

# Scattering Matrix Approach for the Design of Microwave Filters

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**Abstract**—A synthesis procedure, based on a distributed parameter model, for the design of microwave filters is presented in this paper. The frequency response of the filter is described in terms of the characteristic polynomial  $T_{21} = S_{11}/S_{21}$  where  $S_{11}$  and  $S_{21}$  are the scattering parameters of the filter. Starting from the desired polynomial  $T_{21}$ , the design scheme directly yields the scattering parameters of the various junctions, which can be realized by any kind of discontinuity. The capability of synthesizing an arbitrary frequency response allows one to introduce the concept of a “predistorted” characteristic polynomial in order to compensate for the degradations caused by multimodal interactions, frequency dispersion, etc. Comparison with measured data of a Chebyshev-like eight-pole *E*-plane filter confirms the validity of the method also in presence of losses.

**Index Terms**—Electromagnetic modal analysis, filter design, microwave filters, waveguide discontinuities.

## I. INTRODUCTION

THE design of microwave and millimeter-wave devices requires more and more accurate synthesis procedures to satisfy the increasingly stringent specifications of modern communication systems [1]. In particular, as far as microwave filters are concerned, both in the past [2]–[9] and in recent years [10]–[13], a great effort has been devoted to the development of efficient design procedures.

Most of the available synthesis techniques are based on models that do not conveniently describe the physical behavior of the filter (as, e.g., in the case of low-pass prototypes). Generally, the frequency response of the structure designed by these techniques does not match the specifications so that a numerical optimization process is necessary to obtain the final configuration of the filter. In recent times, the availability of high-power computational resources has made this approach possible and has lead to the development of computer-aided design (CAD) tools based on different methods, such as the mode-matching method [14]–[17], the adjoint network method [18], and the space-mapping technique [19].

The synthesis technique presented in this paper, on the contrary, does not require any numerical optimization process because it is based on an accurate model, where the discontinuities are described by their *S*-parameters [20]. As is well known, an  $N$ -resonator waveguide filter can be seen as a cascade of  $N + 1$  discontinuities interconnected by  $N$  transmission lines corresponding to the fundamental waveguide mode. The  $N + 1$  discontinuities can be irises, thick slots, *E*-plane

septa, etc. and more complex structures such as input/output coaxial-waveguide transitions, or even composite junctions, as in the case of multiplexer configurations. Starting from the desired response and on the basis of this distributed model, the synthesis procedure directly yields the scattering parameters of the various discontinuities and the lengths of the resonators. The procedure is based on the properties of the elements of the transmission matrix of the filter:  $T_{21} = S_{11}/S_{21}$  (i.e., the characteristic polynomial) and  $T_{11} = 1/S_{21}$ , where  $S_{11}$  and  $S_{21}$  are the relevant scattering parameters.

The method allows to synthesize an arbitrary frequency response by suitably positioning the  $T_{21}$  roots that correspond to the reflection zeros of the filter. This possibility becomes very useful when the phenomena that at first are ignored in the synthesis procedure cause significant effects. These phenomena are: 1) the frequency dispersion of the *S*-parameters of the various discontinuities; 2) the multimodal interactions between discontinuities; and 3) the material losses. Their effects are generally important in the following cases: filters with large bandwidth, discontinuities with high excitation coefficients of the evanescent modes, input/output junctions with complex configuration (e.g., multiplexers with a rapidly varying frequency response), and finally, when the insertion loss introduced by the filter is significant. In order to find a way to compensate for the degradations produced by these phenomena, the complete design process is phrased in the language of system theory. By means of a system identification technique, the response to be synthesized is predistorted in such a way that the final full-wave analysis of the filter meets the specifications exactly.

## II. EXTRACTION PROCEDURE

As far as the fundamental mode is concerned, an  $N$ -pole microwave filter can be described by the equivalent two-port network shown in Fig. 1.  $\mathbf{S}^{(k)}$  ( $k = 0, N$ ) is the  $2 \times 2$  scattering matrix of the  $k$ th discontinuity and  $l_k$  ( $k = 1, N$ ) is the length of the modal transmission line with propagation constant  $\beta_k$  corresponding to the  $k$ th resonator. Differently from what was assumed in [20], the various resonators can be waveguides with different cross sections (rectangular, circular, coaxial, or with the same geometry, but with different dimensions). For this reason, the propagation constant of the fundamental waveguide mode in each resonator is labeled with the index  $k$ .

Consider the transmission matrix of the  $k$ th discontinuity, defined as

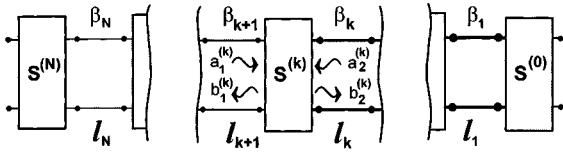
$$\begin{bmatrix} a_1^{(k)} \\ b_1^{(k)} \end{bmatrix} = \mathbf{T}^{(k)} \begin{bmatrix} b_2^{(k)} \\ a_2^{(k)} \end{bmatrix} \quad (1)$$

where  $a_i^{(k)}$  and  $b_i^{(k)}$  are the incident and scattered power waves.

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Fig. 1. Two-port network equivalent circuit of an  $N$ -resonator filter.

