Microwave Secondary Frequency Standards: Stability Limits Due to Intrinsic Fluctuations in Frequency Discriminator

J. G. Hartnett, E. N. Ivanov, and M. E. Tobar

***Abstract*—Limitations imposed on long-term frequency stability of microwave oscillators by intrinsic fluctuations in a frequency discriminator (FD) with RF modulation (Pound scheme) have been studied. It has been found that adding a microwave amplifier before detection, lowers the frequency discriminator noise floor by an amount equal to the amplifier gain. This offers the advantage of achieving high discriminator sensitivity without high levels of incident power or RF modulation index.**

***Index Terms*—Carrier suppression, frequency discriminator, interferometer, pound modulation, stability.**

I. Introduction

FREQUENCY stability of a classical electromagnetic oscillator based on a high-Q resonator is influenced by both external and intrinsic noise sources. The former includes, for example, temperature fluctuations affecting oscillator electronics and the resonator. The latter includes intrinsic noise produced by the oscillator sustaining (i.e., amplifier) and frequency control stages (i.e., frequency discriminator). Characterizing an oscillator's frequency instability in terms of the square root of the Allan Variance or Allan Deviation of fractional frequency fluctuations, the limit due to the frequency discriminator (FD) intrinsic voltage noise (σ_y^{FD}) is given by

$$\sigma_y^{FD} = \frac{1}{S_{FD}} \frac{1\,Volt}{f_{res}} \sigma_u \tag{1}$$

where

- S_{FD} FD sensitivity or conversion ratio (V/Hz);
- f_{res} resonator operational mode frequency;
- σ_u Allan Deviation of fractional voltage fluctuations at the output of FD.

In calculating σ_u, the voltage data were normalized to 1 V, then the square root of the Allan Variance of the resulting data calculated in the usual way [1].

A Pound Frequency Discriminator [2] (subsequently referred to as FD) is commonly used as a frequency sensor in the control electronics of microwave secondary frequency standards. The sensitivity (frequency-to-voltage conversion ratio) of the Pound FD can be found as [3]

$$S_{FD} = \eta K_{amp}^{RF} \gamma_P P_{res} K_{amp}^{\mu w} 4 J_0(\varphi_m) J_1(\varphi_m) \frac{4\beta}{(1+\beta)^2} \frac{Q_0}{f_{res}} \tag{2}$$

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The authors are with The Physics Department, University of Western Australia, Nedlands 6907 WA, Australia.

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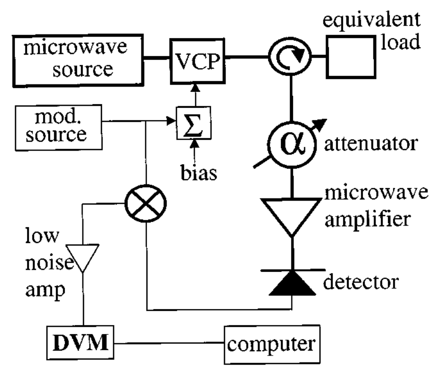

Fig. 1. Schematic of Pound Frequency Discriminator (FD).

where

- η RF mixer conversion ratio;
- K_{amp}^{RF} gain of the low noise RF amplifier at the output of the mixer;
- γ_P diode detector conversion ratio (in volts/Watts);
- P_{res} power incident on the resonator;
- $K_{amp}^{\mu w}$ gain of a microwave amplifier placed in front of the detector;
- Q_0 resonator unloaded Q-factor;
- β resonator coupling coefficient.

J_0 and J_1 are Bessel functions of the phase modulation index φ_m. For the condition $\varphi_m \ll 1$, $J_0(\varphi_m)J_1(\varphi_m) = \varphi_m/2$. If the detector is operating in the small signal regime, γ_P is independent of power and can be assigned a constant value. However, this is not true at high incident detector power where γ_P decreases with power. From (2) it is clear that in order to maximize S_{FD}, coupling (β) should be close to unity, the Q-factor high and $J_0(\varphi_m)J_1(\varphi_m)$ should be maximized. The latter is achieved at $\varphi_m = 1.08$.

