Method for Measuring Properties of High Relative Dielectric Constant Materials in a Cutoff Waveguide Cavity

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***Abstract*—In this paper, a method for measuring properties of ceramic materials with relative dielectric constant value of 20–150 is proposed. It permits us to eliminate the operating $TM_{01\delta}$-mode degeneration due to its frequency coincidence with other modes, including the ones of higher orders. Both that fact and the possibility of precise calculation of an unloaded quality factor for a cavity permit one to execute the accurate measurements of loss tangents values as low as $(1 \div 0,5) \times 10^{-4}$, the error of dielectric constant measurement being equal to or less than 1%. The feasibility of precise measurement of the loaded Q-factor of a cavity by the readings of micrometric probe makes the use of frequency meters unnecessary.**

***Index Terms*—Cutoff waveguide, dielectric constant, measurement, quartz, $TE_{01\delta}$ mode.**

I. INTRODUCTION

METHODS based on utilization of resonance phenomenon in a cutoff waveguides, where metallic end faces of a cavity are connected close to those of a measured sample, are widely used [1]–[3]. They permit one to realize the accurate measurement of relative dielectric constant in ceramic materials, but are not as good for measuring dielectric loss tangent of those materials due to a low unloaded quality (Q)-factor of TE_{011} cavity. Low value of this factor is caused by the loss in the metallic walls of cavity [4]. In this case, the value of the unloaded Q-factor is significantly decreased with the increase of material dielectric constant. That makes it necessary to use two cavities [5]. In one cavity, according to the methods stated, is accomplished the measurement of the dielectric constant. In the other, the dielectric loss tangent is measured on different samples. The use of the cylinder TE_{01n} cavity with a dielectric sample of half-wavelength thickness in it [6] allows us to simultaneously perform the measurement of the dielectric constant and dielectric loss tangent. However, possible degenerations due to highest modes, the number of which sharply increases with the increase of the dielectric constant value, do not ensure a precise measurement of the dielectric loss tangent.

The purpose of this paper is to create a cavity free of the above disadvantages and, at the same time, provide the accurate metering of both the loss tangent up to 5×10^{-5} and the relative dielectric constant in the range of 20–150.

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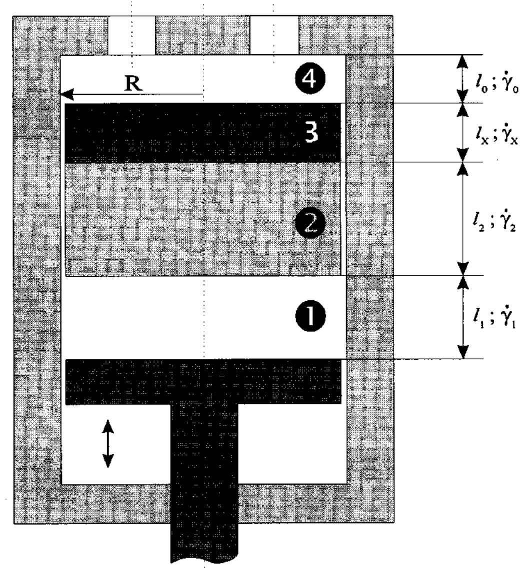

Fig. 1. Cavity configuration.

II. THEORY

Fig. 1 shows a cavity configuration. In cylinder regions 1, 2, and 4, the TE_{01} wave damps, but in region 3, it propagates. Regions 1 and 4 are air filled, while region 2 is charged with quartz (cylinder), and region 3 contains a sample to be measured.

For the cavity under consideration, with the $TE_{01\delta}$ mode being in an excited state [7], a strict solution may be obtained in terms of (1) and (2) as follows:

$$\frac{\dfrac{\tan \dot{\gamma}_0 l_0}{\dot{\gamma}_0} + \dfrac{\tan \dot{\gamma}_X l_X}{\dot{\gamma}_X}}{1 - \dot{\gamma}_X^2 \dfrac{\tan \dot{\gamma}_X l_X}{\dot{\gamma}_X} \times \dfrac{\tan \dot{\gamma}_o l_o}{\dot{\gamma}_o}} = -\frac{\dfrac{\tan \dot{\gamma}_1 l_1}{\dot{\gamma}_1} + \dfrac{\tan \dot{\gamma}_2 l_2}{\dot{\gamma}_2}}{1 - \dot{\gamma}_2^2 \dfrac{\tan \dot{\gamma}_2 l_2}{\dot{\gamma}_2} \times \dfrac{\tan \dot{\gamma}_1 l_1}{\dot{\gamma}_1}} \quad (1)$$