In the case of a reciprocal and lossless structure, this matrix can be written as follows:

$$\begin{aligned} \mathbf{T}^{(k)} &= \begin{bmatrix} 1/S_{21}^{(k)} & -S_{22}^{(k)}/S_{21}^{(k)} \\ S_{11}^{(k)}/S_{21}^{(k)} & -\Delta S^{(k)}/S_{21}^{(k)} \end{bmatrix} \\ &= e^{-j\phi_{21}^{(k)}} \begin{bmatrix} 1 & 0 \\ 0 & e^{j\phi_{11}^{(k)}} \end{bmatrix} \begin{bmatrix} \csc \gamma_k & \cot \gamma_k \\ \cot \gamma_k & \csc \gamma_k \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -e^{j\phi_{22}^{(k)}} \end{bmatrix} \end{aligned} \quad (2)$$

where  $\cos \gamma_k = |S_{11}^{(k)}| = |S_{22}^{(k)}|$ ,  $\sin \gamma_k = |S_{21}^{(k)}|$ , and  $\phi_{ij}^{(k)}$  are the phases of the scattering parameters  $S_{ij}^{(k)}$  and  $\Delta S_{ij}^{(k)}$  is the determinant of  $\mathbf{S}^{(k)}$ . The length of the waveguides corresponding to the  $k$ th cavity can be described by the following transmission matrix:

$$\mathbf{T}^{(k)} = e^{j\beta_k l_k} \begin{bmatrix} 1 & 0 \\ 0 & e^{-2j\beta_k l_k} \end{bmatrix}. \quad (3)$$

According to (2) and (3), the transmission matrix of the whole filter can be expressed as follows:

$$\begin{aligned} \mathbf{T} &= z^{N/2} \exp \left\{ j \left[ \frac{1}{2} (\phi_{11}^{(N)} + \phi_{22}^{(0)}) + \sum_{k=1}^N \psi_k \right] \right\} \\ &\cdot \begin{bmatrix} 1 & 0 \\ 0 & e^{j\phi_{11}^{(N)}} \end{bmatrix} \left\{ \prod_{k=N}^1 \begin{bmatrix} \csc \gamma_k & \cot \gamma_k z^{-1} e^{-j\psi_k} \\ \cot \gamma_k & \csc \gamma_k z^{-1} e^{-j\psi_k} \end{bmatrix} \right\} \\ &\cdot \begin{bmatrix} \csc \gamma_0 & \cot \gamma_0 \\ \cot \gamma_0 & \csc \gamma_0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -e^{j\phi_{22}^{(0)}} \end{bmatrix} \end{aligned} \quad (4)$$

where the complex variable  $z = \exp(j\theta)$  is defined as

$$z \exp(j\psi_k) = -\exp \left\{ j \left[ 2\beta_k l_k - (\phi_{11}^{(k-1)} + \phi_{22}^{(k)}) \right] \right\}. \quad (5)$$

It is to be noted that the complex variable  $z$  is not a function of the index  $k$  and its phase is related to the frequency through the propagation constants  $\beta_k$  and the phases of the scattering parameters  $S_{ii}^{(k)}$  of the junctions. Hence, the unknown phase terms  $\psi_k$  have been introduced in order to compensate for the different phase behavior of the resonators and to guarantee the possibility of synthesizing arbitrary frequency responses, as will be explained later.

Since only the amplitude of the transmission matrix elements is of interest, it is convenient to drop the phase factors in (4) and to regard the following expression:

$$T = \left\{ \prod_{k=N}^1 \begin{bmatrix} \csc \gamma_k & \cot \gamma_k z^{-1} e^{-j\psi_k} \\ \cot \gamma_k & \csc \gamma_k z^{-1} e^{-j\psi_k} \end{bmatrix} \right\} \begin{bmatrix} \csc \gamma_0 & \cot \gamma_0 \\ \cot \gamma_0 & \csc \gamma_0 \end{bmatrix} \quad (6)$$

as the transmission matrix of the whole filter.

From (6), it is easy to recognize that the elements of the transmission matrix  $\mathbf{T}$  are  $N$ -degree polynomials in the complex variable  $z^{-1}$ . In particular,

$$T_{11}(z) = \sum_{k=0}^N a_k z^{-k} \quad T_{21}(z) = \sum_{k=0}^N b_k z^{-k}. \quad (7)$$

$T_{21} = S_{11}/S_{21}$  is the characteristic polynomial of the filter, which efficiently describes the frequency response both in the passband (where  $T_{21} \sim S_{11}$ ) and in the stopband (where  $T_{21} \sim 1/S_{21}$ ). It can be interpreted as the array factor of a linear distribution of radiators or, alternatively, as the response of a digital filter. According to the first interpretation, the filter reflection coefficient in the passband corresponds to the level of the secondary lobes of the array factor, whereas the maximum insertion loss in the stopband corresponds to the main lobe level. According to the second interpretation,  $T_{21}$  can be identified as the  $Z$ -transform of the impulse response of a finite-impulse response (FIR) digital filter. In both cases, well-established synthesis techniques can be used to obtain the desired array factor [21] or the FIR transfer function [22]. Once the polynomial  $T_{21}$  has been defined according to the required specifications, an extraction procedure is applied to determine the scattering matrix of the various junctions.

In order to carry out the extraction procedure, it is necessary to know also the element  $T_{11} = 1/S_{21}$  of the transmission matrix of the filter. For this purpose, one recalls that, in the case of a reciprocal and lossless structure, the difference between the squared magnitudes of  $T_{11}$  and  $T_{21}$  is one for  $|z| = 1$ , i.e., for real values of frequency

$$|T_{11}(z)|^2 = 1 + |T_{21}(z)|^2 \quad \forall |z| = 1. \quad (8)$$

This relationship can be analytically continued in the whole complex  $z$  plane by noting that  $z^* = z^{-1}$  on the circle  $|z| = 1$ . Hence, one obtains

$$T_{11}(z) T_{11}^*(1/z^*) = 1 + T_{21}(z) T_{21}^*(1/z^*) \quad \forall z. \quad (9)$$

On the basis of this equation,  $T_{11}(z)$  can be determined from a specified  $T_{21}(z)$ . It is, in fact, easy to recognize that the  $2N$  roots of the left-hand side of (9) occur in pairs  $\{\alpha_k, 1/\alpha_k^*\}$ , where  $\alpha_k$  are the  $N$  roots of the polynomial  $T_{11}$ . The identification of the  $\alpha_k$  is very simple noting that they are the poles of  $S_{21}$ , ( $T_{11} = 1/S_{21}$ ) and, hence, lie inside the circle  $|z| = 1$  because of the stability condition. The magnitude  $|a_N|$  of the  $N$ th degree coefficient can be determined by evaluating (9) at an arbitrary point, e.g., at  $z = -1$ . As for its phase, it must be equal to that of the coefficient  $b_N$  of the polynomial  $T_{21}$  with the same degree, as explained below.

