Eigenvalue Equations and Numerical Analysis of a Coaxial Cavity with Misaligned Inner Rod

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***Abstract*—Based on the Helmholtz equation, the superposition of cylindrical wave functions, and coordinates transformation, the eigenvalue equation is derived rigorously for a coaxial gyrotron cavity with a misaligned inner rod. It is shown that, due to the existence of the structural misalignment, any single normal mode of a perfect coaxial structure (i.e., without misalignment) no longer simultaneously satisfies both the outer and inner boundary conditions; consequently, the superposition of cylindrical wave functions must be taken into account. A numerical approach of solving the eigenvalue equation is proposed in this paper. As a practical application, analysis is given to the higher mode coaxial cavity employed in a 140-GHz/1.5-MW gyrotron device at the Forschungszentrum Karlsruhe, Karlsruhe, Germany. Result shows that the eigenvalue of the operating mode in a misaligned coaxial cavity is affected noticeably by the structural misalignment.**

***Index Terms*—Eccentric coaxial gyrotron cavity, eigenvalue equations, numerical approach.**

I. Introduction

IN THE PAST years, gyrotrons have become the major subject of high-power high-efficiency radiation sources in the millimeter- and submillimeter-wave ranges. One of their great promises is the application of electron cyclotron resonance heating on tokamaks and stellarators. For the next generation of fusion machines such as the stellarators' large helical device (LHD) in Japan and W7-X in Germany and the Tokamak International Thermonuclear Experimental Reactor (ITER), millimeter-wave generators operating at long pulse up to CW with RF output power in excess of 1 MW per unit at frequencies from 140 to 170 GHz are required (see [1] and cited papers).

As the device operates in long pulse/CW with megawatt level, the ohmic heating of the gyrotron cavity walls will get serious. To solve this problem, large-volume cavities must be employed to keep the ohmic losses at the acceptable value of about 3–4 kW/cm^2 for the current available cooling techniques. However, large volume of a cavity makes the spectrum of eigenvalues become dense, resulting in multimode operation. The competition of the parasitic modes with the desired operating mode may cause unstable operation and deteriorate the performance of the device [2], [3].

In order to get good mode selection of the large-volume cavity in multimode operation, one of the most effective means is to employ coaxial cavities. Insertion of an inner rod into a cylindrical cavity may rarefy the spectrum of the competing modes in the vicinity of the operating mode, and solve the problem of voltage depression if the electron beam is placed close to the inner rod [3]–[5]. Therefore a coaxial structure was widely considered in gyro-devices [3]–[9]. Currently, a coaxial cavity has been employed in a gyrotron oscillator at the Forschungszentrum Research Center Karlsruhe (FZK, formerly the KfK), which is expected to operate in the higher mode $TE_{28,16,1}$ ($TE_{31,17,1}$) with an output power of 1.5 MW at a frequency of 140 GHz (165 GHz) for the next step of fusion experiments such as W7-X and ITER [10]–[12].

From the practical point-of-view, the structure of a coaxial cavity may have a misalignment of the inner rod to the cylindrical cavity wall. Surely the misalignment will change both the distribution of RF fields and the eigenvalues of the cavity. This problem was studied early in several papers by means of the methods such as point-matching, finite differences, and truncation of series [13]–[16]. Generally, these papers considered only the lowest few frequencies, as pointed out by Kuttler [17]. Then, using conformal transformation, Kuttler [17] and Das and Vargheese [18] transformed the Helmholtz equation in annular cross section with eccentric inner rod into the weighted Helmholtz equation in a rectangular cross section [17] or in a perfectly coaxial cross section [18]. More recently, Dumbrajs and Pavelyev [19], [20] studied this problem by expanding the sth mode in azimuthal harmonics in terms of the Graf's formula of Bessel functions.

Making use of the coordinates transformation and the superposition of cylindrical wave functions, where these wave functions are defined as the separated solutions of Maxwell's equations in cylindrical coordinates with the propagation constant and cutoff wavenumber determined later from the boundary conditions, in this paper, we present a comparatively comprehensive consideration of the eigenvalue equation of a coaxial cavity with a misaligned inner rod. This paper is organized as follows. In Section II, we rigorously deal with the eigenvalue problem. In Section III, a numerical approach is proposed to solve the eigenvalue equations. In Section IV, analysis is carried out to the FZK coaxial cavity. Finally, conclusions are drawn in Section V.

