon_{r1} = 2.2$, $\varepsilon_{r2} = 1.05$, $\varepsilon_{r3} = 2.55$, $t_1 = t_3 = 0.8$ mm, and $t_2 = 1.6$ mm

w/h	ε_{eff}			Z_c(Ohms)		
	[4]	New	MoM	[4]	New	MoM
0.6	2.412	1.757	1.782	100.08	117.24	116.85
0.8	2.397	1.774	1.793	89.23	103.74	103.96
1.0	2.384	1.789	1.807	80.81	93.28	94.01
1.5	2.381	1.836	1.838	67.20	76.52	76.73
2.0	2.346	1.867	1.866	58.25	65.30	65.16
2.5	2.317	1.893	1.891	51.56	57.04	56.81
3	2.291	1.915	1.913	46.35	50.71	50.44
4	2.252	1.948	1.955	38.69	41.59	41.27
5	2.223	1.974	1.998	33.29	35.33	34.93

It is seen that in the worst case, the effective permittivity and characteristic impedance from the previous equations differ from the MoM-based data by 35.35% and 14.35%. Calculations show that if $(1+\bar{v}_j)$ is used in (3), the case is even worse. In contrast, the corresponding differences between the results from the improved equations and those from the moment method analysis are only 1.40% and 1.14%, respectively.

IV. Conclusions

Previous design equations for multilayer microstrip lines have been found valid only for up to two layers and improved to become valid also for more layers. The improvement is based on an investigation of a limiting case and verified by a comparison with accurate moment method based data. Although the focus of this discussion is on a multilayer microstrip structure whose signal line resides on the first layer to be used as a stacked-patch antenna feed, the improvement is equally applicable to other multilayer microstrip structures.

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