In general, the determination of the  $2N$  roots of (9) is performed through numerical algorithms and the precision of the computation must be increased as the magnitude range of the polynomial  $T_{21}(z)$  on the circle  $|z| = 1$  increases. However, in the special cases of Butterworth- and Chebyshev-type responses, the roots  $\alpha_k$  can be expressed in closed form. For a Butterworth-type response, where the reflection zeros (roots of  $T_{21}$ ) are all coincident at  $z = -1$ , the roots of  $T_{11}$  are

$$\alpha_k = \left( w_k + j\sqrt{1-w_k^2} \right)^2 \quad (10)$$

where

$$w_k = \sin\left(\frac{\Delta\theta_B}{4}\right) \exp\left\{j\left[\frac{(2k+1)}{2N}\pi\right]\right\}, \quad \text{with } k = 0, \dots, N-1. \quad (11)$$

Here,  $\Delta\theta_B$  is the  $-3$ -dB electrical bandwidth in radians.

In the case of a Chebyshev-type response, the reflection zeros are given by (10), where now

$$w_k = \sin\left(\frac{\Delta\theta_B}{4}\right) \cos\left[\frac{(2k+1)\pi}{2N}\right], \quad \text{with } k = 0, \dots, N-1 \quad (12)$$

and  $\Delta\theta_B$  is the Chebyshev electrical bandwidth. The corresponding filter poles are still defined by (10), but with

$$w_k = \sin\left(\frac{\Delta\theta_B}{4}\right) \cos\left\{\frac{(2k+1)\pi}{2N} - j\frac{1}{N} \cdot \ln\left[\frac{1+\sqrt{1-R^2}}{R}\right]\right\}, \quad \text{with } k = 0, \dots, N-1 \quad (13)$$

where  $R$  is the maximum amplitude of the reflection coefficient in the passband.

It is to be noted that, in order to compensate for the degradation effects produced by the real behavior of the structure, one has to synthesize  $T_{21}$  polynomials, which do not belong to the previous special cases. Hence, the numerical solution of (9) becomes necessary in most practical cases, even if a Chebyshev-like response is required.

Once the polynomials  $T_{11}^{[N]}$  and  $T_{21}^{[N]}$  are determined (the superscript  $[N]$  has been added to emphasize that they refer to the whole structure consisting of  $N$  cavities), it is possible to start the extraction procedure, which leads to the determination of the scattering parameters of the  $N+1$  discontinuities. From (6), one can write the following relationship:

$$\begin{bmatrix} T_{11}^{[N]} \\ T_{21}^{[N]} \end{bmatrix} = \begin{bmatrix} \csc\gamma_N & \cot\gamma_N z^{-1} e^{-j\psi_N} \\ \cot\gamma_N & \csc\gamma_N z^{-1} e^{-j\psi_N} \end{bmatrix} \begin{bmatrix} T_{11}^{[N-1]} \\ T_{21}^{[N-1]} \end{bmatrix} \quad (14)$$

where the polynomials  $T_{11}^{[N-1]}$  and  $T_{21}^{[N-1]}$  correspond to the structure consisting of the first  $N-1$  cavities. By solving the linear system (14), the following relationships are obtained among the coefficients of the four polynomials involved:

$$\begin{aligned} a_k^{[N-1]} &= \csc\gamma_N a_k^{[N]} - \cot\gamma_N b_k^{[N]}, \quad \text{with } k = 0, \dots, N \\ b_k^{[N-1]} &= \left[-\cot\gamma_N a_{k+1}^{[N]} + \csc\gamma_N b_{k+1}^{[N]}\right] e^{j\psi_N}, \\ &\quad \text{with } k = -1, \dots, N-1. \end{aligned} \quad (15)$$

Since the polynomials  $T_{11}^{[N-1]}$  and  $T_{21}^{[N-1]}$  are of degree  $N-1$  in the variable  $z^{-1}$ , the coefficients  $a_N^{[N-1]}$  and  $b_{-1}^{[N-1]}$  must be zero. By enforcing these conditions, two relationships are obtained, both giving the magnitude of  $S_{11}$  for the  $N$ th discontinuity

$$|S_{11}^{[N]}| = \cos\gamma_N = \frac{a_N^{[N]}}{b_N^{[N]}} = \frac{b_0^{[N]}}{a_0^{[N]}}. \quad (16)$$

It is easy to prove that (9) implies the following relationship:

$$\frac{a_N^{[N]}}{b_N^{[N]}} = \left(\frac{b_0^{[N]}}{a_0^{[N]}}\right)^*. \quad (17)$$

Since the ratio (17) must be real and positive, the phase of  $a_N^{[N]}$  must be chosen equal to that of  $b_N^{[N]}$ , as previously stated. Under this condition and because of (17), the second ratio of (16) always coincides with the first one.