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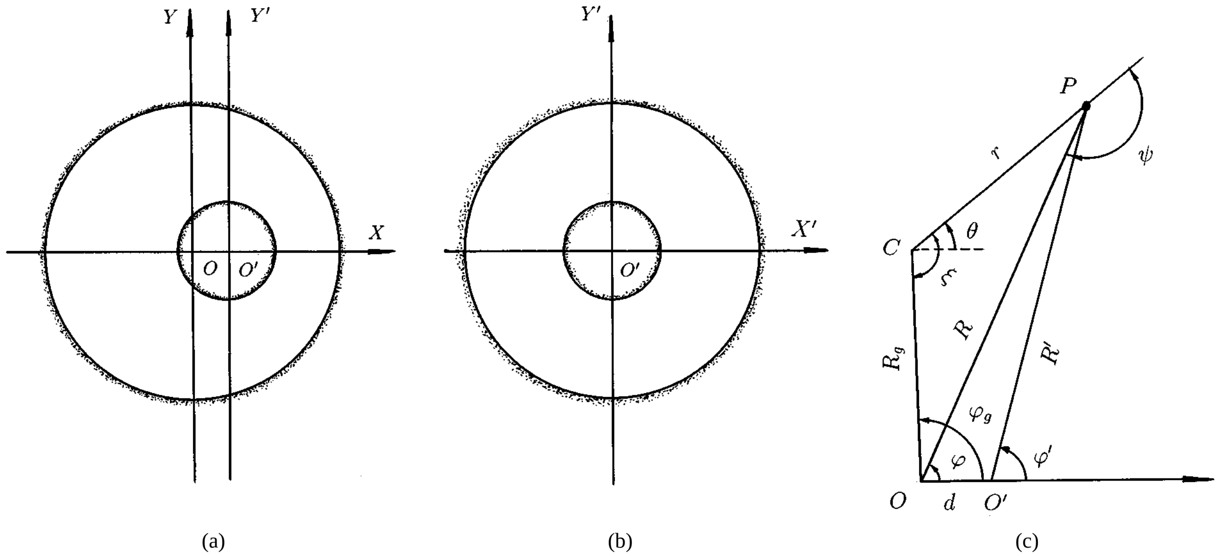

Fig. 1. Cross-sectional view of coaxial cavity and the coordinate systems. (a) Misaligned coaxial structure. (b) Perfectly coaxial structure. (c) Coordinates system employed in this paper.

II. ENIGENVALUE EQUATIONS

A. Perfectly Coaxial Structure

Without loss of generality, we consider a coaxial cavity in which the inner rod has a misalignment to the outer conductor [see Fig. 1(a)]. Before processing the misaligned coaxial structure, it is worthwhile to present a brief analysis of the perfect structure shown in Fig. 1(b). For the convenience of description, it is referred to as system *B*, and the corresponding quantities are marked by a prime.

We assume that the fields governed by the equation of wave are separable cases with time variable t and longitudinal coordinate varible z. It is then well known (see [21]) that the variation of the longitudinal component of the fields is governed by the Helmholtz equation

$$(\nabla^2_{\vec{R}'} + k'^2_\perp)\begin{Bmatrix} H'_z \\ E'_z \end{Bmatrix} = 0 \quad (1)$$

and the *general solution* is

$$H'_z(R', \varphi', z; t) = \sum_m A'_m\Big[J_m(k'_\perp R') - \zeta'_m N_m(k'_\perp R')\Big] \cdot e^{jm\varphi'}\, e^{\pm jk'_z z}\, e^{j\omega t} \quad (2)$$

for the transverse electric (TE) field structure and

$$E'_z(R', \varphi', z; t) = \sum_m A'_m\Big[J_m(k'_\perp R') - \zeta'_m N_m(k'_\perp R')\Big] \cdot e^{jm\varphi'}\, e^{\pm jk'_z z}\, e^{j\omega t} \quad (3)$$