where $\dot{\gamma}_i$ are the complex constants of propagation (damping) in the corresponding region and l_i are the lengths of the corresponding cylinder region. Equation (2) is shown at the bottom of the following page, where $k = 2\pi f_o'/C = 0.209\,44 f_o$ (cm^{-1}),

f_o is the measuring frequency (in gigahertz), f_o' is the measuring frequency (in hertz), Δd is the damping due to loss in dielectric material, $\chi = 3.8317/R$, R is the radius of cavity (in centimeters), ε_i is the relative dielectric constant of the corresponding region, and ε_i'' is the dielectric loss in corresponding region.

With the presence of damping oscillations in specified regions and at $\tan\delta_{\varepsilon x'} \le 0.05$, (1) may be divided in two equations as follows:

$$\frac{\dfrac{\tanh\gamma_0 l_0}{\gamma_0} + \dfrac{\tan\gamma_X l_X}{\gamma_X}}{1-\gamma_X^2 \dfrac{\tanh\gamma_0 l_0}{\gamma_0}\dfrac{\tan\gamma_X l_X}{\gamma_X}} = -\frac{\dfrac{\tanh\gamma_2 l_2}{\gamma_2} + \dfrac{\tanh\gamma_1 l_1}{\gamma_1}}{1+\gamma_2^2 \dfrac{\tanh\gamma_2 l_2}{\gamma_2}\dfrac{\tanh\gamma_1 l_1}{\gamma_1}} \quad (3)$$

where

$$\begin{aligned}\gamma_0 &= \gamma_1 = \sqrt{\chi^2 - k^2}\\ \gamma_2 &= \sqrt{\chi^2 - k^2\varepsilon_2'}\\ \gamma_X &= \sqrt{k^2\varepsilon_X' - \chi^2}\\ \Delta d &= \tan\delta_{\varepsilon_x'} N_X \varepsilon_X' + \tan\delta_{\varepsilon_2'} N_2 \varepsilon_2' \end{aligned} \quad (4)$$

where $N_x\varepsilon_x'$ and $N_2\varepsilon_2'$ are the coefficients equal to the ratio of the electric energy, stored in corresponding regions, to the total energy in the cavity, and $\tan\delta_{\varepsilon x'}$ and $\tan\delta_{\varepsilon 2'}$ are the loss tangents in the corresponding regions filled with dielectric materials

$$\begin{aligned}N_X &= \frac{1}{A}M_X\\ N_2 &= \frac{1}{A}M_2\\ M_2 &= \frac{\sinh 2\gamma_2 l_2 - 2\gamma_2 l_2}{\gamma_2^3 L_2^2(1+\cosh 2\gamma_2 l_2)}(1-\gamma_2^2 C_1^2) + \frac{2}{L_2^2} C_1 C_2 (C_1 + C_2)\\ M_X &= \frac{2\gamma_X l_X - \sin 2\gamma_X l_X}{\gamma_X^3 L_X^2(1+\cos 2\gamma_X l_X)}(1+\gamma_X^2 C_0^2) + \frac{2}{L_X^2} C_0 C_X (C_0 + C_X)\\ L_X &= 1 - \gamma_X^2 C_X C_0\\ C_X &= \frac{\tan\gamma_X l_X}{\gamma_X}\\ C_0 &= \frac{\tanh\gamma_0 l_0}{\gamma_0}\\ L_2 &= 1 + \gamma_2^2 C_1 C_2\end{aligned}$$

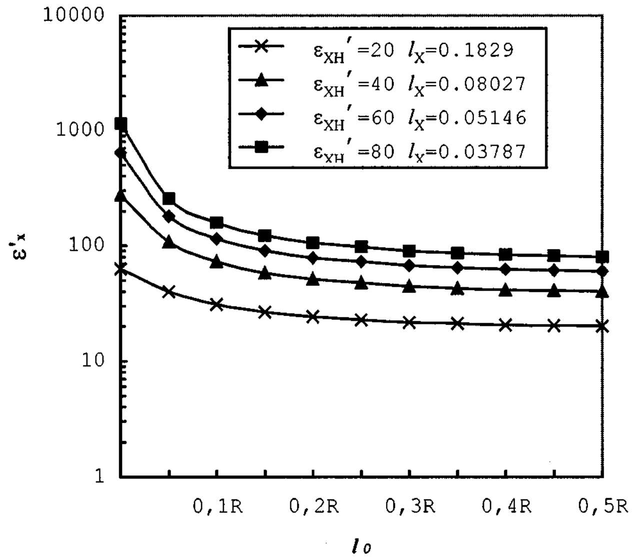

Fig. 2. Dielectric constant versus l_o values.

