Discrete Complex Image Method for Green's Functions of General Multilayer Media

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***Abstract*—The discrete complex image method is extended to efficiently and accurately evaluate the Green's functions of multilayer media for the method of moments analysis. The difficulty associated with the surface-wave extraction for multilayer media is solved by evaluating a contour integral recursively in the complex k_ρ-plane. With this scheme, the discrete complex image method works in both the near- and far-field regions for general multilayer media, whether open or shielded and lossy or lossless.**

***Index Terms*—Discrete complex image method, Green's function, method of moments, multilayer medium, Sommerfeld integral.**

I. INTRODUCTION

THE METHOD of moments (MoM) solution of the integral equation has received intense attention to tackle the multilayer medium problems. A critical factor for the efficient and accurate MoM analysis is the evaluation of multilayer medium's Green's functions, which are expressed in terms of the Sommerfeld integrals (SIs). Generally, the analytical solution of the SI is not available, and the numerical integration is time consuming since the integrand is both highly oscillating and slowly decaying. Several efficient techniques have been proposed to speed up this numerical integration [1]. The recent efforts to further accelerate the computation include the fast Hankel transform (FHT) approach [2], the steepest-descent path (SDP) approach [3], and the discrete complex image method (DCIM) [4]–[8].

The basic idea of the DCIM is to approximate the spectral kernel by a sum of complex exponentials using the Prony method or the generalized pencil-of-function (GPOF) method. The SIs can then be evaluated in closed forms via the Sommerfeld identity. This method has been extensively employed to analyze microstrip structures. However, most of the work is confined to single-layered or double-layered structures since the surface-wave contribution has to be extracted analytically using residue calculus, which makes the DCIM difficult to be extended to multilayer media. As a result, to use the DCIM for multilayer media, the surface waves are often not extracted. This approximation is invalid for multilayer configurations with strong surface-wave modes. Also, for shielded multilayer configurations, there are two kinds of modes: the surface-wave modes guided by the layers and the modes guided by the top and bottom ground planes. The lack of extraction of these modes results in errors in the far-field region since the guided modes behave in the manner of cylindrical waves and it is physically inappropriate to approximate such waves by spherical waves. For a general-purpose algorithm, especially for the MoM analysis of electrically large structures, the guided modes have to be extracted. In this letter, the extraction of guided modes (which refer to surface-wave modes for open multilayer media, and both surface-wave modes and ground-plane guided modes for shielded multilayer media) is carried out numerically by evaluating a contour integral in the complex k_ρ-plane [10]. After the guided-mode extraction, the GPOF is applied to approximate the remaining kernel. All the Green's functions for vector and scalar potentials can be obtained. Numerical results demonstrate the efficiency and accuracy of the DCIM.

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II. FORMULATION

Consider a current source in a multilayer medium. Each layer is characterized by relative permittivity ϵ_r, relative permeability μ_r, and thickness h. The electric field due to the current can be expressed in a mixed potential form as

$$\mathbf{E} = -j\omega\mu_0\langle \bar{\mathbf{G}}^A, \mathbf{J}\rangle + \frac{1}{j\omega\epsilon_0}\nabla\langle G^\Phi, \nabla'\cdot\mathbf{J}\rangle \tag{1}$$

where $\mathbf{J}$ denotes the electric current density of the source and $\bar{\mathbf{G}}^A$ and G^Φ are the Green's functions for the vector and scalar potentials, respectively. The detailed discussion in [9] shows that it is preferable to choose $\bar{\mathbf{G}}^A$ as

$$\bar{\mathbf{G}}^A = \begin{bmatrix} G^A_{xx} & 0 & G^A_{xz} \\ 0 & G^A_{xx} & G^A_{yz} \\ G^A_{zx} & G^A_{zy} & G^A_{zz} \end{bmatrix} \tag{2}$$

and G^Φ as the scalar potential for a horizontal electric dipole (HED).

In general, the Green's function for a multilayer medium is expressed in terms of a SI, which can be written as

$$G(\rho, z|z') = \frac{1}{2\pi}\int_0^\infty \tilde{G}(k_\rho, z|z')J_0(k_\rho\rho)k_\rho\, dk_\rho \tag{3}$$

where $\tilde{G}$ is the spectral domain counterpart of G.

To evaluate the SI, we begin with the derivation of the spectral domain Green's functions for an arbitrary dipole in multilayer media, which can be accomplished by constructing equivalent transmission lines. Therefore, the original problem to find electric and magnetic fields is converted to the problem of obtaining the voltage and current on the corresponding transmission lines. From the electric and magnetic fields, the Green's functions for the vector potential $\tilde{\bar{\mathbf{G}}}^A$ and the scalar potential $\tilde{G}^{\Phi}$ can be derived. Once the spectral domain Green's functions are obtained, the DCIM is applied to rapidly evaluate the SIs.

Let us rewrite the spectral domain Green's function in a simple form as

$$\tilde{G} = A\frac{F}{2jk_{zm}} \tag{4}$$

where A is a constant. As the first step of the DCIM, the primary field term F_{pr} is extracted from F when the source and observation points are in the same layer. Note that there is no primary field term in $\tilde{G}^A_{zt}$. The static contributions F_{st}, which dominate as $k_\rho \to \infty$, are also extracted, which makes the remaining kernel decay to zero for sufficiently large k_ρ. It happens only when both source and field points are on the interface of two different layers.