for the transverse magnetic (TM) field structure, where

$$\omega = k'c,\, k'^2 = k'^2_z + k'^2_\perp \quad (4)$$

where c is the speed of light in vacuum, ω is the wave frequency, k' is the total wavenumber, and J_m and N_m are the first kind of Bessel function and Neumann function of order m, respectively. The following are four independent constants for each value of m: A'_m, k'_z (or k'), $k'_\perp$ and ζ'_m, which are to be determined by the initial conditions and boundary conditions. Physically speaking, m is referred to as the azimuthal mode index, each value of which indicates an elementary mode; A'_m denotes the field amplitude of the mode, which is related to the *initial condition*; $k'_\perp$ (being named as the cutoff wavenumber or transverse wavenumber) and ζ'_m are determined by the *outer and inner boundary conditions*; and k'_z is known as the propagation constant or the axial wave number. In this paper, we define $k'_\perp R_{\text{out}}$ as the eigenvalue of the elementary mode (where R_{out} denotes the radius of the outer conductor); but for the convenience of description, we also call the cutoff wavenumber $k'_\perp$ as the eigenvalue since the constant R_{out} does not change the physical meaning.

The particular solution of the transverse electric wave (TE mode) in system *B* can be obtained by substituting the general solution into the inner boundary condition

$$\left.\frac{\partial H'_z}{\partial n}\right|_{|\vec{R}'|=R_{\text{in}}} = 0 \quad (5)$$

and the outer boundary condition

$$\left.\frac{\partial H'_z}{\partial n}\right|_{|\vec{R}'|=R_{\text{out}}} = 0 \quad (6)$$

to determine ζ'_m and the eigenvalue $k'_\perp R_{\text{out}}$, where $\partial H'_z/\partial n$ is the normal gradient of H'_z and R_{in} is the radius of the inner conductor, respectively. The inner boundary condition gives the coefficient ζ'_m

$$\zeta'_m = \frac{J'_m(k'_\perp R_{\text{in}})}{N'_m(k'_\perp R_{\text{in}})}, \qquad m = 0, 1, 2, \cdots \quad (7)$$

where J'_m and N'_m are, respectively, the derivatives of J_m and N_m to the whole argument. Substituting (7) into (2), then into the outer boundary condition (6), we obtain

$$\sum_m A'_m\left[J_m(k'_\perp R_{\text{out}}) - \frac{J'_m(k'_\perp R_{\text{in}})}{N'_m(k'_\perp R_{\text{in}})}\, N_m(k'_\perp R_{\text{out}})\right] e^{jm\varphi'} = 0. \quad (8)$$

In order to ensure that not all of A'_m is to be zero and to keep the validity for all φ', we must have

$$J'_m(k'_\perp R_{\rm out}) - \frac{J'_m(k'_\perp R_{\rm in})}{N'_m(k'_\perp R_{\rm in})} N'_m(k'_\perp R_{\rm out}) = 0, \quad m = 0, 1, 2, \cdots. \tag{9}$$

The nth nontrivial root of (9) denotes the eigenvalue of the TE_{mn} mode, where m and n are the azimuthal and radial mode indexes, respectively. Equation (9) is referred to as the eigenvalue equations of the TE_{mn} mode. It indicates the well-known fact that, generally speaking, a single mode can exist in a perfectly coaxial waveguide and their cutoff wavenumbers are different from each other for different values of m. For example, in a cylindrical waveguide with a radius R_w, the eigenvalue $k'_\perp R_w$ of the TE_{01} mode is 3.832, whereas the eigenvalue of the TE_{11} mode is 1.841.

Similarly, the inner and outer boundary conditions of the TM wave

$$E'_z|_{|\vec{R}'|=R_{\rm in}} = 0 \tag{10}$$

and

$$E'_z|_{|\vec{R}'|=R_{\rm out}} = 0 \tag{11}$$

yield the eigenvalue equations of TM mode

$$J_m(k'_\perp R_{\rm out}) - \frac{J_m(k'_\perp R_{\rm in})}{N_m(k'_{\perp m} R_{\rm in})} N_m(k'_\perp R_{\rm out}) = 0, \quad m = 0, 1, 2, \cdots \tag{12}$$

where the nth nontrivial root of (12) denotes the eigenvalue of the TM_{mn} mode, m and n are, respectively, the azimuthal and radial mode indexes.