The procedure described can be repeated to obtain the  $S_{11}$ -parameter of the  $(N-1)$ th discontinuity as follows:

$$\begin{aligned} |S_{11}^{[N-1]}| &= \cos\gamma_{N-1} \\ &= \frac{b_0^{[N-1]}}{a_0^{[N-1]}} \\ &= \frac{-\cos\gamma_N a_1^{[N]} + b_1^{[N]}}{a_0^{[N]} - \cos\gamma_N b_0^{[N]}} e^{j\psi_N}. \end{aligned} \quad (18)$$

From this expression, the role played by the phase term  $\psi_N$  introduced in (5) appears clearly. Its value must in fact be chosen so that the ratio (18) is real and positive. By iterating the previous steps, all the parameters  $\gamma_k \in [0, \pi/2]$ ,  $(k = 0, \dots, N)$  and  $\psi_k$  ( $k = 1, \dots, N$ ) can be determined. Formally, the extraction scheme is as follows:

$$p = N, \dots, 1$$

$$|S_{11}^{[p]}| = \cos\gamma_p = \frac{b_0^{[p]}}{a_0^{[p]}}$$

$$\psi_p = \mathcal{ARG} \left\{ \frac{a_0^{[p]} - \cos\gamma_p b_0^{[p]}}{-\cos\gamma_p a_1^{[p]} + b_1^{[p]}} \right\}$$

$$k = 0, \dots, p-1$$

$$a_k^{[p-1]} = \csc\gamma_p a_k^{[p]} - \cot\gamma_p b_k^{[p]}$$

$$b_k^{[p-1]} = \left[-\cot\gamma_p a_{k+1}^{[p]} + \csc\gamma_p b_{k+1}^{[p]}\right] e^{j\psi_p}$$

$$p = 0$$

$$|S_{11}^{[0]}| = \cos\gamma_0 = \frac{b_0^{[0]}}{a_0^{[0]}}$$

As in all extraction procedures (see, e.g., [23]), particular attention must be paid to the numerical implementation of the algorithm. If the number of cavities increases and the bandwidth decreases so that the range of the polynomial  $T_{21}$  on the circle  $|z| = 1$  becomes large, roundoff errors can greatly affect the extraction procedure, especially in the case of the last cells. In these cases, the use of multiple precision software may be necessary.

In order to show the behavior of the  $S$ -parameters of the various junctions as a function of the filter specifications, this synthesis procedure has been applied to a five- and seven-pole Chebyshev-type response. Fig. 2(a) and (b) show the amplitude of the coefficients  $S_{21}^{(k)}$  of the various discontinuities versus the relative electrical bandwidth  $\Delta\theta_B/\pi$  and for two levels of the return loss in the passband: 20 dB (solid lines) and 30 dB (dashed

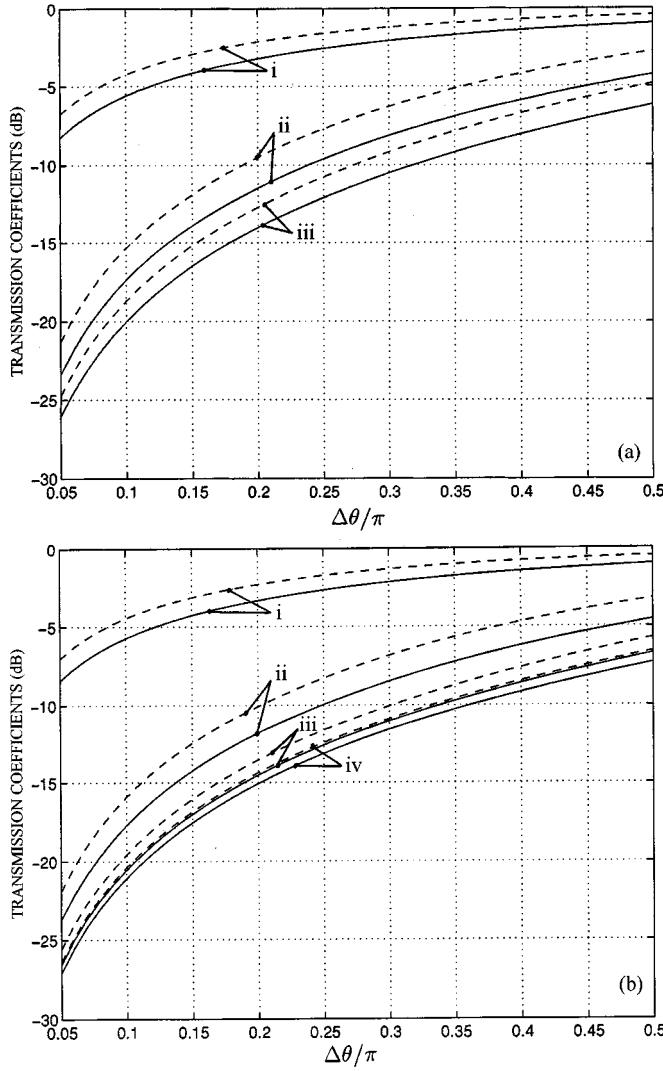


Fig. 2. Amplitude of the transmission coefficients  $S_{21}^{(k)}$  of the discontinuities in the case of a Chebyshev filter as a function of the relative bandwidth. Return loss 20 dB: solid line. Return loss 30 dB: dashed line. (a) Five-pole filter: i)  $S_{21}^{(1)} = S_{21}^{(6)}$ , ii)  $S_{21}^{(2)} = S_{21}^{(5)}$ , and iii)  $S_{21}^{(3)} = S_{21}^{(4)}$ . (b) Seven-pole filter: i)  $S_{21}^{(1)} = S_{21}^{(8)}$ , ii)  $S_{21}^{(2)} = S_{21}^{(7)}$ , iii)  $S_{21}^{(3)} = S_{21}^{(6)}$ , and iv)  $S_{21}^{(4)} = S_{21}^{(5)}$ .

lines). As is to be expected, the transmission coefficients increase with the bandwidth and the return loss, and the largest are those of the input/output junctions. Note that in the case of a Chebyshev-type response, the phase terms  $\psi_k$  are zero and the filter is symmetric.