The next step is to extract the guided-mode contributions F_{gm}, which can be written as

$$F_{gm} = 2jk_{zm}\sum_i \frac{2k_{\rho i}\mathrm{Re}\, s_i}{k_\rho^2 - k_{\rho_i}^2} \tag{5}$$

where $\mathrm{Re}\, s_i$ is the residue for the pole $k_{\rho i}$. When transformed to the spatial domain, the guided-mode contributions to the Green's function become

$$G_{gm} = \frac{A}{4\pi}\left(-2\pi j\sum_i \mathrm{Re}\, s_i H_0^{(2)}(k_{\rho i}\rho)k_{\rho i}\right) \tag{6}$$

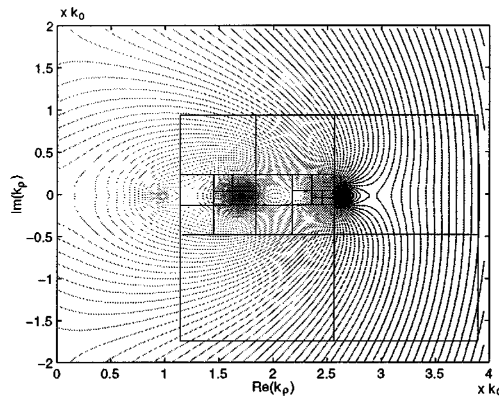
where $H_0^{(2)}$ denotes the zeroth-order Hankel of the second kind.

In the previous work on the DCIM, $\mathrm{Re}\, s_i$ is calculated analytically by using residue calculus, which makes it difficult to be extended to multilayer cases. Here, we obtain $\mathrm{Re}\, s_i$ by evaluating a contour integral numerically in the complex k_ρ-plane recursively. The integration begins with a rectangular contour enclosing the region of interest. If the calculated value is nonzero, then we subdivide the contour into four contours and evaluate the contour integral along each of them. This process is repeated until the location $k_{\rho i}$ and residue $\mathrm{Re}\, s_i$ for all the poles are found. The process can be illustrated by the following example, which is a four-layer medium. The magnitude of $\tilde{G}^{\Phi}$ in the first and fourth quadrants is plotted in Fig. 1. The contour integral is repeated until we find the surface-wave poles at $k_\rho = 1.736\,k_0$ and $2.435\,k_0$, respectively. Since the top layer is the unbounded free space, there is a branch cut associated with k_{z0}, where the radiation modes lie on [11]. This branch cut is known and can be deliberately avoided. For shielded multilayer media, both of the modes guided by the layers and the ground planes are extracted. The poles on the imaginary axis correspond to the evanescent modes, which are not extracted in this method.

After these extractions, the remainder of F can be approximated as a sum of complex exponentials using the GPOF method. With the aid of the Sommerfeld identity, we can obtain the closed-form Green's functions.

	ε_r	h
5	1.0	∞
4	2.1	0.7
3	12.5	0.3
2	9.8	0.5
1	8.6	0.3

(a)



(b)

Fig. 1. Magnitude of $\tilde{G}^{\Phi}$ in the first and fourth quadrants of the complex k_ρ-plane for a four-layer medium. The frequency is 30 GHz. Unit for thickness is mm.

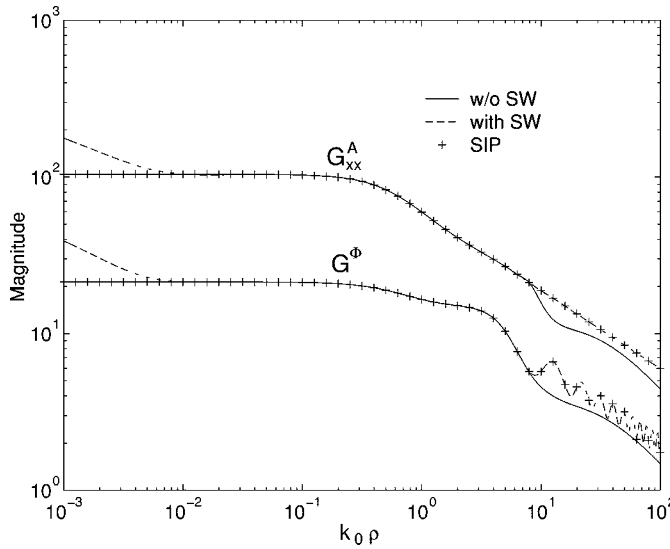


Fig. 2. Magnitude of Green's functions G^A_{xx} and G^{Φ}, for a four-layer medium. $z' = 0.4$, $z = 1.4$.

With the guided-mode extraction, the DCIM works well in the far-field region. However, a problem is encountered for the near-field evaluation. As we know, when $z \neq z'$, the Green's function is not singular at $\rho = 0$; however, the guided-mode term carries the singularity. This phenomenon is shown in Fig. 2, where a four-layer medium problem is investigated for $z \neq z'$. In the figure, the dashed and solid lines represent those obtained by the DCIM with and without the guided-mode extraction, respectively. The crosses represent those obtained by direct numerical integration along the Sommerfeld integration path (SIP).

