# Commutation Quality Factor of Two-State Switchable Devices

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***Abstract*—An unified way to characterize a two-state one-port switchable network is discussed in this paper. A figure-of-merit of the two-state one-port, called the commutation quality factor (CQF), is introduced. It can be applied to different types of switching devices (semiconductor, ferroelectric, superconductor, etc.) and used for a design of two-state components with optimal characteristics. The CQF is defined as the ratio of the input impedances of a lossless reciprocal two-port terminated in the impedance pair $Z_1$ and $Z_2$, provided the imaginary parts of both input impedances are zero. A simple formula is derived for a calculation of the CQF. The invariance of the CQF with respect to lossless reciprocal transformation is shown. The applicability of the CQF does not depend on the physical nature of the device. The CQF is a working tool for a selection of switching devices while designing electronically controlled microwave components and subsystems. The CQF is recommended to be used for determining the available minimum of insertion loss of a switching microwave component. Thus, the introduced CQF can be used for optimization of the switching microwave component designed.**

***Index Terms*—Microwave phase shifter, microwave switch, quality factor, switchable devices.**

## I. Introduction

In many microwave systems, two-state one-ports are widely used in the design of controllable components such as digital phase shifters [1], switches [2], modulators, and switchable multiplexers [3], [4]. The commonly used p-i-n diode switch is characterized by two quantized states and, consequently, may be considered as a two-state one-port [1]. The two quantized states can as well be realized by a FET switch [5], and by a superconducting S–N switch based on the transition between superconducting (S) and normal (N) states in a thin superconducting film bridge [6]–[8]. The semiconductor varactor diode [9] and the ferroelectric planar capacitor [10]–[12] are continuously tunable, but both of them can be characterized by the properties in two different states and, in this respect, may be considered as two-state one-ports. We shall use the term "switching device" as a synonym of the two-state one-port.

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Throughout three decades, several attempts to define a numerical figure-of-merit for a two-state switching device have been undertaken. In an early fundamental investigation [13], [14], such a useful definition was presented for semiconductor p-i-n diodes. Unfortunately, this definition was presented in a mathematical style of classical circuit theory and has not been accepted by the microwave community. Twenty-five years ago, a figure-of-merit of tunable ferroelectric devices was formulated [15], [16]. The recent attempts of finding a reasonable characterization of the ferroelectric devices are given in [17] and [18].

The goal of this paper is to define a way of characterizing a two-state one-port network, which can be applied to any switching device and used for optimizing switching circuits. Below, we distinguish between a switching device and a switching circuit component, which includes the switching device.

In this paper, we focus on the following subjects.

1) General design considerations that concern switchable components such as phase shifters, modulators, switches, etc.
2) A discussion about a figure-of-merit of the switchable components. For digital phase shifters, it concerns the ratio between the phase shift and the average insertion loss measured in *degrees/decibels*: $(\varphi_1 - \varphi_2)/[0.5(L_1 + L_2)]$ and for a single-pole single-through (SPST) switch, it concerns the insertion and return losses in the two states.
3) Introduction of a figure-of-merit for the switching device is useful for optimizing the switchable circuit components. We suggest to call this figure-of-merit the commutation quality factor (CQF).

Using the principles partially formulated earlier [13], [14], [19], it will be shown that the CQF of any switching device as defined below is invariant with respect to a lossless reciprocal impedance transformation. The CQF does not depend on the physical nature of the two-state one-port. The figure-of-merit of the microwave component (phase shifter, etc.) is a distinct function of the CQF. Knowing the CQF together with other parameters of the switching device allows optimizing the design of microwave system components.

## II. Formulation of the Problem

Let us consider a one-port switchable device, which can be in two states characterized by the impedance pair

$$Z_1 = R_1 + jX_1 \text{ and } Z_2 = R_2 + jX_2. \tag{1}$$

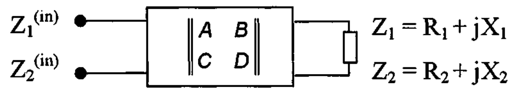


Fig. 1. Reciprocal lossless two-port described by $ABCD$ matrix transforming the $Z_1$, $Z_2$ impedance pair into $Z_{\text{in}}^{(1)}$, $Z_{\text{in}}^{(2)}$ impedance pair.