In the case of lossy structures, where the correspondence between the amplitudes of  $S_{11}$  and  $S_{21}$  does not hold, it is necessary to refer to the quantity  $\sin \gamma_k = |S_{21}^{(k)}|$  rather than to  $\cos \gamma_k = |S_{11}^{(k)}|$ . It will be shown in the following section that the synthesis procedure also maintains its validity in this case.

As far as the computation of the length of the cavities and the correspondence between electrical and angular bandwidth is concerned, it is necessary to refer to the expression of the phase of the complex variable defined in (5). In particular, referring to the function

$$\theta_k = \pi + 2\beta_k(\omega)l_k - 2\phi_k(\omega) - \psi_k \quad (19)$$

where, for brevity, the following variable has been introduced:

$$\phi_k(\omega) = \frac{1}{2} [\phi_{11}^{(k-1)}(\omega) + \phi_{22}^k(\omega)] \quad (20)$$

the phase of the complex variable  $z$  is defined from (19) as

$$\theta = \pi + 2\beta(\omega)l - 2\phi(\omega) - \psi \quad (21)$$

where  $\beta(\omega)$ ,  $l$ ,  $\phi(\omega)$ , and  $\psi$  are the mean values on the index  $k$  of the corresponding quantities  $\beta_k(\omega)$ ,  $l_k$ ,  $\phi_k(\omega)$ , and  $\psi_k$ , respectively. In particular, the value of  $l$  is obtained by summing the two values of expression (21) evaluated at  $\omega_2$  and  $\omega_1$ , where  $\omega_1$  and  $\omega_2$  are the angular frequencies of the two limits of the passband, corresponding to the angle  $\theta_1 = \pi - \Delta\theta_B/2$  and  $\theta_2 = \pi + \Delta\theta_B/2$ , respectively,

$$l = \frac{\psi + \phi(\omega_2) + \phi(\omega_1)}{\beta(\omega_2) + \beta(\omega_1)}. \quad (22)$$

As for the length of the  $k$ th resonator, by equating (19)–(21), one has

$$l_k = \frac{\bar{\beta}l + \frac{1}{2}\psi_k - \frac{1}{2}\psi + \bar{\phi}_k + \bar{\phi}}{\bar{\beta}_k} \quad (23)$$

where the quantities  $\bar{\beta}$ ,  $\bar{\phi}$ ,  $\bar{\beta}_k$ , and  $\bar{\phi}_k$  are the mean values of the corresponding functions  $\beta(\omega)$ ,  $\phi(\omega)$ ,  $\beta_k(\omega)$ , and  $\phi_k(\omega)$ , respectively, evaluated on the frequency interval  $[\omega_1, \omega_2]$ . Finally, the angular bandwidth  $\Delta\theta_B$ , necessary to define the polynomial  $T_{21}(z)$  that must be synthesized, is obtained by subtracting the two values of (21) evaluated at  $\omega_2$  and  $\omega_1$

$$\Delta\theta_B = 2[\beta(\omega_2) - \beta(\omega_1)]l - 2[\phi(\omega_2) - \phi(\omega_1)]. \quad (24)$$

From this expression, it is clear that, for the definition of the angular bandwidth  $\Delta\theta_B$ , the two values  $\phi(\omega_1)$  and  $\phi(\omega_2)$  must be estimated at the beginning of the synthesis procedure.

### III. COMPENSATION PROCEDURE

In some cases, the frequency dispersion of the  $S$ -parameters of the junctions, the multimodal interaction between them, the different frequency behavior of the resonators, the losses, and the presence of other filters in the case of multiplexer configurations can produce degradation effects in the frequency response of the filter such that the specifications are not completely satisfied. In order to compensate for these effects optimization techniques are generally applied. In this paper, an alternative procedure, based on linear system concepts, is proposed. Consider the process consisting of the following steps: the application of the extraction procedure presented above, the definition of the filter geometry, and the final full-wave analysis. At least in the band of interest, this process can be viewed as a linear system where the input signal is the desired  $T_{21}$  curve and the output is the  $T_{21}$  obtained by the full-wave analysis [see Fig. 3(a)]. On the basis of this point-of-view, a “predistorted” polynomial to be synthesized is obtained, so that the full-wave analysis applied to the new filter geometry satisfies the desired frequency response [see Fig. 3(b)]. To illustrate this concept, let us refer to a simple example of an equiripple four-pole  $E$ -plane metal-insert filter in WR28 rectangular waveguide with central frequency 30

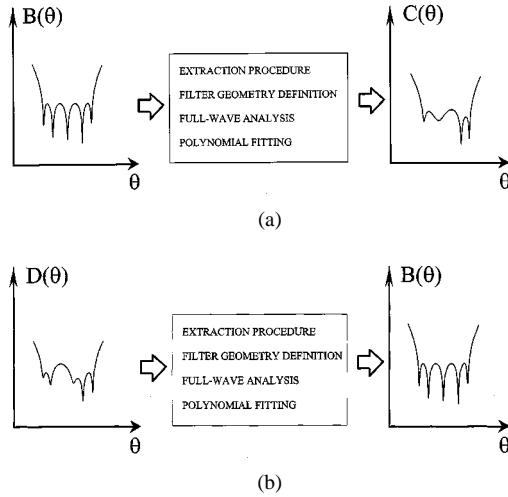


Fig. 3. Linear system interpretation of the synthesis-analysis process. (a) Identification of the linear system. (b) Predistortion of the polynomial to be synthesized to obtain the desired frequency response.

