A Synthesis-Oriented Conditional Stability Criterion for Microwave Multidevice Circuits with Complex Termination Impedances

Francesco Centurelli, *Member, IEEE*, Giuseppe Scotti, Pasquale Tommasino, and Alessandro Trifiletti

***Abstract*—In this work, a new conditional stability criterion for multidevice circuits is proposed, in order to guarantee stability in spite of input and output termination variations in regions surrounding complex nominal values. A check of this criterion can be implemented in a commercial CAD environment, and this allows imposing stability from within an optimization routine. A design methodology to synthesize multidevice circuits with complex termination impedances, which are stable under both process parameters and terminations variations, is proposed.**

***Index Terms*—CAD, MMIC, stability, yield.**

I. INTRODUCTION

STABILITY is a key requirement in the design of every circuit which has to be used as an amplifier, since its lack makes the designed circuit unusable. Microwave and millimeter-wave multidevice circuits are typically designed for a given value of input and output terminations, using nominal device parameters. However, if a satisfactory yield is desired, both circuit performance and stability have to be guaranteed not only for the nominal design, but also under termination impedances and device parameters variations, caused by the nonideality of the fabrication process or coupling between MMIC package and interconnections.

A multidevice amplifier can be designed according to different stability requirements with respect to terminations: stability for the nominal values, unconditional stability (i.e., stability for all the passive input and output loads), and stability for all the input and output termination values which are placed in prefixed regions of the Smith chart around the nominal ones (conditional stability). This third approach leaves more degrees of freedom in the design of the amplifier to satisfy performance requirements, allowing a trade-off between performance and stability. The conditional stability so represents the best choice if the maximum spread of input and output terminations can be estimated *a priori*.

Rigorous methods to check the stability of multidevice circuits for a given couple of terminations have been presented in [1]–[3]. The stability of the circuit for different values of input and output loads can be studied by means of the stability circles [4] or the stability envelope technique [5]. These methods require to visually inspect the polar plots of properly chosen functions, therefore allow performing the stability check only at the end of the design phase, often resulting in a tedious trial and error design process.

In [6] a methodology to design high yield multidevice circuits has been presented which allows including stability requirements among the goals of a circuit synthesis performed by CAD tools optimizer. However this procedure can be applied only if the nominal input and output loads are assumed to be positive resistances, and are used to normalize S parameters.

Anyway in many cases, such as in the design of optical receiver front-ends or distributed amplifiers, the input and/or output terminations can be complex impedances.

In this work we propose a new stability criterion which is able to guarantee stability of multidevice circuits for all the values of input and output terminations placed in circular regions of the Smith chart centered in the complex nominal termination values and with a given radius. Based on this new stability criterion, a design methodology to synthesize multidevice circuits, which are stable under both process parameters and terminations variations, is proposed starting from the methodology presented in [6].

II. STABILITY CRITERION

Consider a linear active two-port terminated by impedances Z_1 and Z_2. Let (a_1, b_1) and (a_2, b_2) be the incident and reflected waves at port 1 and 2 respectively, defined by

$$a_i = \frac{V_i + I_i Z_{oi}}{2\sqrt{Z_{oi}}} \qquad (i = 1, 2), \tag{1}$$

$$b_i = \frac{V_i - I_i Z_{oi}}{2\sqrt{Z_{oi}}} \qquad (i = 1, 2) \tag{2}$$

where Z_{oi} is the reference impedance at port i. Z_{o1} and Z_{o2} are assumed here to be positive real resistances. Using the steady-state S-parameters (S_{ij}'s) that relate a_i's and b_i's, the signal flow graph for a twoport is depicted in Fig. 1, where

$$\Gamma_i = \frac{Z_i - Z_{oi}}{Z_i + Z_{oi}} \qquad (i = 1, 2). \tag{3}$$

Typically, due to process parameter variations or to spurious couplings, the value of impedances Z_i's is not well defined and varies in a region surrounding the nominal one.

Denoting with $\overline{Z}_1$ and $\overline{Z}_2$ the nominal values of input and output terminations we obtain

$$\overline{\Gamma}_i = \frac{\overline{Z}_i - Z_{oi}}{\overline{Z}_i + Z_{oi}} \qquad (i = 1, 2). \tag{4}$$

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The authors are with the Dipartimento di Ingegneria Elettronica, Università di Roma “La Sapienza”, 00184 Roma, Italy (e-mail: trifiletti@die.ing.uniroma1.it).

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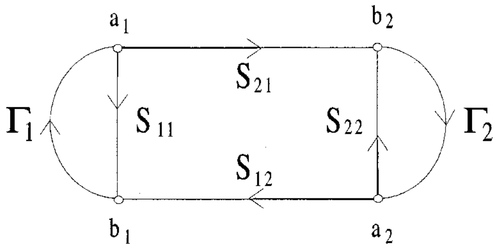

Fig. 1. Signal flow graph of a twoport.