The axial wavenumber k'_z of a standing wave in a perfect oscillator cavity with a length of L, for instance, the $\mathrm{TE}_{m,n,q}$ mode, can be determined by the axial boundary condition of z, resulting in

$$k'_z L = q\pi, \qquad q = 1, 2, 3, \cdots \tag{13}$$

where q is referred to as the axial mode index, and the eigenfunction of z has been selected as $\sin(k_z z)$. In this case, the oscillating frequency of the $\mathrm{TE}_{m,n,q}$ mode, ω, is determined by the cutoff wavenumber and the axial wavenumber

$$f = \frac{\omega}{2\pi} = \frac{c}{2\pi}\sqrt{k_\perp^2 + k_z^2}. \tag{14}$$

For a traveling-wave propagating in a waveguide, the axial wavenumber is determined by the cutoff wavenumber and the given wave frequency

$$k'_z = \sqrt{\left(\frac{\omega}{c}\right)^2 - k_\perp^2}. \tag{15}$$

Therefore, the eigenvalue may be expressed in terms of the axial wavenumber since they can transfer each other when a wave frequency in (15) is given.

We can now find the following properties of the eigenvalue equations of the perfectly coaxial cavity.

1) Each eigenvalue equation (index m) defines a subset of eigenvalues $k_{\perp,mn}$, $n = 0, 1, 2, \cdots$.
2) Single cylindrical wave functions can satisfy both the inner and outer boundary conditions simultaneously, consequently, they can exist individually, defining the modes of the waveguide. These basic modes are called the normal modes [22].
3) Each eigenvalue depends on the inner-rod radius $R_{\rm in}$, the outer-wall radius$R_{\rm out}$, and the azimuthal mode index m, and the radial mode index n.

B. Misaligned Coaxial Structure

Let us turn our attention to the misaligned coaxial structure, as is shown in Fig. 1(a), which has a deviation d from the inner rod axis to the outer wall axis. We call it system *A*. Noting that the transformation

$$\vec{R} = \vec{R}' + \vec{d} \qquad (\text{i.e., } x = x' + d;\ y = y') \tag{16}$$

does not change the form of the Helmholtz equation, one may obtain the *general solution* of TE waves in system *A* by choosing the *inverse transformation* of the general solution (2) of system *B* as follows:

$$H_z(R, \varphi, z; t) = \sum_m \sum_{p=-\infty}^{\infty} A_m J_p(k_\perp d)\Big[J_{m+p}(k_\perp R) - \zeta_m N_{m+p}(k_\perp R)\Big] \cdot e^{j(m+p)\varphi} e^{\pm jk_z z} e^{j\omega t} \tag{17}$$

where the Graf's addition formulas [23] in the triangle $\Delta OPO'$

$$J_m(k_\perp R')\, e^{jm\varphi'} = \sum_{p=-\infty}^{\infty} J_p(k_\perp d) J_{m+p}(k_\perp R)\, e^{j(m+p)\varphi} \tag{18}$$

$$N_m(k_\perp R')\, e^{jm\varphi'} = \sum_{p=-\infty}^{\infty} J_p(k_\perp d) N_{m+p}(k_\perp R)\, e^{j(m+p)\varphi} \tag{19}$$

have been used. Mathematically speaking, the *general solution* of system *A* has much choice. The reason why we choose the inverse transformation of system *B* is the similarity of these two systems. Obviously, this choice meets the physical requirement that, at a fixed point in space, the field must be single valued [21]. The equivalence of the *general solutions* [i.e., (2) and (17)] implies that the fields in a misaligned coaxial waveguide may be expanded in terms of the superposition of cylindrical wave functions, where these wave functions are defined as the separated solutions of Maxwell's equations in cylindrical coordinates with the propagation constant k_z and cutoff wavenumber $k_\perp$ to be determined later from the boundary conditions. This treatment is similar with that employed by Collin [22]. He described that in a cylindrical waveguide with finite conducting walls or with discontinuity region, as a single normal mode no longer satisfies the boundary conditions, the arbitrary field can be represented as an infinite series of the normal modes (cf. [22, chs. 5.3, 5.6, and 5.7]). As to the misaligned coaxial waveguide, however, this time the particular solution must be determined by the boundary conditions of system A

