Resonance Degeneration and Spurious Mode Suppression in a Cryogenic Whispering Gallery Mode Sapphire Resonator

O. Di Monaco, Y. Kersale', and V. Giordano

***Abstract*—A method to solve the main resonance degeneration and the spurious mode problem for a cryogenic Whispering Gallery Mode Sapphire Resonator (WGMSR) is reported. Two thin metal wires are deposited on top of the sapphire disk. With an appropriate choice of the relative radial direction and the orientation of these wires with respect to the coupling probe, a dominant perturbation for one of two excited resonance modes has been realized. Moreover, the spurious modes around the selected resonance mode are being suppressed in a frequency span of the order of 300 MHz, without modification of the main resonance performance.**

In this letter, the efficiency of this method is demonstrated experimentally for a cryogenic WGMSR realized with a very high purity sapphire monocrystal operating near 7 GHz.

***Index Terms*—Cryogenic sapphire resonator, microwave signal source, whispering gallery mode.**

I. INTRODUCTION

AN ultrastable microwave signal source is the fundamental instrument for the development of microwave systems in a lot of different application fields, such as time-frequency metrology, space, radar, and telecommunications.

In low phase noise applications, the microwave source is usually realized by means of a quartz crystal oscillator multiplied to the desired frequency [1]. Unfortunately, the performance of these conventional signal sources is limited by the phase fluctuation enhancement due to the multiplication.

A convenient solution to these problems is the use of a Whispering Gallery (*WG*) mode sapphire resonator cooled to liquid nitrogen temperature [2]. In fact, the excitation of a higher order resonance mode (WG mode) in a very high quality sapphire monocrystal cooled at 77K, allows to obtain, directly at the microwave frequencies, a resonance mode with a Q-factor value of the order of 30 10^6 [1], [2].

A Whispering Gallery Mode Sapphire Resonator (WGMSR) prototype operating near 7 GHz has been realized. A sapphire disk with a diameter of 50 mm and 20 mm in height is placed in the center of a copper cavity with a diameter of 90 mm and 40 mm high, in order to limit the radiation losses and the effect of the exterior perturbations. These dimensions are chosen to work with a high order $WG_{m,0,0}$ mode $(m > 5)$, corresponding to a Q-factor value limited only by the intrinsic losses of the dielectric sapphire. Moreover, to ensure ultrahigh resonator performance, a high quality sapphire monocrystal, with very low intrinsic losses ($tg\delta \approx 4\ 10^{-6}$ at 300K and $tg\delta \approx 3\ 10^{-8}$ at 77K), is used. The coupling to the resonator is accomplished with two electrical probes: the penetration length, and the relative angular position of these probes enable adjustment of the coupling strength.

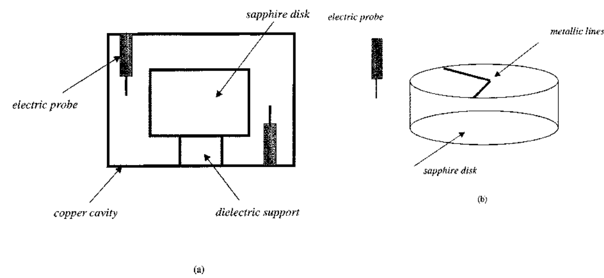

Fig. 1. (a) WGMSR prototype near 7 GHz and (b) design of the sapphire resonator acting on the $WGH_{9,0,0}$ mode.