$$\begin{aligned}C_1 &= \frac{\tanh\gamma_1 l_1}{\gamma_1}\\ C_2 &= \frac{\tanh\gamma_2 l_2}{\gamma_2}\\ A &= \frac{\sinh 2\gamma_1 l_1 - 2\gamma_1 l_1}{\gamma_1^3 L_2^2(1+\cosh 2\gamma_1 l_1)}(1-\gamma_2^2 C_2^2) + M_X\varepsilon_X' + M_2\varepsilon_2' + \frac{\sinh 2\gamma_0 l_0 - 2\gamma_0 l_0}{\gamma_0^3 L_X^2(1+\cosh 2\gamma_0 l_0)}(1+\gamma_X^2 C_X^2).\end{aligned}$$

When dielectric loss magnitudes in regions 2 and 3 are virtually nil, unloaded attenuation of the cavity may be determined from (5)

$$d_c = \frac{1}{Q_c} = \frac{\Delta\left[\dfrac{R}{k^2}(\gamma_2^2 + H_4^2\gamma_0^2) + 2B\dfrac{\chi^2}{k^2}\right]}{2\times R\times B_\varepsilon} \quad (5)$$

where

Q_c unloaded Q-factor of the cavity (unloaded Q_c) with zero loss in regions 2 and 3;

Δ $= 100 \times \sqrt{1/\pi f_0' \mu\sigma}$—thickness of skin-layer or depth of field penetration into metallic surface of the cavity (in centimeters);

μ $= \mu_o = 4\pi 10^{-7}$ H/m—magnetic permeability;

σ conductance of cavity walls (in siemens per meter) (in calculations, $\sigma = 5.80\times 10^7$ S/m)

$$\begin{aligned}B &= D_1 + H_2^2 D_2 + 2H_2H_3D_{23} + H_3^2 D_3 + H_4^2 D_4\\ B_\varepsilon &= \varepsilon_2' D_1 + \varepsilon_X'(H_2^2 D_2 + 2H_2H_3D_{23} + H_3^2 D_3) + H_4^2 D_4\\ D_1 &= \frac{\sinh\gamma_2 l_2 \cosh\gamma_2 l_2}{2\gamma_2} - \frac{l_2}{2}\end{aligned}$$

$$\dot{\gamma}_i = \sqrt{k^2\left(1-\frac{\Delta d^2}{4}\right)\varepsilon_i' + k^2\Delta d\varepsilon_i'' - \chi^2 + jk^2\Delta d\varepsilon_i' - jk^2\left(1-\frac{\Delta d^2}{4}\right)\varepsilon_i''} \quad (2)$$

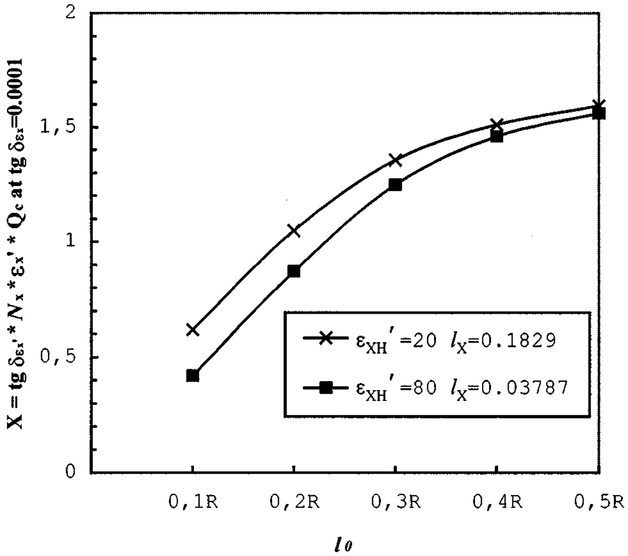

Fig. 3. Dependence of X from l_o values.