$$\left.\frac{\partial H_z}{\partial n}\right|_{|\vec{R}-\vec{d}|=R_{\rm in}} = 0 \tag{20}$$

and

$$\left.\frac{\partial H(R,\,\varphi,\,z;\,t)}{\partial n}\right|_{|\vec{R}|=R_{\rm out}} = 0. \tag{21}$$

Substituting (17) into the inner boundary condition (20) yields

$$\zeta_m = \frac{J'_m(k_\perp R_{\rm in})}{N'_m(k_\perp R_{\rm in})}, \qquad m = 0,\,1,\,2,\,\cdots. \tag{22}$$

Inserting (22) into (17), then substituting (17) into the outer boundary condition (21), we obtain

$$\sum_m \sum_{p=-\infty}^{\infty} k_\perp A_m J_p(k_\perp d)\Big[J'_{m+p}(k_\perp R_{\rm out}) - \frac{J'_m(k_\perp R_{\rm in})}{N'_m(k_\perp R_{\rm in})} \cdot N'_{m+p}(k_\perp R_{\rm out})\Big]\, e^{j(m+p)\varphi} = 0. \tag{23}$$

Letting

$$\begin{aligned} m + p &= i \\ p &= i - m \end{aligned} \tag{24}$$

we rewrite (23) as

$$\sum_{i-m=-\infty}^{\infty} \sum_m k_\perp A_m J_{i-m}(k_\perp d)\Big[J'_i(k_\perp R_{\rm out}) - \frac{J'_m(k_\perp R_{\rm in})}{N'_m(k_\perp R_{\rm in})} \cdot N'_i(k_\perp R_{\rm out})\Big]\, e^{ji\varphi} = 0. \tag{25}$$

This equation must be valid for all φ, giving a set of eigenvalue equations of TE waves as follows:

$$\sum_m A_m J_{i-m}(k_\perp d) \cdot \left[J'_i(k_\perp R_{\rm out}) - \frac{J'_m(k_\perp R_{\rm in})}{N'_m(k_\perp R_{\rm in})}\, N'_i(k_\perp R_{\rm out})\right] = 0, \quad i = 0,\,\pm 1,\,\pm 2,\,\cdots. \tag{26}$$

Similarly, one can obtain a set of eigenvalue equations of TM waves

$$\sum_m A_m J_{i-m}(k_\perp d) \cdot \left[J_i(k_\perp R_{\rm out}) - \frac{J_m(k_\perp R_{\rm in})}{N_m(k_\perp R_{\rm in})}\, N_i(k_\perp R_{\rm out})\right] = 0, \quad i = 0,\,\pm 1,\,\pm 2,\,\cdots. \tag{27}$$

C. Eigenvalue Equation

Equations (26) and (27) include the effect of the misalignment on eigenvalues. When $d = 0$, eigenvalue equations (26) and (27) of the misaligned structure are restored to the eigenvalue equations (9) and (12) of the perfect structure by the property of the Bessel function [23]

$$J_{i-m}(k_\perp d)|_{d=0} = \begin{cases} 1, & \text{for } m = i \\ 0, & \text{for } m \neq i. \end{cases} \tag{28}$$

Now let us show that a single normal mode solution in a perfectly coaxial waveguide no longer individually satisfies both the outer and inner boundary conditions of the misaligned coaxial structure. For the case of a single normal mode solution, in the above derivation, the sum to m, $\sum_m$, should be moved away in (17) and, consequently, the result, i.e., (26), becomes

$$J_{i-m}(k_\perp d)\left[J'_i(k_\perp R_{\rm out}) - \frac{J'_m(k_\perp R_{\rm in})}{N'_m(k_\perp R_{\rm in})}\, N'_i(k_\perp R_{\rm out})\right] = 0, \qquad i = 0,\,\pm 1,\,\pm 2,\,\cdots. \tag{29}$$

Obviously, *no* single-valued $k_\perp$ can satisfy the above equation for various values of i even if i may be finite when the misalignment d is very small. In other words, a single normal mode solution must conflict the boundary conditions of the misaligned coaxial structure.