II. Experimental Results and Discussion

In order to evaluate the noise contribution of the FD electronics to the oscillator frequency instability, a nondispersive equivalent load was substituted for the high-Q resonator as shown in Fig. 1. A microwave amplifier was introduced before the detector to study its effect on the FD output voltage noise. The input power to the amplifier was varied between -40 and -26 dBm by making use of the attenuator (α). Allan Deviation of fractional voltage fluctuations at the output of the FD (σ_u), with two different detectors, measured as a function of input power to the detector ($P_{\rm det}$) is shown in

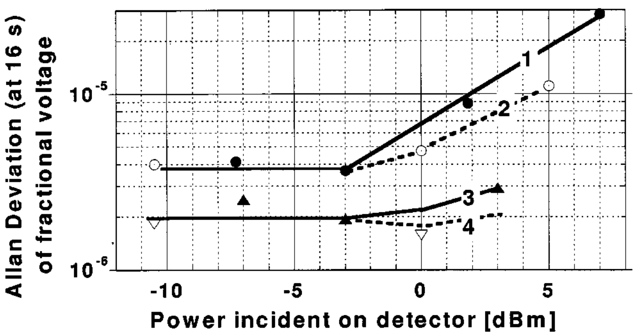

Fig. 2. Allan Deviation (at 16 s integration time) of voltage fluctuations of the FD electronics using detector #1 with a microwave amplifier before the detector (curve 1) and without the amplifier (curve 2). Measurements were repeated with the same microwave amplifier and detector 2 (curve 3) and without the amplifier (curve 4). All measurements were taken using the same voltage controlled phase shifter (VCP) and phase modulation index.

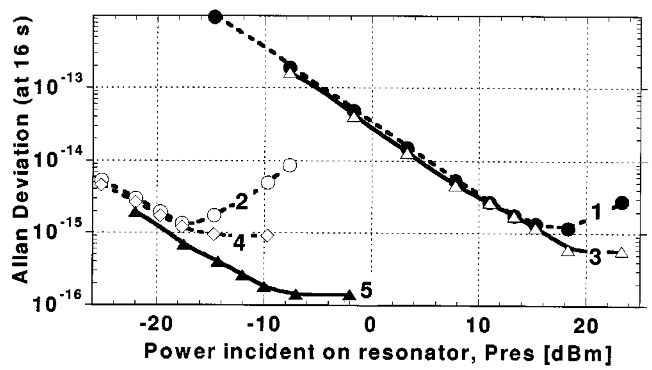

Fig. 3. Allan Deviation (at 16 s integration time) of frequency fluctuations due to the FD electronics as function of power incident on the resonator.

Fig. 2. The measurement time was chosen to be 16 s. Even though detector 2 had lower power-to-voltage conversion efficiency than detector 1, it was found to be intrinsically less noisy (curves 3 and 4). The power levels at which detectors 1 and 2 enter the regime of saturation are -3 dBm and 0 dBm, respectively. In such a regime, the power-to-voltage efficiency of the detector is no longer constant and reduces with power. Apart from that, intensity of voltage fluctuations at the output of the FD increases with power (see Fig. 2). This was found to be mainly due to detector excess noise with $1/f$ spectral density. The effect of amplifier flicker noise on Allan Deviation of fractional voltage fluctuations in the saturation regime was evaluated by removing the microwave amplifier from the FD and monitoring the output voltage noise while maintaining the same level of power incident on the detector. For instance, the data given in curves 1 and 2 in Fig. 2 were measured with the FD based on detector 1. Curve 1 corresponds to the FD with the microwave amplifier installed and curve 2 corresponds to the conventional FD without the microwave amplifier. As seen in Fig. 2, introducing the microwave amplifier into the FD results in a 2–4 dB increase in Allan Deviation of the fractional voltage fluctuations, when power -3 dBm $< P_{\rm det} <$ 3 dBm. Voltage noise measurements were performed at a modulation frequency of 1 MHz to minimize the contribution of flicker noise of the detector. Repeated measurements at a higher modulation frequency of 10 MHz did not reveal any significant difference.