As a first step, the stability of the circuit for $\Gamma_i = \overline{\Gamma}_i$ ($i = 1, 2$) has to be ascertained: this can be done by means of a method based on Nyquist criterion [1]–[3] or by means of conditions reported in [6] which allow imposing prefixed stability margins on a multidevice circuit from within the optimization routine performed by a CAD tools optimizer. Stability of the circuit for $\Gamma_i = \overline{\Gamma}_i$ ($i = 1, 2$) ensures that the scattering parameters have no RHP poles.

We then consider all the termination values placed in circular regions of the Smith chart with center in $\overline{\Gamma}_i$ and a proper radius r_i (Fig. 2) with the aim of taking into account the spread of input and output loads. In order to consider only passive loads, the radii r_i's have to be chosen according to the inequality

$$\left|\overline{\Gamma}_i\right| + r_i < 1. \tag{5}$$

Theorem 1: Provided that the S-parameters defined for at least one pair of positive constant reference impedances have no RHP poles, a linear active twoport is stable for any $|\Gamma_i - \overline{\Gamma}_i| < r_i$ ($i = 1, 2$) if the following conditions are fulfilled:

$$\mu'_{S_C}(S_{ij}, \overline{\Gamma}_1, r_1, \overline{\Gamma}_2, r_2) > 1 \tag{6}$$

$$|S_{22}| < \frac{1}{\left|\overline{\Gamma}_2\right| + r_2} \tag{7}$$

where (see (8) and (9), shown at the bottom of the page).

Proof: When the scattering parameters, Γ_1 and Γ_2 have no RHP poles, the necessary and sufficient condition for the system in Fig. 1 to be stable is that the graph determinant of the system

$$\Delta = 1 - S_{11}\Gamma_1 - S_{22}\Gamma_2 + D\Gamma_1\Gamma_2 = \Delta_1\Delta_2 \tag{10}$$

has no RHP zeros [7], where

$$\Delta_1 = 1 - \Gamma_2 S_{22}, \tag{11}$$

$$\Delta_2 = 1 - \Gamma_1\left(S_{11} + \frac{S_{12}S_{21}\Gamma_2}{1 - \Gamma_2 S_{22}}\right) = 1 - \Gamma_1\Gamma_{\text{in}}. \tag{12}$$

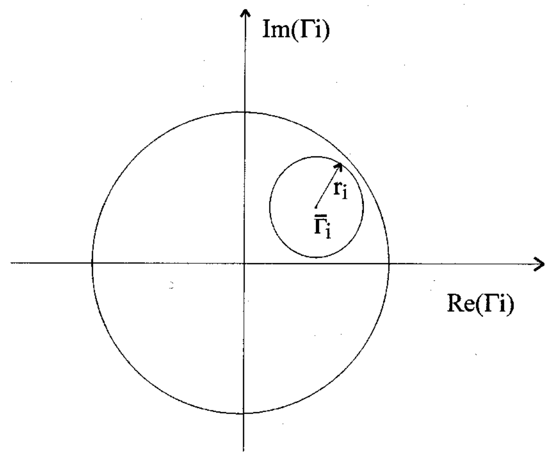

Fig. 2. Variability regions for input and output terminations.

By applying Nyquist criterion as in [8], it can be shown that the fulfillment of the following conditions:

$$|\Gamma_2(\omega)S_{22}(\omega)| < 1 \tag{13}$$

$$|\Gamma_1(\omega)\Gamma_{\text{in}}(\omega, \Gamma_2)| < 1 \tag{14}$$

ensures that Δ has no RHP zeros.

The transformation

$$\Gamma_{\text{in}} = \frac{1}{S_{22}}\left(D + \frac{S_{12}S_{21}}{1 - S_{22}\Gamma_2}\right) \tag{15}$$

maps the region $|\Gamma_2 - \overline{\Gamma}_2| < r_2$ into the region

$$|\Gamma_{\text{in}} - C_3| \gtrless r_3 \tag{16}$$

where

$$C_3 = \frac{S_{11}w_o^* - D\left[\overline{\Gamma}_2 + S_{22}^*\left(r_2^2 - \left|\overline{\Gamma}_2\right|^2\right)\right]}{|w_o|^2 - |S_{22}|^2 r_2^2} \tag{17}$$

$$r_3 = \frac{|S_{12}S_{21}|r_2}{\left||w_o|^2 - |S_{22}|^2 r_2^2\right|}; \tag{18}$$

$$w_o = 1 - S_{22}\overline{\Gamma}_2 \tag{19}$$

depending on $|w_o|^2 - |S_{22}|^2 r_2^2 \lessgtr 0$.