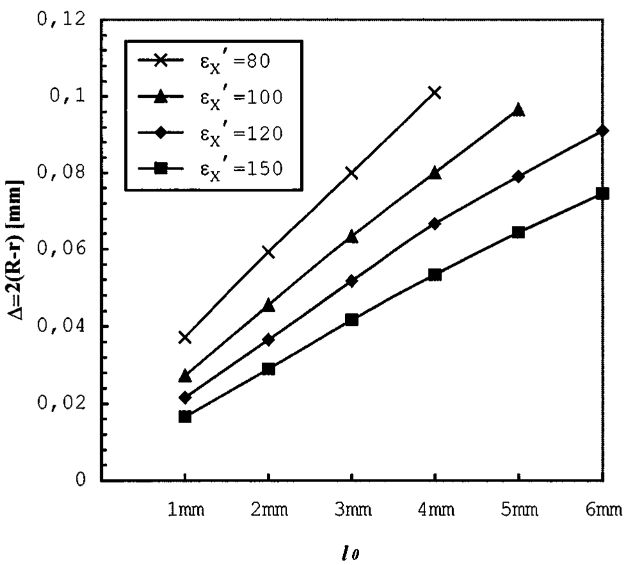

Fig. 4. Dependence of allowable gap from l_o values.

$$
\begin{aligned}
D_2 &= \frac{l_X}{2} - \frac{\sin 2\gamma_X l_X}{4\gamma_X} \\
D_{23} &= \frac{\sin^2 \gamma_X l_X}{2\gamma_X} \\
D_3 &= \frac{l_X}{2} + \frac{\sin 2\gamma_X l_X}{4\gamma_X} \\
D_4 &= \frac{\sinh \gamma_0 l_0 \cosh \gamma_0 l_0}{2\gamma_0} - \frac{l_0}{2} \\
H_2 &= \frac{\gamma_2}{\gamma_X} \cosh \gamma_2 l_2 \\
H_3 &= \sinh \gamma_2 l_2 \\
H_4 &= \frac{1}{\sinh \gamma_0 l_0} \\
&\quad \cdot \left[\frac{\gamma_2}{\gamma_X} \cosh \gamma_2 l_2 \sin \gamma_X l_X + \sinh \gamma_2 l_2 \cos \gamma_X l_X \right].
\end{aligned}
$$

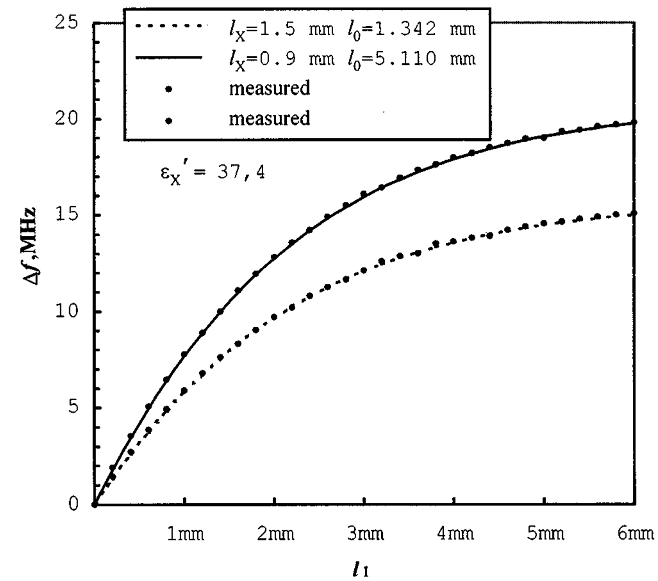

Fig. 5. Calculated curves and experimental data for cavity resonant frequency versus lengths of region l_1.

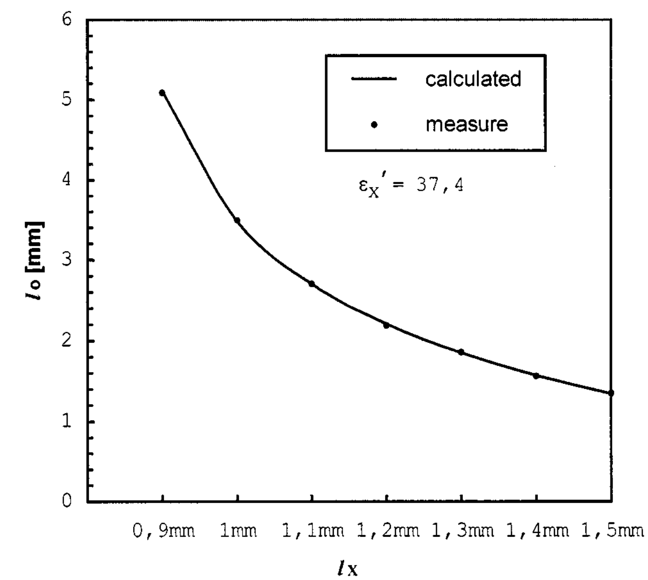

Fig. 6. Calculated curves and experimental data for length of the l_o region versus sample thickness l_x.