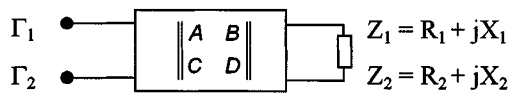


Fig. 2. Reflection-type 180° phase shifter based on switching the device impedance pair between $Z_1$ and $Z_2$.

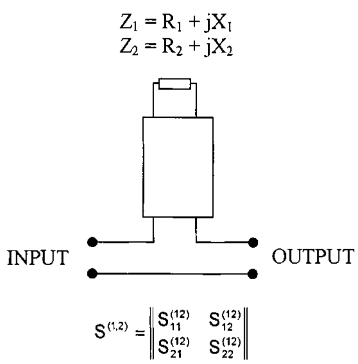


Fig. 3. SPST switch based on switching the device impedance pair between $Z_1$ and $Z_2$.

Using the impedance's $Z_{1,2}$ as loads of a reciprocal lossless two-port network (Fig. 1), one obtains the input impedance of the two-port

$$Z_{\text{in}}^{(1)} = R_{\text{in}}^{(1)} + jX_{\text{in}}^{(1)} \text{ and } Z_{\text{in}}^{(2)} = R_{\text{in}}^{(2)} + jX_{\text{in}}^{(2)}. \tag{2}$$

The transformed impedance pair should be changed in a way that provides a maximum influence on the microwave circuit containing the transforming lossless two-port and the two-state switchable device.

In order to determine the requirements on $Z_{\text{in}}^{(1)}$ and $Z_{\text{in}}^{(2)}$, let us consider suitable two-ports used in either a reflection-type circuit (e.g., a 180° phase shifter, Fig. 2) or in a transmission-type circuit (SPST, Fig. 3).

For the reflection-type phase-shifter circuit (Fig. 2), we want to obtain two different reflection coefficients equal in magnitude and 180° phase shifted ($\Gamma_1 = -\Gamma_2$). The simplest way to design such a phase shifter is to assume

$$X_{\text{in}}^{(1)} = X_{\text{in}}^{(2)} = 0 \tag{3}$$

and

$$R_{\text{in}}^{(1)} \ll Z_0 \text{ and } R_{\text{in}}^{(2)} \gg Z_0. \tag{4}$$

The second circuit (Fig. 3) is an SPST switch, which should provide a maximum large $|S_{12}|$ and $|S_{11}| \ll 1$ in one state, and a maximum large $|S_{11}|$ and $|S_{12}| \ll 1$ in the other state. Such characteristics of the SPST switch can be realized if conditions (3) and (4) are fulfilled.

The constraints (3) are always possible to realize in practice. However, it will be shown below that the conditions (3) are not the only possible conditions for a synthesis of the transforming two-port, but they simplify considerably the analysis procedure. Using other possible constraints will not change the fundamental characteristics of the switchable circuits considered.

### *A. Definition of the CQF of a Switchable Device*

When the transformer meets the conditions (3), the impedance pair $Z_{\text{in}}^{(1)}$ and $Z_{\text{in}}^{(2)}$ turns into a pair of resistances $R_{\text{in}}^{(1)}$ and $R_{\text{in}}^{(2)}$. The larger is the change of all components of $Z_1$ and $Z_2$; the larger will be the ratio of the resistances $R_{\text{in}}^{(1)}$ and $R_{\text{in}}^{(2)}$. We suggest the ratio of the resistances $R_{\text{in}}^{(1)}$ and $R_{\text{in}}^{(2)}$ as the CQF ($K$) of the two-state switchable device. Hence,

$$K = R_{\text{in}}^{(2)} / R_{\text{in}}^{(1)} \tag{5}$$

assuming the equalities (3) are fulfilled. Not disturbing the generalization, one may put

$$K > 1. \tag{6}$$

### *B. Reflection-Type Phase Shifter*

Fig. 2 shows a schematic of a reflection-type phase shifter. The reflection coefficients for two states are

$$\Gamma_1 = \frac{R_{\text{in}}^{(1)} - Z_0}{R_{\text{in}}^{(1)} + Z_0}$$
$$\Gamma_2 = \frac{R_{\text{in}}^{(2)} - Z_0}{R_{\text{in}}^{(2)} + Z_0} \tag{7}$$

where $Z_0$ is the characteristic impedance of the transmission line.