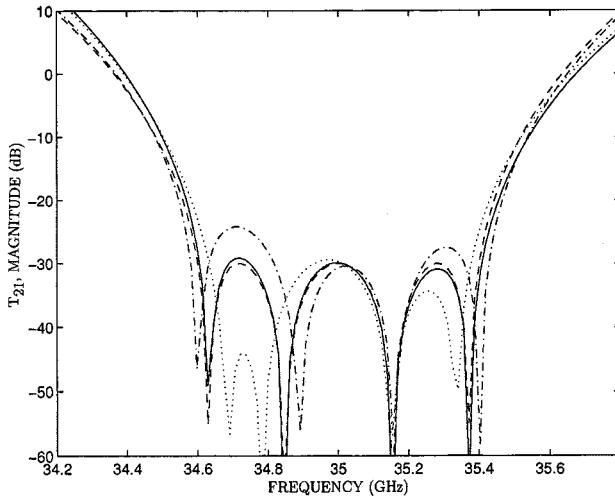


Fig. 4. Design example of a Chebyshev four-pole filter.  $T_{21} = S_{11}/S_{21}$  versus frequency corresponding to the various steps of the design. Chebyshev polynomial (dashed line), full-wave analysis: 1° step (dotted line), new polynomial to be synthesized (dash-dotted line), full-wave analysis: 2° step (solid line).

GHz, bandwidth 800 MHz, return loss 30 dB, septum thickness 0.5 mm. As far as this filter configuration is concerned, a number of papers have been published (see, e.g., [24]–[27]). Fig. 4 shows several plots of the parameter  $T_{21}$  corresponding to the various steps of this technique. In particular, the dashed curve is the Chebyshev polynomial

$$B(z) = \sum_{k=0}^N b_k z^{-k} \quad (25)$$

to be synthesized, which satisfies the specifications. The dotted curve shows  $T_{21} = S_{11}/S_{21}$  computed by the full-wave analysis applied to the configuration obtained by the procedure previously presented. It is to be noted that this curve is close to the desired one, even if it does not satisfy the bandwidth requirement and equiripple conditions. This confirms the validity of the extraction procedure proposed in this paper. The parameter

$T_{21} = S_{11}/S_{21}$  obtained by the full-wave analysis is, of course, not a polynomial, but can be fitted, at least in the band of interest, with a polynomial with the same degree  $N$  as the goal  $B(z)$

$$\frac{S_{11}}{S_{21}} \simeq C(z) = \sum_{k=0}^N c_k z^{-k}. \quad (26)$$

Now the Chebyshev polynomial  $B(z)$  and the fitting polynomial  $C(z)$  are interpreted as the input and output signals of the linear system of Fig. 3(a). Furthermore, it is worthwhile to observe that the sequences of the polynomial coefficients  $\{b_k\}$  and  $\{c_k\}$ , given in (25) and (26), can be interpreted as the discrete spectra of the periodic signals  $B(\theta)$  and  $C(\theta)$ , respectively. As a consequence, one can write

$$h_k = c_k/b_k, \quad \text{with } k = 0, \dots, N \quad (27)$$

where the sequence  $\{h_k\}$  is the transfer function of the linear system of Fig. 3 evaluated on the spectrum of the input and output signals.

It is now possible to define a new input polynomial  $D(z)$  (dash-dotted curve of Fig. 4 with coefficients that, in view of (27), are given by

$$d_k = b_k^2/c_k, \quad \text{with } k = 0, \dots, N. \quad (28)$$

These coefficients define the input signal that must be synthesized in order to obtain  $B(z)$  as output signal [see Fig. 3(b)].

As it appears from Fig. 4, this new polynomial can be viewed as a predistorted input signal to compensate for the multimodal interaction between the septa and frequency dispersion of their  $S$ -parameters. Now, the extraction procedure is carried out again, starting from this new input polynomial  $D(z)$ . The new  $T_{21}$ , computed by a full-wave analysis of the configuration obtained directly by the synthesis procedure, satisfies the specifications, as shown by the solid line in Fig. 4. In general, the zeros of  $D(z)$  can be placed anywhere in the complex  $z$ -plane. Hence, the possibility of synthesizing an arbitrary frequency response assumes great importance. Thanks to this peculiarity, if necessary, the whole process can be iterated. For example, this happens in the case of a multiplexer configuration where each filter must be designed taking into account the load produced by the other filters.

#### IV. RESULTS

The method presented in the previous section has been used to design several filters. In the following, an example of realization of an equiripple eight-pole  $E$ -plane metal insert filter in a brass WR90 waveguide (insert thickness 0.52 mm) is described. This kind of material was selected to emphasize the degradations caused by losses. The bandwidth of 800 MHz is centered at 11 GHz, with a return loss of 25 dB. Starting from a Chebyshev polynomial to represent  $T_{21}$  (see dashed curve of Fig. 5), the synthesis procedure yielded the following data:  $\gamma_k = \{50.7778^\circ, 19.9927^\circ, 13.8806^\circ, 12.7208^\circ, 12.4859^\circ\}$ , with  $k = 0, \dots, 4$ ;  $\gamma_k = \gamma_{9-k}$  with  $k = 5, \dots, 9$  and  $\psi_k = 0$  with  $k = 1, \dots, 8$ . The full-wave analysis of the structure selected by this synthesis step and the linear system identification

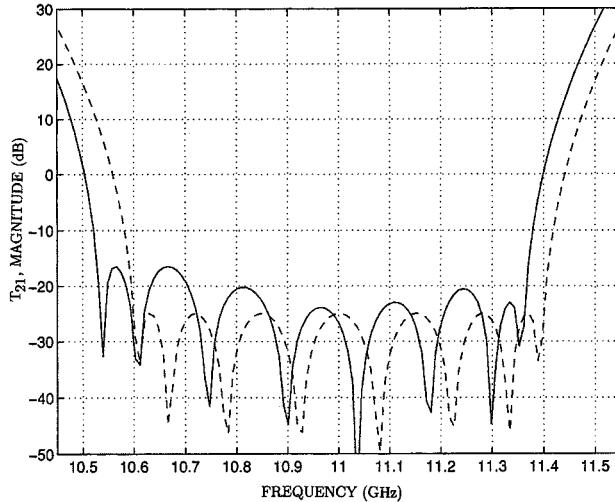


Fig. 5. Design of an eight-pole  $E$ -plane metal-insert WR90 waveguide filter.  $T_{21}$  polynomials to be synthesized. Chebyshev polynomial (dashed line) and predistorted polynomial (solid line).

of the whole process allowed us to define the predistorted polynomial  $T_{21}$  that had to be synthesized (Fig. 5, solid curve). Note that this polynomial is significantly different from the nominal one. The central frequency is lowered, the bandwidth is slightly reduced, and the passband reflection coefficient has reached  $-17$  dB.