To eliminate the influence of the highest modes, it is necessary to impose certain requirements on the thickness of the disk under test lx (in centimeters) as follows:

$$
\begin{aligned}
l_X &= \frac{\pi}{\gamma_X} - \frac{1}{\gamma_X} \arctan(Y), \qquad \text{at } Y > 0 \\
l_X &= -\frac{1}{\gamma_X} \arctan(Y), \qquad \text{at } Y < 0 \\
Y &= \frac{C_1 + C_2 + C_O(1 + \gamma_2^2 C_1 C_2)}{1 + \gamma_2^2 C_1 C_2 - \gamma_X^2 C_O (C_1 + C_2)} \gamma_X.
\end{aligned} \tag{6}
$$

When the thickness of the sample meets the requirements of (6), resonant frequencies of highest modes will be greater than resonant frequency of the $\mathrm{TE}_{01\delta}$ mode, and their influence on the result of dielectric loss measurement will be completely eliminated.

TABLE I
RESULTS OF MEASURING DIELECTRIC PARAMETERS OF SEVERAL MATERIALS

Designation	Composition	ε_X'	$\tan\delta_{\varepsilon_X}$'	Sample thickness, mm	Unloaded quality factor - Q_c
MCT 20	Ca-Ti-O	20.1	2×10^{-3}	2.47	11595
MCT 50	Ca-Ti-O	47.7	2×10^{-4}	0.85	12223
MCT 50	Ca-Ti-O	50.2	3×10^{-4}	0.85	10835
MCT 80	Ca-Ti-O	86.0	2×10^{-4}	0.44	12802
MCT 100	Ca-Ti-O	98.4	4×10^{-4}	0.44	9385
T5HB	Ti-Ba-Nd-Bi	70.2	2×10^{-4}	0.56	12103
TST-8	Zr-Sn-Ti	42.4	4×10^{-4}	1.00	11460
BZT-40	Zr-Ti-O	39.4	1×10^{-4}	0.85	19729

III. NUMERICAL RESULTS

Fig. 2 shows the dependence of the dielectric constant at $R = 1.3$ cm, $f_o = 6.0$ GHz, $l_2 = 1.2$ cm, $\varepsilon_2' = 3.814$; $l_1 = 0$, and of several ε'_{XH} values from l_o value expressed in terms of the cavity radius R. For each chosen ε'_{XH} value at $l_o = 0.5R$, the thickness of the measured disk is determined from (6). When l_o varies, the thickness remains unchanged. As is seen from the given curves, variations of ε_x', when l_o changes from $0.5R$ to zero, are rather pronounced. However, for several reasons, it is rather difficult to use all the range of possible ε_x' variations. First, having such a great range of ε_x' changes it, and is impossible to exclude degeneracy of the measured mode by the lowest modes ($\mathrm{TE}_{11\delta}$ and $\mathrm{TM}_{01\delta}$). Second, unloaded Q_c decreases considerably at $l_o < 0.1 \times R$. Third, the uncertainty in the determination of the reactance of the coupling elements leads to inaccuracy of the ε_x' measurement. Therefore, we further limited the range of l_o variation to 0.1 through $0.5R$.

The ratio of attenuation inserted by the sample to unloaded attenuation of the cavity provides objective estimation of cavity quality. In our case, this ratio is determined by the following expression:

$$X = \tan\delta_{\varepsilon_x'} N_X \varepsilon'_X Q_c. \tag{7}$$

In Fig. 3, the dependence of X from l_o for different thicknesses of the measured sample at $\tan\delta_{\varepsilon x'} = 1\times10^{-4}$ is shown. When plotting the curves, the following conditions have been accepted: $R = 1.3$ cm, $l_2 = 1.2$ cm, and $\varepsilon_2 = 3.814$, $l_1 = 0$. As is seen from Fig. 3, the measured loss magnitude depends only slightly on inaccuracy in finding the unloaded Q_c when $l_o > 0.2 \times R$. In this case, 10% error in measuring the conductance of cavity walls causes the error in $\tan\delta_{\varepsilon x'}$ evaluation of about $\pm10^{-5}$.

$N_x\varepsilon_x'$ becomes four times the corresponding $N_2\varepsilon_2'$ value.