Combining data in Fig. 2 with (1) and (2) enabled the calculation of the limit imposed on the fractional frequency fluctuations of a cryogenic oscillator by the FD electronic noise. Here we assumed a 9 GHz resonator with a Q-factor of 10^7, typical for a frequency-temperature compensated 77K sapphire resonator [4]. The phase modulation index was held constant at 0.07 during all measurements. The RF mixer conversion ratio (η) was measured at 0.58 V/V. The small signal gain of the RF (K^{RF}_{amp}) and the microwave ($K^{\mu w}_{amp}$) amplifier were measured to be 40 dB and 33 dB, respectively. The detector conversion ratio (γ_P) was measured to be 743 mV/mW and 640 mV/mW for detectors 1 and 2, respectively, at $P_{\rm det} = -10$ dBm.

The value of σ^{FD}_y (16 s) for different FD configurations and components is plotted in Fig. 3 as a function of power incident on the resonator (P_{res}). For instance, curve 1 shows σ^{FD}_y (16 s) for a conventional Pound FD based on detector 1, that is, the circuit in Fig. 1 without the microwave amplifier or attenuator. Curve 2 corresponds to σ^{FD}_y (16 s) of this FD with the addition of an amplifier having a small signal gain of 33 dB. In both cases, a resonator coupling coefficient $\beta = 0.8$ was assumed. This corresponded to a total power (carrier plus sidebands) reflected from the resonator of approximately 18 dB below P_{res}.

In the absence of a microwave amplifier, the detector becomes saturated when P_{res} ≈15 dBm, which explains the knee in the plot of σ^{FD}_y (16 s) versus P_{res} occurring at this power level (curve 1). If the amplifier with 33 dB gain is introduced, P_{res} must be kept below -18 dBm to avoid detector saturation and to minimize its effect on oscillator frequency stability. As follows from a comparison of curves 1 and 2, the same minimum value of σ^{FD}_y (16 s), close to 10^{-15}, can be attained at power levels limited by saturation of the detector. Increasing or decreasing the modulation index simply translates the curves horizontally. The above calculations were repeated for a FD based on detector 2, first, without the microwave amplifier (curve 3) and then with the microwave amplifier introduced (curve 4). Finally, σ^{FD}_y (16 s) was calculated for the FD comprising the 33 dB amplifier, detector 2, and a critically coupled resonator (curve 5). In such a case, values of σ^{FD}_y (16 s) close to 2×10^{-16} were obtained. It can be shown that to achieve such a performance a circulator with more than 26 dB of isolation is required, to avoid detector saturation by signal leakage.

Data in Fig. 3 clearly demonstrate that the addition of the amplifier reduces the Allan Deviation of fractional frequency fluctuations, at the same level of incident resonator power, by the amount of the amplifier gain, provided the small signal operation of the detector is ensured. This is due to a reduction of FD effective noise temperature.

III. INTERFEROMETER

The benefits of using an amplifier in conjunction with the critically coupled cavity are quite apparent (compare curves 3 and 5 in Fig. 3). In practice, the equivalent of a critically coupled resonator is obtained by suppressing the reflected carrier in an interferometer configuration [5]. The carrier at the input of the amplifier can be made more than 100 dB [6] less than the power incident on the resonator, resulting in the minimum obtainable Allan Deviation. However, introducing interferometric carrier suppression will cause additional oscillator frequency fluctuations due to temperature induced fluctuations of the interferometer phase mismatch ($\delta\Psi$). The latter are related to temperature fluctuations (δT) by

$$\delta\Psi = \frac{\omega}{c}\,\Delta l \frac{1}{\varepsilon}\,\frac{\partial\varepsilon}{\partial T}\,\delta T \tag{3}$$

where

- Δl the arm length mismatch;
- c speed of light;
- ω angular frequency of the oscillator;
- ε relative dielectric permittivity of the dielectric material used in the transmission lines of the interferometer arms and $(1/\varepsilon)(\partial\varepsilon/\partial T)$ the Temperature Coefficient Permittivity (TCP).