Assuming $R_{\text{in}}^{(1)} \ll Z_0 \ll R_{\text{in}}^{(2)}$, one obtains the phases of the reflection coefficient $\varphi_1 = 180°$ and $\varphi_2 = 0°$. Requiring the magnitudes of the reflection coefficients equal $|\Gamma_1| = |\Gamma_2|$ provides equal return losses in both states. This is obtained if

$$Z_0 = \sqrt{R_{\text{in}}^{(1)} R_{\text{in}}^{(2)}}. \tag{8}$$

If the condition (8) is fulfilled, one obtains

$$\left|S_{11}^{(1,2)}\right| = \frac{\sqrt{K} - 1}{\sqrt{K} + 1} \cong 1 - \frac{2}{\sqrt{K}} \tag{9}$$

where $K$ is the CQF as defined by (5).

If the condition (8) is not fulfilled, the magnitudes of the reflection coefficients are not equal: $|S_{11}^{(1)}| \neq |S_{11}^{(2)}|$. The average loss for the two states can be defined as

$$L_{\text{av}} = \left[\frac{1}{2}\left(\left|S_{11}^{(1)}\right|^2 + \left|S_{11}^{(2)}\right|^2\right)\right]^{1/2}. \tag{10}$$

It can be proven that the average loss in the two states is minimum if the condition (8) is fulfilled.

Equation (9) determines the minimum insertion loss of the reflection-type phase shifter. If, in the case of practical realization, one uses the two-state device with the CQF being equal to $K$ and the measured loss of the phase shifter coincides with the value given by (9), one can assert that the phase-shifter characteristic is optimum [6], [8] and the smaller insertion loss cannot be obtained. This statement was generalized for the reflection-type phase shifter for any value of the phase shift $\Delta\varphi$

$$\left|S_{11}^{(1,2)}\right| = 1 - \frac{2}{\sqrt{K}} \cdot \sin\frac{\Delta\varphi}{2}. \tag{11}$$

The same is regarded to transmission-type phase shifters based on switchable devices.

### C. SPST Switch

Fig. 3 shows a schematic of the SPST switch considered. The impedance pair of the switchable device is transformed into the pair of resistances $R_{\text{in}}^{(1)}$ and $R_{\text{in}}^{(2)}$, which are connected in series with the transmission line. The components of the scattering matrix corresponding to the pair of resistances $R_{\text{in}}^{(1)}$ and $R_{\text{in}}^{(2)}$ are

$$\begin{aligned} S_{11}^{(1)} &= \frac{R_{\text{in}}^{(1)}}{R_{\text{in}}^{(1)} + 2Z_0} \\ S_{12}^{(1)} &= \frac{2Z_0}{R_{\text{in}}^{(1)} + 2Z_0} \\ S_{11}^{(2)} &= \frac{R_{\text{in}}^{(2)}}{R_{\text{in}}^{(2)} + 2Z_0} \\ S_{12}^{(2)} &= \frac{2Z_0}{R_{\text{in}}^{(2)} + 2Z_0}. \end{aligned} \tag{12}$$

Let us assume that the insertion loss in the state $Z_1$ and the return loss in the state $Z_2$ are equal. Thus,

$$\left|S_{12}^{(1)}\right| = \left|S_{11}^{(2)}\right| \tag{13}$$

which is obtained if

$$Z_0 = \frac{1}{2}\sqrt{R_{\text{in}}^{(1)} R_{\text{in}}^{(2)}}. \tag{14}$$

We obtain the following formula for the insertion and return loss, respectively:

$$\left|S_{12}^{(1)}\right| = \left|S_{11}^{(2)}\right| = \frac{\sqrt{K}}{1+\sqrt{K}} \cong 1 - \frac{1}{\sqrt{K}}. \tag{15}$$

Likewise the case of the reflection-type phase shifter discussed above, one can say if the experimental performance of an SPST exhibits the results covered by (15), the design of the SPST is optimum.

Thus, we may conclude that the $K$ factor is the governing factor in designing a switchable component. The principle of an estimation of the CQF of a switchable device used for design of switchable components does not depend on physical nature of the switchable device.