Letting

$$a_{i,\,m}(k_\perp) = J_{i-m}(k_\perp d)\left[J'_i(k_\perp R_{\rm out}) - \frac{J'_m(k_\perp R_{\rm in})}{N'_m(k_\perp R_{\rm in})} \cdot N'_i(k_\perp R_{\rm out})\right] \tag{30}$$

for TE waves, and

$$a_{i,\,m}(k_\perp) = J_{i-m}(k_\perp d)\left[J_i(k_\perp R_{\rm out}) - \frac{J_m(k_\perp R_{\rm in})}{N_m(k_\perp R_{\rm in})} \cdot N_i(k_\perp R_{\rm out})\right] \tag{31}$$

for TM waves, one can rewrite (26) and (27) as

$$\sum_m a_{im}(k_\perp) A_m = 0, \qquad i = 0,\,\pm 1,\,\pm 2,\,\cdots. \tag{32}$$

As a matter of fact, (32) is a set of linear equations to A_m. If (32) could be simplified as a solvable matrix equation to A_m, then in order to ensure not all of A_m to be zero, it is necessary that the coefficient determinant of A_m must be zero as follows:

$$\begin{vmatrix} \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & a_{i-1,\,m-1}(k_\perp) & a_{i-1,\,m}(k_\perp) & a_{i-1,\,m+1}(k_\perp) & \cdots \\ \cdots & a_{i,\,m-1}(k_\perp) & a_{i,\,m}(k_\perp) & a_{i,\,m+1}(k_\perp) & \cdots \\ \cdots & a_{i+1,\,m-1}(k_\perp) & a_{i+1,\,m}(k_\perp) & a_{i+1,\,m+1}(k_\perp) & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \end{vmatrix} = 0. \tag{33}$$

The above equation is the eigenvalue equation of a misaligned coaxial waveguide and determines the cutoff wavenumber $k_\perp$ of the propagation wave. When $d = 0$, all the elements except those on the main diagonal line of the determinant are zero, and (33) is restored to the eigenvalue equations (9) and (12) of the perfect structure by the property of Bessel functions (28). For a given d, the higher order values of $J_x(k_\perp d)$ (i.e., the coefficient of $a_{i,m}$) tend to be small; consequently, the eigenvalue equation (33) may be truncated as a determinant with finite order, and the eigenvalue $k_\perp$ can be solved by computer.

Finally, it is worthwhile to mention the substantial difference of our method with Dumbrajs and Pavelyev's treatment in [19] and [20]: their derivation was based on the expansion of a *single mode* (i.e., the sth mode in their papers, where the eigenvalue of the normal mode in a *perfect* structure was involved in their eigenvalue equation [19, eq. (4)] and [20, eq. (5)]), whereas our general solution is constructed by the superposition of cylindrical wave functions and, thus, the eigenvalue of the normal mode in a *perfect* structure is not involved. This substantial difference certainly leads to modified conclusions in both mathematical and physical aspects.

III. Numerical Approach of Eigenvalue Equation

Without loss of generality, we consider the case of a small misalignment. We assume that a $\mathrm{TE}_{u,n}$ traveling wave is injected or a $\mathrm{TE}_{u,n,p}$ standing wave is excited in a perfectly coaxial structure. This wave must be modified when the structure has a misalignment d. If the misalignment d is so small that finite terms $J_0(k_\perp d)$, $J_{\pm 1}(k_\perp d)$, $J_{\pm 2}(k_\perp d), \cdots, J_{\pm v}(k_\perp d)$ in (33) survive and higher order terms vanish, only the $(u-v)$th, $(u-v+1)$th, $\cdots$, $(u-1)$th, uth, $(u+1)$th, $\cdots$, $(u+v-1)$th, $(u+v)$th cylindrical wave functions will be involved, and the eigenvalue equation (33) is truncated as a determinant with order of $(2v+1)$

$$\begin{vmatrix} a_{u-v,\,u-v}(k_\perp) & \cdots & a_{u-v,\,u}(k_\perp) & \cdots & a_{u-v,\,u+v}(k_\perp) \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{u,\,u-v}(k_\perp) & \cdots & a_{u,\,u}(k_\perp) & \cdots & a_{u,\,u+v}(k_\perp) \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{u+v,\,u-v}(k_\perp) & \cdots & a_{u+v,\,u}(k_\perp) & \cdots & a_{u+v,\,u+v}(k_\perp) \end{vmatrix} = 0. \quad (34)$$