The corresponding new data set obtained by this new synthesis step was  $\gamma_k = \{48.6337^\circ, 19.1474^\circ, 14.0873^\circ, 13.0088^\circ, 12.7972^\circ\}$ , with  $k = 0, \dots, 4$ ,  $\gamma_k = \gamma_{9-k}$  with  $k = 5, 9$  and  $\psi_k = \{1.6558^\circ, 2.2345^\circ, 2.4179^\circ, 2.4580^\circ\}$ , with  $k = 1, \dots, 4$ ;  $\psi_k = \psi_{9-k}$  with  $k = 5, \dots, 8$ . One should note that this synthesis step yielded nonzero values of  $\psi_k$  to compensate for the different behavior of the phase of the reflection coefficients  $S_{11}^{(k)}$  of the septum discontinuities. The geometry of the final configuration is as follows: septum lengths  $s_1 = s_9 = 1.083$  mm,  $s_2 = s_8 = 6.609$  mm,  $s_3 = s_7 = 8.514$  mm,  $s_4 = s_6 = 9.008$  mm, and  $s_5 = 9.110$  mm, and resonator lengths  $l_1 = l_8 = 11.515$  mm,  $l_2 = l_7 = 11.529$  mm, and  $l_3 = l_4 = l_5 = l_6 = 11.527$  mm. Fig. 6 shows the full-wave analysis for this configuration (solid line) and the measurements (dashed line) of the prototype. The excellent agreement between the measured and predicted results confirms the validity of the whole design procedure. The analysis is carried out by applying the method of moments with weighted Gegenbauer polynomials as basis functions to represent the aperture field distribution with the right edge conditions. The projections of the basis functions onto the modal sets is carried out in the spectral domain by exploiting the analytical knowledge of the Fourier transform of the basis functions. The material losses are taken into account by introducing the relevant impedance boundary conditions in the moment-method application. In this way, the loss phenomena are accurately modeled. In particular, the extra losses produced by the reactive fields excited by the discontinuities are accounted for. It should be noted that, even though the synthesis procedure is based on the hypothesis of lossless devices, the linear system identification

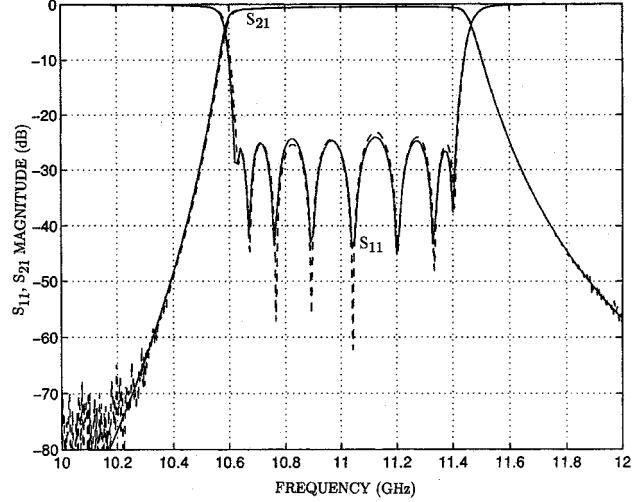


Fig. 6. Eight-pole  $E$ -plane metal-insert WR90 waveguide filter designed by the synthesis procedure described in this paper. Measurements of the prototype made of brass (dashed line). Full-wave analysis (solid line). The geometry is reported in this paper.

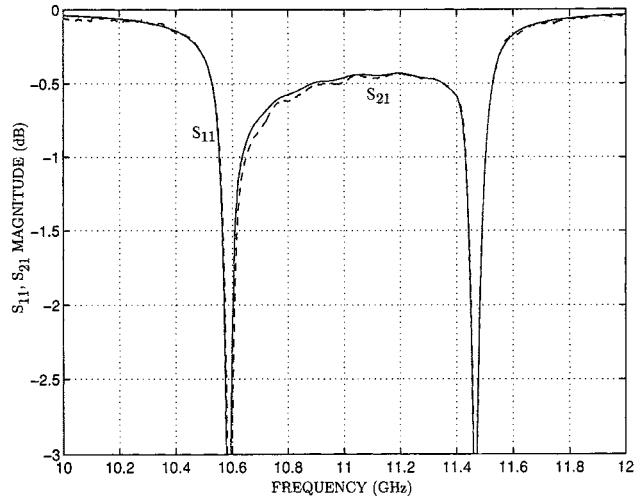


Fig. 7. Detail of the frequency response of Fig. 6. Measured (dashed line) and prediction (solid line). The geometry is reported in this paper.

process allows one to also compensate for the degradation produced by the losses. Fig. 7 shows in expanded scale the comparison between predicted and measured data of the frequency response of Fig. 6. The excellent agreement proves the accuracy of our design/analysis tool, also for the prediction of the insertion losses.