## III. Transforming an Impedance Pair ($Z_1$, $Z_2$) into a Resistance Pair ($R_{\text{in}}^{(1)}$, $R_{\text{in}}^{(2)}$)

It is convenient to describe the transforming two-port by the $ABCD$ matrix

$$\|A\| = \left\| \begin{matrix} A & B \\ C & D \end{matrix} \right\|.$$

Let it be

$$\begin{aligned} A &= a \\ B &= jb \\ C &= jc \\ D &= d \end{aligned} \tag{16}$$

where $a$, $b$, $c$, and $d$ are real numbers and $j$ is imaginary unit. The equalities of (16) are valid for the lossless two-port. The reciprocity of the two-port is provided by the following equation:

$$ad + cb = 1. \tag{17}$$

The impedance pair of the switchable device $Z_{1,2} = R_{1,2} + jX_{1,2}$ is transformed into the input impedance's of the two-port

$$Z_{\text{in}}^{(1,2)} = \frac{a(R_{1,2} + iX_{1,2}) + ib}{ic(R_{1,2} + iX_{1,2}) + d}. \tag{18}$$

For the real and imaginary parts of the impedances, one has

$$R_{\text{in}}^{(1,2)} = \frac{R_{1,2}}{d^2} \cdot \frac{1}{1 - 2\frac{c}{d}X_{1,2} + \left(\frac{c}{d}\right)^2 \left(R_{1,2}^2 + X_{1,2}^2\right)} \tag{19}$$

and

$$X_{\text{in}}^{(1,2)} = \frac{1}{d^2} \cdot \frac{bd + (ad - bc)X_{1,2} - ca\left(R_{1,2}^2 + X_{1,2}^2\right)}{1 - 2\frac{c}{d}X_{1,2} + \left(\frac{c}{d}\right)^2 \left(R_{1,2}^2 + X_{1,2}^2\right)}. \tag{20}$$

Assuming $X_{\text{in}}^{(1,2)} = 0$, one obtains the system of equations with respect to the elements of the $ABCD$ matrix

$$\begin{aligned} bd + (ad - bc)X_1 - ca(R_1^2 + X_1^2) &= 0 \\ bd + (ad - bc)X_2 - ca(R_2^2 + X_2^2) &= 0. \end{aligned} \tag{21}$$

From (21), one obtains

$$R_{1,2}^2 + X_{1,2}^2 = \frac{bd + (ad - bc)X_{1,2}}{ca}. \tag{22}$$

Substituting (22) into (19) yields

$$R_{\text{in}}^{(1,2)} = \frac{R_{1,2}}{d} \cdot \frac{a}{1 - \frac{c}{d}X_{1,2}}. \tag{23}$$

On introducing

$$\begin{aligned} \frac{a}{d} &= n^2 \\ \frac{c}{d} &= y \\ \frac{b}{d} &= z \end{aligned} \tag{24}$$

the system of (21) can be rewritten as follows:

$$\begin{aligned} z + (n^2 - yz)X_1 - yn^2\left(R_1^2 + X_1^2\right) &= 0 \\ z + (n^2 - yz)X_2 - yn^2\left(R_2^2 + X_2^2\right) &= 0. \end{aligned} \tag{25}$$

One may see from (23) and (24) that, in the case $X_{\rm in}^1 = X_{\rm in}^2 = 0$, the introduced parameter $n$ is a transformation coefficient, which can be selected to obtain desired values of $R_{\rm in}^{(1,\,2)}$.

If $R_1$, $X_1$, $R_2$, and $X_2$ are known and the transformation coefficient $n$ is selected, (23) and (25) make it possible to find all elements of the $ABCD$ matrix. Now the point is, how to synthesize the two-port network transforming the impedance pair ($Z_1$, $Z_2$) into the pair of resistances ($R_{\rm in}^{(1)}$, $R_{\rm in}^{(2)}$). For a complete formulation of the problem, the frequency dependence should be taken into account.

From (17) and (25), one obtains

$$y^2\Big[\left(R_1^2+X_1^2\right)X_2-\left(R_2^2+X_2^2\right)X_1\Big] -y\Big[\left(R_1^2+X_1^2\right)-\left(R_2^2+X_2^2\right)\Big]+(X_1-X_2)=0. \tag{26}$$

The solution to (26) is a first step in synthesizing the $ABCD$ matrix. It is interesting to note that (26) does not contain the transformation coefficient $n$.