This estimation may be made by using the eigenvalue of the perfect structure. In this way, one can calculate the eigenvalue $k_\perp$, which is modified by the misalignment d.

IV. Analysis of the FZK Coaxial Cavity

In this section, we give a numerical analysis of the influences of a possible misalignment on the eigenvalues and eigenfrequencies by using the following design parameters: the radii of the inner and outer conductors are 8.128 and 29.81 mm ($R_{\mathrm{out}}/R_{\mathrm{in}} = 3.6677$) and the cavity length is 20 mm. According to (13), the axial wavenumber k_z can be figured out, which is $k_z = q\pi/L \approx 0.1571$ mm^{-1}. The oscillating frequency of the $\mathrm{TE}_{28,16,1}$ mode is figured out from (14).

A main goal of the gyrotron development program at the Research Center Karlsruhe, FZK, is the design, construction, and test of high-power gyrotron oscillators for electron cyclotron wave application and diagnostics of magnetically confined plasma in controlled thermonuclear fusion research. A coaxial-cavity gyrotron oscillator is under development, which is expected to operate in the $\mathrm{TE}_{28,16,1}$ mode at the frequency of 140 GHz with an output power of 1.5 MW [10].

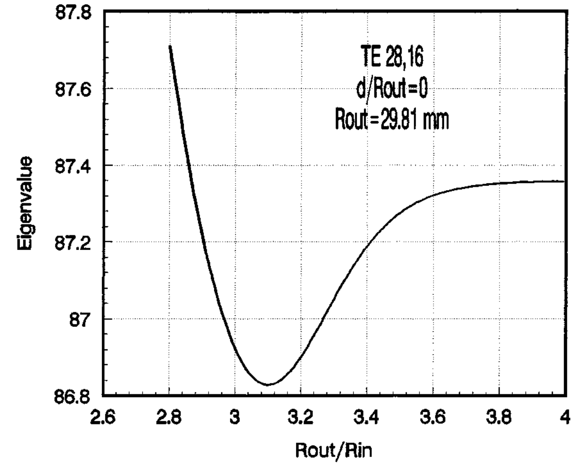

Fig. 2. Dependence of the eigenvalues on the ratio of the outer wall radius to the inner rod radius for $\mathrm{TE}_{28,16}$ traveling wave when the coaxial structure has no misalignment.

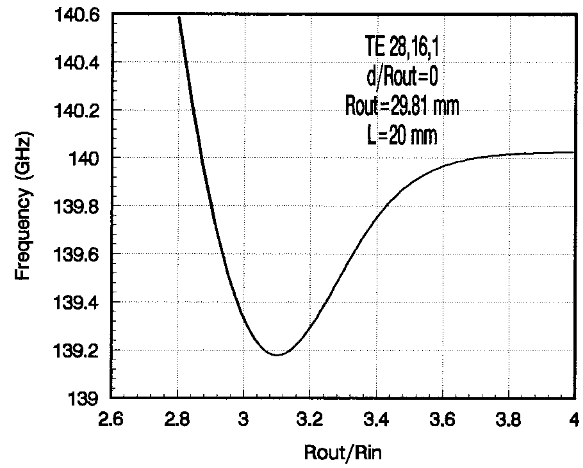

Fig. 3. Dependence of the eigenfrequency on the ratio of the outer wall radius to the inner rod radius for $\mathrm{TE}_{28,16,1}$ standing wave when the coaxial structure has no misalignment, where the outer wall radius is 29.81 mm and the cavity length is 20 mm.