Through the bilinear transformation (15), condition (14), which has to hold for any $|\Gamma_i - \overline{\Gamma}_i| < r_i$ ($i = 1, 2$), gives the following condition on the plane Γ_{in}:

$$|C_3| + r_3 < \frac{1}{\left|\overline{\Gamma}_1\right| + r_1}. \tag{20}$$

Substituting (17) and (18) into (20) and rearranging, we get condition (6).

$$\mu'_{S_C} = \frac{1 - |S_{22}|^2\left(r_2^2 - \left|\overline{\Gamma}_2\right|^2\right) - 2\text{Re}\left(\overline{\Gamma}_2 S_{22}\right)}{\left[\left|S_{11} - D\overline{\Gamma}_2 - S_{22}^*\left[S_{11}\overline{\Gamma}_2^* + D\left(r_2^2 - \left|\overline{\Gamma}_2\right|^2\right)\right]\right| + |S_{12}S_{21}|r_2\right]\left[\left|\overline{\Gamma}_1\right| + r_1\right]} \tag{8}$$

$$D = S_{11}S_{22} - S_{12}S_{21}. \tag{9}$$

The maximum value for $|\Gamma_2|$ when Γ_2 varies within the prefixed region $|\Gamma_2 - \overline{\Gamma}_2| < r_2$ is obviously given by $|\overline{\Gamma}_2| + r_2$, so (13) implies (7), thus concluding the Proof of Theorem 1.

It is interesting to note that if we express Δ in (10) as

$$\Delta = 1 - S_{11}\Gamma_1 - S_{22}\Gamma_2 + D\Gamma_1\Gamma_2 = \Delta_1'\Delta_2' \qquad (10')$$

where

$$\Delta_1' = 1 - \Gamma_1 S_{11}, \qquad (11')$$
$$\Delta_2' = 1 - \Gamma_2\left(S_{22} + \frac{S_{12}S_{21}\Gamma_1}{1 - \Gamma_1 S_{11}}\right) = 1 - \Gamma_2\Gamma_{\rm out} \qquad (12')$$

we obtain conditions dual to (6) and (7), in the sense that the we have to exchange subscripts 1 and 2.

A. Case of Real Termination Impedances.

If the impedances Z_i's are positive resistances we can assume $\overline{Z}_1$ and $\overline{Z}_2$ as reference impedances Z_{o1} and Z_{o2} in order to have $\overline{\Gamma}_1 = 0$ and $\overline{\Gamma}_2 = 0$. In this particular case conditions (6) and (7) can be rewritten as

$$\mu_S'(S_{ij}, r_1, r_2) > 1, \qquad (21)$$
$$|S_{22}| < \frac{1}{r_2} \qquad (22)$$

where

$$\mu_S' = \frac{1 - |S_{22}|^2 r_2^2}{|S_{11} - S_{22}^* D r_2^2| r_1 + |S_{12}S_{21}| r_2 r_1}. \qquad (23)$$

It can be easily shown that condition (21) implies (22), and a single parameter can be used to ascertain stability when the terminations vary in the prefixed regions specified by r_1, r_2.

If both r_i's are chosen equal to 1, μ_S' is equal to the unconditional stability factor μ' derived in [9], and theorem 1 becomes [8, Th. 2A] for unconditional stability.

III. Design Methodology

A design methodology to design high yield microwave multidevice circuits has to take into account stability requirements under both process parameters and terminations variations. To avoid a time-consuming trial and error procedure, stability conditions are needed that can be automatically checked inside the CAD optimization routine. The conditional stability criterion presented in Section II, together with conditions presented in [6], that assure stability with a prefixed gain or phase margin, can be used.

The resulting design methodology manages in a similar way circuit performance and stability: a CAD tools optimizer finds a set of optimal design factors (nominal values) starting from performance specifications (type A specs) and stability requirements, such as stability margins (m_g, m_ϕ) and the radii of the circles for terminations variations (type B specs). Then a Monte Carlo analysis is performed on the optimized circuit in order to estimate both type A and type B (i.e., m_g, m_ϕ, μ_{S_C}') circuit responses variability under parameters and terminations variations. If the circuit results always stable, and with a satisfactory yield for type A responses, design procedure ends, otherwise it is necessary to re-perform the optimization step, modifying type A and/or type B specs.

IV. Conclusions

A new criterion to guarantee the stability of a multidevice circuit when input and output terminations vary in regions surrounding complex nominal values has been proposed. A check of this criterion can be implemented in a commercial CAD environment, and this allows imposing stability from within an optimization routine.

A new design methodology based on this criterion has been proposed to design multidevice circuits with complex termination impedances which are stable under both process parameters and terminations variations. The procedure takes into account the effects of variations on stability during the optimization step, resulting in a simpler yield-oriented design flow.

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