An obvious transformation yields

$$X_1-X_2=\frac{\Big[R_1^2(1-yX_2)-R_2^2(1-yX_1)\Big]\cdot y}{(1-yX_2)\cdot(1-yX_1)}. \tag{27}$$

## IV. Commutation Quality Factor of a Switching Device

From (23) and (24), one obtains

$$R_{\rm in}^{(1,\,2)}=R_{1,\,2}\cdot\frac{n^2}{1-yX_{1,\,2}}. \tag{28}$$

Using the definitions of (5) and (28), we get

$$K=\frac{R_1}{R_2}\cdot\frac{1-yX_2}{1-yX_1} \tag{29}$$

or after a transformation

$$K=\frac{R_1}{R_2}+\frac{R_1}{R_2}\cdot\frac{X_1-X_2}{1-yX_1}\cdot y. \tag{30}$$

Analogously, one may find

$$\frac{1}{K}=\frac{R_2}{R_1}+\frac{R_2}{R_1}\cdot\frac{X_2-X_1}{1-yX_2}\cdot y. \tag{31}$$

The summation of (30) and (31) yields

$$K+\frac{1}{K}=\frac{R_1}{R_2}+\frac{R_2}{R_1}+\frac{\Big[R_1^2(1-yX_2)-R_2^2(1-yX_1)\Big]}{(1-yX_1)\cdot(1-yX_2)}\cdot\frac{X_1-X_2}{R_1R_2}. \tag{32}$$

Equation (32) is symmetrical with respect to $R_{\rm in}^{(1)}$ and $R_{\rm in}^{(2)}$. From the quadratic equation (32), one may select the solution that gives $K > 1$. Substituting (27) into (32) leads to

$$K+\frac{1}{K}=\frac{R_1}{R_2}+\frac{R_2}{R_1}+\frac{(X_2-X_1)^2}{R_1R_2}. \tag{33}$$

Equation (33) is a basic and comprehensive formula for the CQF of a switching device.

## V. Invariance of the CQF of a Switching Device with Respect to a Lossless Reciprocal Impedance Transformation

In the pioneering investigations of a two-state one-port [13], [14], the following figure-of-merit associated with switching properties was introduced:

$$M=\left|\frac{Z_1-Z_2}{Z_1+Z_2^*}\right| \tag{34}$$

where $Z_1$, $Z_2$ is the impedance pair of the two-state one-port (switching device) considered. The asterisk denotes the complex conjugate. From (33), one obtains

$$M^2=\frac{(R_1-R_2)^2+(X_1-X_2)^2}{(R_1+R_2)^2+(X_1-X_2)^2}. \tag{35}$$

Let us consider two impedance pairs ($Z_1$, $Z_1$) and ($Z_1'$, $Z_2'$). The first one is the original pair that was used for obtaining (24) and (25), and the second one is a result of impedance transformation by a reciprocal nondissipative two-port. The two-port is described by an $ABCD$ matrix obeying the equalities (16) and (17). Thus, the pair ($Z_1'$, $Z_2'$) can be obtained by application of (18). Substituting the components ($R_1'$, $R_2'$, $X_1'$, $X_2'$) of the pair ($Z_1'$, $Z_2'$) into (35), one finds that all $a$, $b$, $c$, and $d$ components of the $ABCD$ matrix are canceled. That means that the parameter $M$ is invariant with respect to the lossless reciprocal impedance transformation.

We have the following relation between $M$ and $K$:

$$K+\frac{1}{K}=2\cdot\frac{1+M^2}{1-M^2}. \tag{36}$$

By substituting (35) into (36) and yielding (33), one can check the validity of (36). Solution to the quadratic equation (36) with respect to $K$ yields two roots; one of them complying with (34) is

$$K=\frac{1+M}{1-M}. \tag{37}$$

It was shown above that the parameter $M$ is invariant with respect to the lossless reciprocal impedance transformation. The equation is followed by the conclusion that the parameter $K$ is invariant as well. Thus, the CQF of a switching device in the form of (33) is invariant with respect to any lossless reciprocal impedance transformation.

In order to clarify the statement about invariance of the CQF, let us consider the circuit shown in Fig. 4. Here, $Z_1$, $Z_2$ is the impedance pair of the switching device and $Z_1^{(0)}$, $Z_2^{(0)}$ is the impedance pair obtained after transformation by a transformer $A_1$. The combined transformation produced by two transformers $A_1$ and $A_2$ leads to the resistance pair $R_1^{(\rm in)}$, $R_2^{(\rm in)}$, which have already been considered. One may interchange the transformers $A_1$ and $A_2$, keeping the same input resistance pair. Thus, we may obtain a wide range of impedance pairs $Z_1^{(0)}$, $Z_2^{(0)}$, which are characterized by the same CQF. If both transformers $A_1$ and $A_2$ are lossless and reciprocal, the substitution into (33) of the active and reactive components from any of impedance pairs $Z_1$, $Z_2$, $Z_1^{(0)}$, $Z_2^{(0)}$, and $Z_1^{(\rm in)}$, $Z_2^{(\rm in)}$ yields the same value of $K$. Among these