It is possible to eliminate the degeneration of the $\mathrm{TE}_{01\delta}$ mode under influence of the $\mathrm{TE}_{11\delta}$ and $\mathrm{TM}_{01\delta}$ modes by the choice of cavity radius. When the latter is 1.3–1.4 cm and the frequency is equal to 6.0 GHz, the above degeneration will be absent over the whole range of l_o variations (0.1 through $0.5R$), as well as over the whole range of measured dielectric constants (20–150).

Fig. 4 shows the dependence of the allowable gap $\delta = 2(R - r)$ from l_o for different dielectric constants of the sample (r is the disk radius measured). For gaps equal to or less than those illustrated in Fig. 4, the degenerations between $\mathrm{TE}_{01\delta}$ and $\mathrm{TE}_{21\delta}$ modes are nil.

IV. EXPERIMENTAL RESULTS

Fig. 5 shows the calculated curves and experimental results of the cavity resonant frequency variation versus the length of region l_1. The variations stated have been determined for ceramic samples with a dielectric constant ε_x' of 37.4 and for two thicknesses of 1.5 and 0.9 mm, respectively. A slight increase of experimental values of resonant frequency variations at small l_1 values can be attributed to the nonideal form of quartz cylinder (departure of cavity end faces from perpendicular to its axis).

Relatively small variation of the cavity resonant frequency at the significant displacement of the plunger permits one to carry out an accurate measurement of cavity passband with a disk inserted.

Cavity attenuation is measured, and the loss tangent of that measured is determined by (8) coupled with (4) and (5) as follows:

$$\tan\delta_{\varepsilon_x'} = \frac{1}{N_X\varepsilon'_X}\left[\frac{\Delta d_\Sigma}{1+2\beta} - \tan\delta_{\varepsilon_2'} N_2\varepsilon_2' - d_c\right] \tag{8}$$

where Δd_Σ is the loaded attenuation of the cavity, $\beta = 1/(2(10^{\Delta\alpha/20} - 1))$, and $\Delta\alpha$ is the cavity attenuation at resonance (in decibels).

Experimental data and calculated curves (Fig. 6) show the region l_o length dependence of the sample thickness. The curves are given for the dielectric constant ε_x' of 37.4. The experimental data are well conformed with the calculated curve.

Experimental data is obtained by measuring the corresponding parameters at $f_o = 6.0$ GHz. The cavity radius was $R = 1.3$ cm, the quartz cylinder length was $l_2 = 1.2$ cm, and the dielectric constant of quartz was $\varepsilon_2' = 3.814$.

The results of measuring dielectric parameters of several materials are shown in Table I. Compositions of materials are specified. In the identical compositions, quantitative proportions of constituents are being changed.

V. CONCLUSIONS

On the base of a test cavity, the instrument for measuring the dielectric material parameters was designed. The meter comprises the cavity, microwave generator, and selective amplifier. A generator is tuned to the frequency of 6.0 GHz, which permits one to smoothly change output power level to –60 dB (1 mW). The selective amplifier has sensitivity with 10 μV/scale limit provides the recording of the detected signal from the cavity. The instrument allows us to measure the dielectric constant parameter in the range of 20–150 and dielectric loss tangent from 5×10^{-5} to 2×10^{-3}. Test samples have a diameter of 26 mm and a thickness from 0.26 to 2.5 mm depending on the dielectric constant value.

REFERENCES

[1] B. W. Hakki and P. D. Colemak, "A dielectric resonator method of measuring inductive capacities in the millimeter range," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-8, pp. 402–410, July 1960.

[2] W. E. Courtney, "Analyses and evaluation of a method of measuring the complex permittivity and permeability of microwave insulation," *IEEE Trans. Microwave Theory Tech*, vol. MTT-18, pp. 476–485, Aug. 1970.

[3] Y. Kobayashi and S. Tanaka, "Resonant modes of a dielectric rod resonator short-circuited at both ends by parallel conducting plates," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-28, pp. 1077–1085, Oct. 1980.

[4] Y. Kobayashi, T. Aoki, and Y. Kabe, "Influence of conductor shields on the Q-factors of a TE_0 dielectric resonator," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-33, pp. 1361–1366, Dec. 1985.

[5] *Dielectric Resonators*, Trans-Tech, Adamstown, MD, 1997.

[6] M. N. Afsar, J. R. Birch, and R. N. Clarke, "The measurement of the properties of materials," *Proc. IEEE*, vol. 74, Jan. 1986.

[7] V. V. Nikolski, *Electrodynamics and Radiowave Propagation*. Moscow, Russia: Nauka, 1973.

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