## V. CONCLUSIONS

The synthesis technique presented in this paper is based on a scattering description of the various discontinuities that constitute the filter. The generality of the method allows one to synthesize arbitrary frequency responses. This fact is of substantial importance because the use of a numerical optimization process for the determination of the final configuration can be avoided. The key point is the capability of synthesizing a predistorted frequency response in order to compensate for the effects due to multimodal interactions,

frequency dispersions, and losses. The comparison between measurements and predicted results confirms the validity of the approach also in the presence of losses. This method can be applied as well to the design of multiplexers. In this case, the filter–multiplexer junction and the loads produced by the other filters are included in the scattering matrix that describes the input/output discontinuity of the filter.

## REFERENCES

- [1] C. Kudsia, R. Cameron, and W.-C. Tang, "Innovations in microwave filters and multiplexing networks for communication satellite systems," *IEEE Trans. Microwave Theory Tech.*, vol. 40, pp. 1133–1149, June 1992.
- [2] S. B. Cohn, "Direct-coupled-resonator filters," *Proc. IRE*, vol. 45, pp. 187–196, Feb. 1957.
- [3] L. Young, "Direct-coupled cavity filters for wide and narrow bandwidths," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-11, pp. 162–178, May 1963.
- [4] G. L. Matthaei, L. Young, and E. M. T. Jones, *Microwave Filters, Impedance Matching Networks, and Coupling Structures*. New York: McGraw-Hill, 1964.
- [5] R. Levy, "Theory of direct-coupled-cavity filters," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-15, pp. 340–348, June 1967.
- [6] A. E. Atia and A. E. Williams, "Narrow-bandpass waveguide filters," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-20, pp. 258–265, Apr. 1972.
- [7] J. D. Rhodes, *Theory of Electrical Filters*. New York: Wiley, 1976.
- [8] R. J. Cameron, "General prototype network-synthesis methods for microwave filters," *ESA J.*, vol. 6, no. 2, pp. 193–206, 1982.
- [9] *IEEE Trans. Microwave Theory Tech.*, vol. MTT-30, Sept. 1982.
- [10] *IEEE Trans. Microwave Theory Tech.*, vol. 30, July 1994.
- [11] A. Abdelmonem, J.-F. Liang, H.-W. Yao, and K. A. Zaki, "Full-wave design of spurious free D.R. TE mode band pass filters," *IEEE Trans. Microwave Theory Tech.*, vol. 43, pp. 744–752, Apr. 1995.
- [12] R. Levy, "Direct synthesis of cascaded quadruplet (CQ) filters," *IEEE Trans. Microwave Theory Tech.*, vol. 43, pp. 2940–2945, Dec. 1995.
- [13] *Int. J. Microwave Millimeter Wave Computer-Aided Eng.*, vol. 7, no. 2/3, Mar./May 1997.
- [14] U. Papziner and F. Arndt, "Field theoretical computer-aided design of rectangular and circular iris coupled rectangular or circular waveguide cavity filters," *IEEE Trans. Microwave Theory Tech.*, vol. 41, pp. 462–471, Mar. 1993.
- [15] M. Guglielmi, "Simple CAD procedure for microwave filters and multiplexers," *IEEE Trans. Microwave Theory Tech.*, vol. 42, pp. 1347–1352, July 1994.
- [16] F. Alessandri, M. Dionigi, and R. Sorrentino, "A full-wave CAD tool for waveguide components using a high speed direct optimizer," *IEEE Trans. Microwave Theory Tech.*, vol. 43, pp. 2046–2052, Sept. 1995.
- [17] F. Arndt, Th. Sieverding, T. Wolf, and U. Papziner, "Optimization-oriented design of rectangular and circular waveguide components with the use of efficient mode-matching simulators in commercial circuit cad tools," *Int. J. Microwave Millimeter Wave Computer-Aided Design*, vol. 7, no. 1, pp. 37–51, Jan. 1997.
- [18] F. Alessandri, M. Mongiardo, and R. Sorrentino, "New efficient full wave optimization of microwave circuits by the adjoint network method," *IEEE Microwave Guided Waves Lett.*, vol. 3, pp. 414–416, Nov. 1993.
- [19] J. W. Bandler, R. M. Biernacki, S. H. Chen, P. A. Grobelny, and R. H. Hemmers, "Space mapping technique for electromagnetic optimization," *IEEE Trans. Microwave Theory Tech.*, vol. 42, pp. 2536–2544, Dec. 1994.
- [20] R. Tascone, P. Savi, D. Trinchero, and R. Orta, "A procedure for the design of microwave filters based on a distributed parameter model," in *IEEE MTT-S Int. Microwave Symp. Dig.*, June 8–13, 1997, pp. 643–646.
- [21] R. S. Elliot, *Antenna Theory and Design*. Englewood Cliffs, NJ: Prentice-Hall, 1981.
- [22] T. W. Parks and C. S. Burrus, *Digital Filter Design*. New York: Wiley, 1987.
- [23] R. Levy, "Tables of element values for the distributed low-pass prototype filter," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-13, pp. 514–523, Sept. 1965.
- [24] L. Q. Bui, D. Ball, and T. Itoh, "Broad-band millimeter-wave *E*-plane bandpass filters," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-32, pp. 1655–1658, Dec. 1984.
- [25] R. Vahldieck, "Quasi-planar filters for millimeter-wave applications," *IEEE Trans. Microwave Theory Tech.*, vol. 37, pp. 324–334, Feb. 1989.
- [26] J. Bornemann and F. Arndt, "Modal *S*-matrix design of metal finned waveguide components and its application to transformers and filters," *IEEE Trans. Microwave Theory Tech.*, vol. 40, pp. 1528–1537, July 1992.
- [27] D. Budimir, "Optimized *E*-plane bandpass filters with improved stop-band performance," *IEEE Trans. Microwave Theory Tech.*, vol. 45, pp. 212–220, Feb. 1997.



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