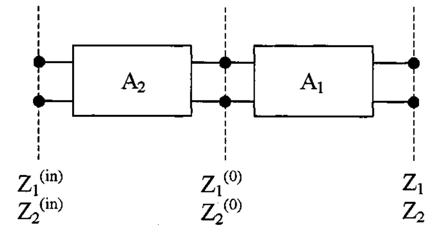


Fig. 4. Impedance transformation realized by two reciprocal lossless transformers as used to discuss the invariance of the CQF of a two-state device.

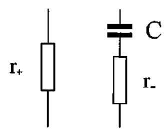


Fig. 5. Equivalent circuit of a p-i-n diode at microwave frequency.

impedance pairs, one may select the pairs with nonzero $X_1^{(0)}$, $X_2^{(0)}$ that removes the constraints of (3).

Any switchable component suitable for practical applications has to have a rather large CQF, i.e., $K > 1000$. For such a case, (33) can be simplified.

a) For the case when the real part of the switching device is changed ($R_2 > R_1,\ X_2 = X_1$)

$$K = \frac{R_2}{R_1}. \tag{38}$$

b) For the case when the imaginary part of the switching device is changed ($R_2 \cong R_1,\ X_2 > X_1$)

$$K = \frac{(X_1 - X_2)^2}{R_1 R_2}. \tag{39}$$

Equation (39) is valid for $K > 200$.

## VI. Commutation Quality Factor of Switching Devices of Different Types

### A. p-i-n Diode

Fig. 5 shows the equivalent circuit of an unpackaged p-i-n diode [1], [19]. Three parameters are used to describe the p-i-n diode: resistance $r_+$ for the on state (forward-biased diode) and resistance $r_-$ and capacitance $C$ in series for the off state (zero-biased diode). Thus, the impedance pair of the p-i-n diode is

$$\begin{aligned} R_1 &= r_+ \\ X_1 &= 0 \\ R_2 &= r_- \\ X_2 &= \frac{1}{\omega C}. \end{aligned} \tag{40}$$

Substituting (40) with $r_+ \cong r_-$ into (39) yields

$$K = \frac{1}{(\omega C)^2 r_+ r_-}. \tag{41}$$

In the case of a packaged p-i-n diode, the equivalent circuit of the package should be taken into consideration [1]. The package can be described as a lossless reciprocal network, and in accordance with Section V does not change the value of $K$ determined by (41).

As a numerical example of the parameters of a p-i-n diode [1], we assume $C = 0.3$ pF, $r_+ = r_- = 0.7\ \Omega$. For the frequency $F = 10$ GHz, using (41), one obtains $K = 5 \cdot 10^3$.

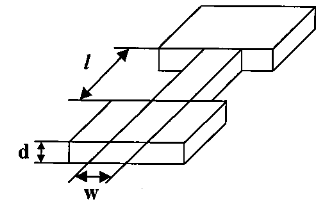


Fig. 6. Generally used configuration of a superconducting bridge. Typical dimensions of the bridge made from YBCO thin film are: $d = 0.3\ \mu$m, $w = 20\ \mu$m, and $l = 300\ \mu$m.

### B. Superconducting Bridge Exhibiting S–N Transition

Fig. 6 shows a typical configuration of a superconducting bridge [6]–[8]. Three parameters are used to describe the microwave properties of the thin-film superconducting bridge: The resistance $R_S$ and inductance $L$ connected in series in the $S$ state (no dc current through the bridge) and resistance $R_N \gg R_S$ and inductance $L$ in series in the $N$ state. Thus, impedance pair of the superconducting bridge is

$$\begin{aligned} R_1 &= R_S \\ R_2 &= R_N \\ X_2 &= X_1 = \omega L. \end{aligned} \tag{42}$$

Substituting (42) into (38) yields

$$K = \frac{R_N}{R_S}. \tag{43}$$

It is known [6]–[8] that a thin-film superconducting bridge is characterized by the following resistance pair:

$$\begin{aligned} R_N &= \frac{1}{\sigma_N d} \cdot \frac{l}{w} \\ R_S &= (\omega\mu_0)^2 \cdot \frac{\lambda_L^4}{d} \cdot \sigma_S \cdot \frac{l}{w} \end{aligned} \tag{44}$$

where $l$, $d$, and $w$ are geometrical parameters of the bridge (Fig. 6), $\lambda_L$, $\sigma_S$, and $\sigma_N$ are London penetration depth and normal conductivity of the film material in $S$ and $N$ states, respectively.