Figs. 2 and 3 show the dependence of the eigenvalue and oscillating frequency on the ratio of the outer wall radius to the inner rod radius in a perfect structure. From these figures, we see that the FZK cavity had eigenvalue of 87.3377 and oscillating frequency of 139.9929 GHz, which is in agreement with the observation in the experiment [10].

Figs. 4 and 5 displays the influences of small misalignment of the inner rod to the cavity wall on the eigenvalue $k_\perp R_{\mathrm{out}}$ and the eigenfrequency shift $f(d) - f(d=0)$. In the calculation, the eigenvalue equation is truncated as a determinant of order 13. It appears that the misalignment may decrease the eigenvalue and eigenfrequency.

Finally, it should be pointed out that the modification of the eigenvalue based on the existence of the misalignment affects not only the eigenfrequency, but also the distribution of the RF fields since the eigenvalue is involved in the arguments of the Bessel functions and Neumann functions of the RF field expressions. Consequently, the interaction of the electron beam with

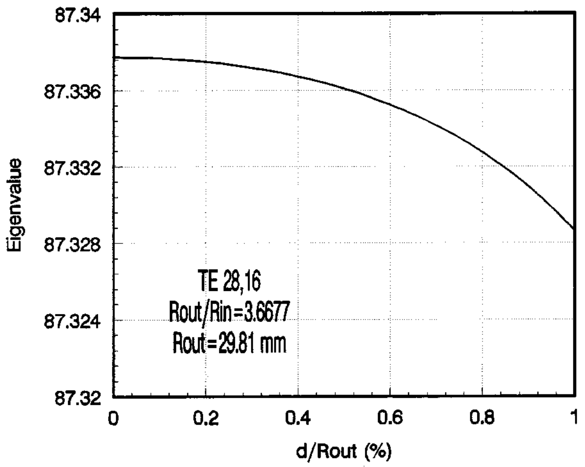

Fig. 4. Eigenvalue of $\mathrm{TE}_{28,16}$ traveling wave modified by small misalignment, where $R_{\rm out}/R_{\rm in} = 3.6677$.

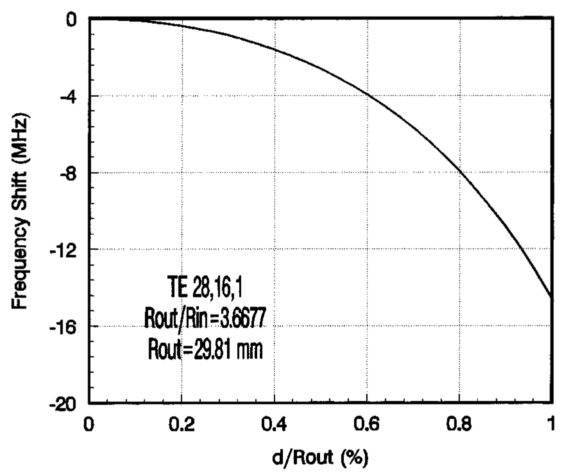

Fig. 5. Modification of the eigenfrequency of $\mathrm{TE}_{28,16,1}$ standing wave by small misalignment, where $R_{\rm out}/R_{\rm in} = 3.6677$, the outer wall radius is 29.81 mm and cavity length is 20 mm.

the RF fields in a coaxial-cavity gyro-device must be modified by the structure misalignment. This topic is under consideration and will be published in a future paper.

V. Conclusions

In this paper, we have derived the rigorous eigenvalue equation (33) for the coaxial structure with a misalignment between the outer wall and inner rod, proposed a numerical approach to solve the modification of the eigenvalue due to the existence of the structural misalignment, and carried out the analysis of the higher mode coaxial cavity employed in a 140-GHz/1.5-MW gyrotron oscillator of FZK. The following general conclusions can be drawn: 1) single cylindrical wave function can no longer satisfy both the outer and inner boundary conditions of a misaligned coaxial structure (waveguide or cavity), and taken individually, these functions cannot define the modes of the structure; however, the modes can be written as a *superposition* of cylindrical wave functions, and are of the TE or TM (or TEM) type and 2) the eigenvalues of a misaligned coaxial structure must be calculated on the basis of this superposition of cylindrical wave functions and are modified noticeably by the structural misalignment.

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