TABLE I
TRANSMISSION SPECTRUM PERFORMANCE OF THE $WGH_{9,0,0}$ RESONANCE MODE, MEASURED BEFORE AND AFTER METALLISATION OF THE SAPPHIRE DISK, AT 77K

$WGH_{9,0,0}$ Resonance mode		*f_0 GHz*	*I.L. dB*	*Q_L*	*β_1*	*β_2*	*Q_0*
before metallisation	m=9- m=9+	7.318023 7.317984	~-12 ~-10	~14 10^6 ~14 10^6	~0.5 ~0.4	~0.5 ~0.4	28 10^6 25 10^6
after metallisation	m=9- m=9+	---------- 7.317887	------- ~-11	---------- 11 10^6	----- ~0.6	----- ~0.4	-------- 22 10^6

At liquid nitrogen temperature, the use of the WG modes as reference frequencies presents two inconveniences: the high density of spurious modes and the splitting of the operational mode in two resonances. The presence of these unwanted resonances can compromise a stable operation of a WGMSR in an oscillator configuration [3], [4].

In this letter, we propose an efficient method to solve these problems simultaneously. We demonstrate that this method does not alter the very high WG mode performance, thus allowing the realization of a very stable microwave signal source [5].

II. MAIN RESONANCE DEGENERATION AT CRYOGENIC TEMPERATURE

The Whispering Gallery modes $WG_{m,n,l}$ of a cylindrical dielectric resonator are the higher order resonances, characterized by three integers m, n, l, which represent the electromagnetic

Manuscript received March 2, 2000; revised June 9, 2000.
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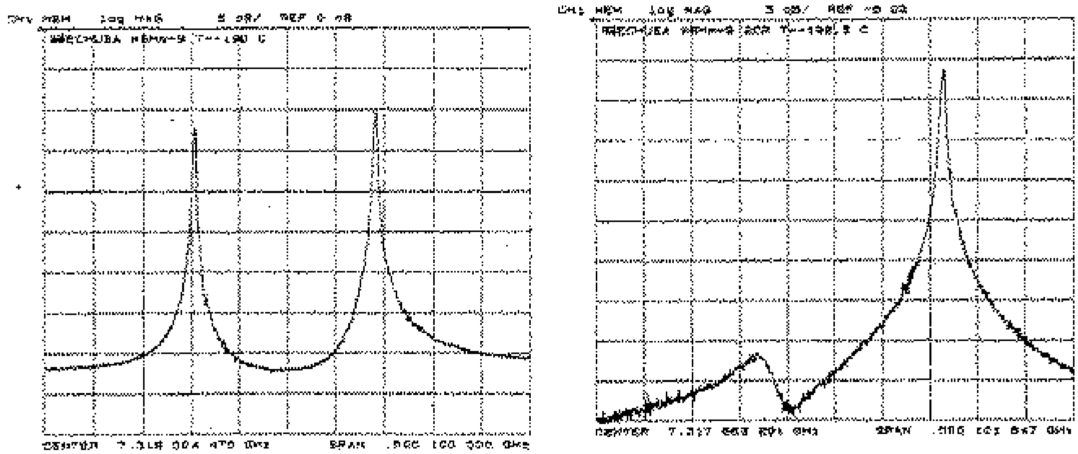
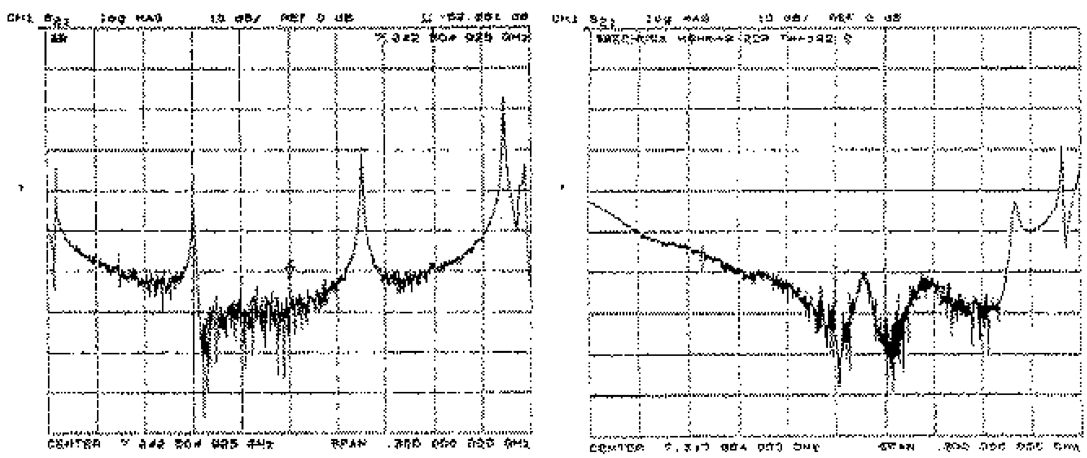
Publisher Item Identifier S 1051-8207(00)08453-1.