The TCP for Teflon at 77K was estimated to be 4×10^{-4} from its thermal expansion coefficient [7]. To estimate the effect of interferometer phase mismatch on oscillator instability, the Allan Deviation of fractional temperature fluctuations $(\sigma_T(\tau))$ of a liquid nitrogen bath (77K) was measured, yielding

$$\sigma_T(\tau) = \begin{cases} 1.25\times 10^{-5} & 1<\tau<16 \text{ s} \\ 3.6\times 10^{-5}\sqrt{\tau} & \tau>16 \text{ s,} \end{cases} \tag{4}$$

which is equivalent of a bath temperature instability $\delta T \sim 1$ mK ($\tau < 16$ s). Knowing $\sigma_T(\tau)$, the oscillator fractional frequency stability due to temperature induced fluctuations of phase mismatch can be estimated [8]

$$\sigma_y(\tau) = \frac{1-\beta^2}{4\beta}\,\frac{f_{res}}{Q_0}\,\frac{2\pi}{c}\,\Delta l \frac{1}{\varepsilon}\,\frac{\partial\varepsilon}{\partial T}\,T_0\sigma_T(\tau). \tag{5}$$

Substituting (4) into (5), with unloaded bandwidth $f_{res}/Q_0 = 10^3$ Hz, $\Delta l = 5$ cm, $T_0 = 77$K and $\beta = 0.8$, results in $\sigma_y(\tau)$ (curve 1 in Fig. 4). It is more than an order of magnitude greater than the limit imposed on the oscillator frequency stability by the FD electronics (curve 2). Here, it is assumed $P_{res} = 15$ dBm and detector #2 is used. As follows from (5), active temperature control of the interferometer at a level of $\delta T \sim 20$ μK at $T_0 = 77$K is needed to reach the limit set by the FD electronics noise (curve 2). This would result in an oscillator fractional frequency instability of a few parts in 10^{-15} for $\tau < 16$ s. At this level, the implementation of the interferometric technique would offer the advantage of near perfect carrier suppression.

IV. Conclusion

Providing the small signal operation of the detector, adding an amplifier to the FD of a Pound stabilized oscillator reduces the Allan Deviation due the electronics voltage noise by an amount equal to the power gain of the amplifier. A short-term frequency stability due to FD electronics noise of 2×10^{-16} is potentially achievable when the resonator is critically coupled. Interferometric carrier suppression techniques can be considered as a means of implementing a critically coupled resonator. However, temperature induced fluctuations of interferometer phase mismatch necessitate active temperature control of the interferometer at the level of 10–20 μK at 77K, which is not trivial. As part of our ongoing research, a temperature control system will be investigated, that is capable of achieving this level of temperature stability over the temperature range 52–77K, which is accessible with liquid or solid nitrogen.

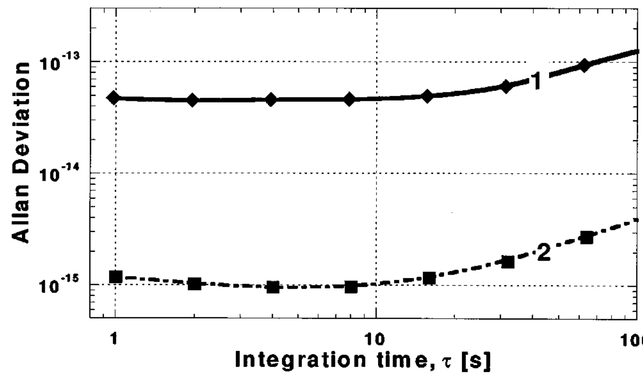

Fig. 4. Allan Deviation limit imposed by phase mismatch in cryogenic interferometer arms (curve 1) in a liquid nitrogen bath. Allan Deviation limit imposed by the electronics of a conventional Pound FD (curve 2) where $P_{res} = 15$ dBm.

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