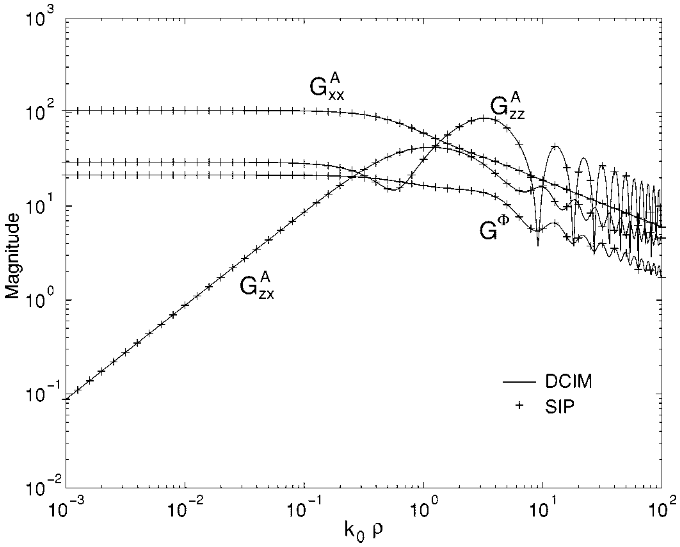


Fig. 3. Magnitude of Green's functions G^A_{xx}, G^A_{zx}, G^A_{zz}, and G^Φ, for a four-layer medium. $z' = 0.4$, $z = 1.4$.

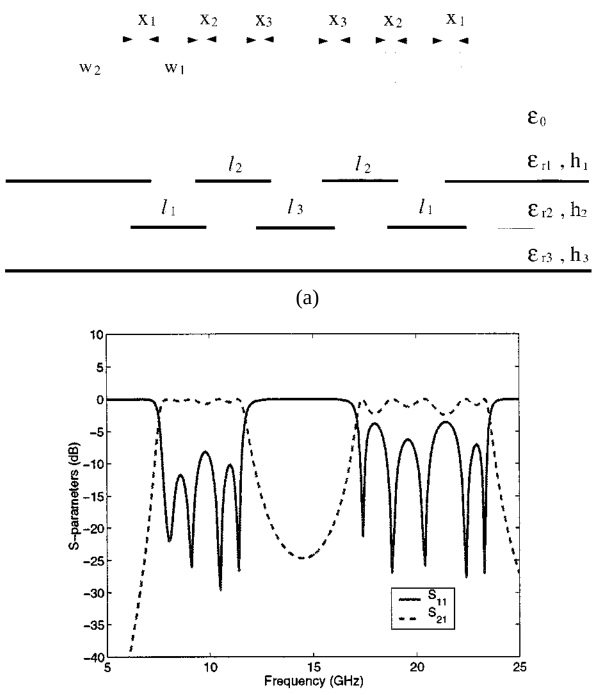


Fig. 4. S-parameters of an overlap-gap-coupled microstrip filter. $\epsilon_{r1} = 1.8$, $\epsilon_{r2} = 9.8$, $\epsilon_{r3} = 2.2$, $h_1 = h_2 = h_3 = 0.254$ mm, $w_1 = 0.812$ mm, $w_2 = 0.458$ mm, $l_1 = 6.990$ mm, $l_2 = 6.457$ mm, $l_3 = 7.242$ mm, $x_1 = 1.311$ mm, $x_2 = 0.386$ mm, $x_3 = 0.269$ mm.

To overcome this difficulty, a transition point is introduced, as is done in [4], which divides the near- and far-field regions. Therefore, the DCIM is applied twice: once with the guided-mode extraction and the other without the guided-mode extraction. The first yields the Green's function for the near-field region, and the second gives the Green's function for the far-field region. From Fig. 2, we can see that the two approaches overlap in the middle region and the transition point can be easily picked up, say, at $\log_{10} k_0\rho = 0$. With this modification, the DCIM works for all the regions. The magnitudes of four Green's functions G^A_{xx}, G^A_{zx}, G^A_{zz}, and G_Φ are shown in Fig. 3. Compared to the result of SIP, the DCIM with the guided-mode extraction has an average relative error of 0.13% in the far-field region, whereas the number for the DCIM without the guided-mode extraction is over 104%.

Using the DCIM described above, we have successfully developed an MoM to simulate microstrip circuits and antennas in multilayer media. Fig. 4 shows the S-parameter of an overlap-gap-coupled microstrip filter embedded in a three-layer medium.

III. Conclusion

The DCIM with guided-mode extraction is presented to efficiently and accurately evaluate the Green's functions of general multilayer media. The difficulty to extract surface-wave contributions for multilayer media in the previous DCIM is overcome by numerically evaluating a contour integral recursively in the complex k_ρ-plane. With this scheme, the DCIM works in both the near- and far-field regions for general multilayer media, whether open or shielded and lossy or lossless.

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