(a) (b)

Fig. 2. Transmission spectrum around the $WGH_{9,0,0}$ mode (a) without and (b) with metallization, in a frequency span of 100 kHz.



(a) (b)

Fig. 3. Transmission spectrum around the $WGH_{9,0,0}$ mode (a) without and (b) with metallization, in a frequency span of 300 MHz.

field component variations along the azimutal, radial and axial directions, respectively.

In this work we are interested in the $WGH_{m,0,0}$ modes, which are the quasi-TM modes corresponding to a very weak variation of the electromagnetic field components in the radial and axial directions $(n \approx 0, l \approx 0)$.

From the Maxwell equations, the expression of the longitudinal field component E_z for a $WGH_{m,0,0}$ mode excited in a sapphire monocrystal disk, is given by [5], [6]

$$E_z = BJ_m(k_E r)\begin{pmatrix}\cos m\varphi \\ \sin m\varphi\end{pmatrix}\cos\beta z \tag{1}$$

where $k_E = \sqrt{(k_0^2\varepsilon_Z - (\varepsilon_Z/\varepsilon_t)\beta^2}$ is the radial propagation constant parallel to the C-axis of the dielectric disk and ε_Z and ε_t are the relative permittivity components parallel and perpendicular to the C-axis of the sapphire resonator, respectively [7]. β is the longitudinal dielectric propagation constant, m is the azimutal number, and J_m is the Bessel function of the first kind.

As shown in (1), two ortogonal polarizations corresponding to the same azimutal number can exist for the same resonance frequency. Theoretically, for an ideal dielectric resonator, only one of the two polarizations is excited. Practically, the geometrical imperfections and the orientation defects of the sapphire disk determine the resonance conditions of the two modes for two different angular positions, $\phi - \alpha$ and $\phi + \alpha$, where α is the angle characterizing the perturbation. Consequently, because of the finite spatial dimension of the coupling probe, two modes $\mathrm{WG}_{m\pm,0,0}$ are excited simultaneously. This phenomenon, hidden by the line widths of the resonance mode at room temperature, is very well visible under cryogenic conditions, where the resonance bandwidths become very narrow (high Q-factor).

III. The Spurious Modes Problem at Cryogenic Temperature

Another problem to solve, when working with the WG modes, is the presence of a consistent number of parasite modes surrounding the selected resonance.

These modes correspond to the empty cavity modes perturbed by the sapphire disk and to other high order hybrid modes with n and/or $l \neq 0$.

At room temperature, we have solved this problem with a modal selection method proposed in [8]. This method is based on the deposition of $n = 2m$ thin metal wires on the top of the sapphire disk. The radial direction of these metal wires corresponds to the meridian planes which act as electrical short circuits for a $WGH_{m,0,0}$ resonance mode. Consequently, with an appropriate coupling between the sapphire resonator and the external probe, these lines don't alter the electrical field configurations for the chosen resonance, but strongly modify that of the modes with a different azimutal variation of the electric field components.

At liquid nitrogen temperature, this method is not efficient any more. Experimental data demonstrate that, for a $WGH_{m,0,0}$ resonance mode, with $m > 5$, the deposition of $n = 2m$ metal wires degrades the Q-factor of the liquid nitrogen cooled resonator by approximately six times. Particularly, for the $WGH_{9,0,0}$ resonance mode, the unloaded Q-factor value obtained before metallization is equal to 25 10^6, whereas, after deposition of $n = 18$ metal wires, Q_0 is approximately 4 10^6.

These experimental results demonstrate that, at liquid nitrogen temperature, the presence of a very large number of metal wires becomes a critical source of losses for the operational WG mode performance.

Other modal suppression techniques have been proposed previously [9], [10]. Particularly, in [10], a slotted metallic cavity containing the sapphire was designed to suppress parasitic modes coupled well to the cavity. This method was implemented at 77K to realize two WGMSR, one used as the frequency-determining element in a loop oscillator and the second used as a frequency discriminator to measure the oscillator phase noise [4]. This technique allowed to obtain a high Q factor of 60 million, but didn't suppress the WG mode degeneracy, which degraded the phase noise performance of the sapphire oscillator. In the next section we describe a modal selection technique which can be advantageous in applications where the mode degeneracy is a problem.

IV. Partial Modal Selector for the WGMSR Functioning at 77K

A new design for the WGMSR functioning on a $\mathrm{WGH}_{m,0,0}$ mode the liquid nitrogen temperature is proposed. From the considerations of the previous paragraph and using the approximation relation

$$P_n = \frac{n}{2m} P_{2m} \qquad \text{for } n = 1, 2, \cdots, 2m \tag{2}$$

the maximal number of metal wires n, corresponding to the condition $P_n < P_d$, where P_n is the power dissipated in n metal wires and P_d is the power dissipated in the sapphire disk, can be determined. The radial direction of these lines must correspond with n electrical short circuits planes for the selected resonance.

Fig. 1 shows the total device and the design realized for the sapphire resonator acting on the $WGH_{9,0,0}$ mode: two chromium metal wires, 20 μm wide and 0.15 μm in thickness are located on the sapphire disk with a relative orientation of 200°. With this number of metal wires, the condition $P_n < P_d$ is satisfied ($P_n \approx 0.5 P_d$). The radial direction of these lines has been determined experimentally: before sputtering the metallization on the dielectric sapphire, a set of printed test structures was realized for every m possible combinations of the two metal wires. The chosen configuration corresponded to the best rejection of the parasitic modes.

V. Experimental Results

Table I summarizes the results obtained with the resonator before and after the metallization of the sapphire disk, at 77K. In the first case, the $WGH_{9,0,0}$ mode degenerates in two resonances which have a similar microwave performance. On the contrary, with the chromium lines added to the sapphire as shown in Fig. 1, a dominant perturbation has been created for the $WGH_{9-,0,0}$ mode, a sign that it is practically not excited. Moreover, the resonance performance of the $WGH_{9+,0,0}$ mode is not very strongly altered. The tolerance in the angular position of metal wires within which no noticeable change in the Q-factor of the $WGH_{9+,0,0}$ mode is observed is $\pm 1°$.

Fig. 2 shows the transmission spectrum obtained for the two WGMSR configurations, in a frequency span of 100 kHz, where the degeneration of the $WGH_{9,0,0}$ mode can be seen.

Finally, the same transmission spectrum has been measured, in a larger frequency span, in order to verify the effect of this method on the spurious modes surrounding our resonance mode. Fig. 3 compares this result with that obtained before metallization. The potential of our method is obvious: the appropriate coupling between the external probe and the two lines has allowed the suppression of the spurious modes in a very large frequency span, of the order of 300 MHz.

VI. Conclusions

A resonance mode with an unloaded Q-factor value better than 20 10^6 and isolated in a frequency span of 300 MHz has been obtained at 77K for a WGMSR operating at 7.3 GHz. The results presented in this letter demonstrate that a proper design of the resonator allows to use the resonance of a WG mode instead of a classical dielectric resonator operating on a fundamental mode in order to realize an ultrastable microwave signal source.

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