Substituting (44) into (43) yields

$$K = \frac{1}{(\omega\mu_0)^2 \lambda_L^4 \sigma_N \sigma_S}. \tag{45}$$

The $K$ parameter is a characteristic of the superconducting film material and does not depend on the bridge dimensions.

The following parameters of a typical high-temperature superconductor (HTS) film [6]–[8] can be used: $\lambda_L = 0.25\ \mu$m, $\sigma_N = \sigma_S = 10^6\ 1/\Omega{\cdot}$m. For 10 GHz, one obtains $K = 4 \cdot 10^4$.

### C. Ferroelectric Capacitor

Fig. 7 shows the equivalent circuit of a ferroelectric capacitor [10]–[12]. Four parameters are used to describe the microwave

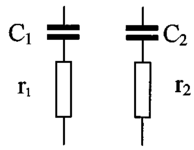


Fig. 7. Equivalent circuit of a ferroelectric capacitor.

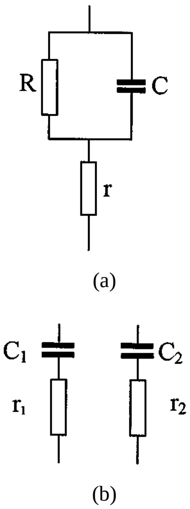


Fig. 8. Equivalent circuit of a semiconductor varactor diode.

properties, resistance $r_1$, and capacitance $C_1$ connected in series in the nonbiased state (no biasing dc voltage applied), and $r_2$ and $C_2$, respectively, in the biased state. Thus, impedance pair of the ferroelectric capacitor becomes

$$\begin{aligned} R_1 &= r_1 \\ R_2 &= r_2 \\ X_1 &= \frac{1}{\omega C_1} \\ X_2 &= \frac{1}{\omega C_2}. \end{aligned} \tag{46}$$

Since resistances $R_1$ and $R_2$ are of the same order of magnitude, (39) can be used. Substituting (46) into (39) yields

$$K = \frac{\left(\dfrac{C_1}{C_2} - 1\right)^2}{(\omega C_1)^2 r_1 r_2}. \tag{47}$$

The following characteristics are commonly used to describe the ferroelectrics:

$$\begin{aligned} n &= \frac{C_1}{C_2} \\ \tan\,\delta_1 &= \omega C_1 r_1 \\ \tan\,\delta_2 &= \omega C_2 r_2 \end{aligned} \tag{48}$$

where the tunability of the material $n$ and loss tangents for two states are denoted. Using the notation (48), one may transform (47) into

$$K = \frac{(n-1)^2}{n \cdot \tan\,\delta_1 \cdot \tan\,\delta_2}. \tag{49}$$

In this case, as in the previous one, the $K$ parameter is a characteristic of the material and does not depend on the dimensions of the switchable component.

For a typical ferroelectric film [7], [17], [18] $n = 2$, and at 5-GHz $\tan\,\delta_1 = 0.015$, $\tan\,\delta_2 = 0.007$. Using (49), one obtains $K = 5 \cdot 10^3$.

### *D. Semiconductor Varactor Diode*

Fig. 8(a), shows the equivalent circuit of a semiconductor varactor diode [9]. The following four parameters are used to describe the microwave properties of the varactor diode: the capacitance $C_1$ in a zero bias state, $C_2$ in biased state, a series resistance $r$ and a resistance $R$ shunting the capacitor for both states. Fig. 8(b) shows the transformed equivalent circuit of the varactor diode where

$$\begin{aligned} r_1 &= r + \frac{1}{(\omega C_1)^2 R} \\ r_2 &= r + \frac{1}{(\omega C_2)^2 R}. \end{aligned} \tag{50}$$

Thus, the impedance pair of the varactor diode is characterized by the capacitance's $C_1$ and $C_2$, and the resistances $r_1$ and $r_2$. For a typical varactor diode [9], one has $C_1 = 1.6$ pF, $C_2 = 0.4$ pF, $R = 300\ \Omega$, and $r = 1\ \Omega$. For 10 GHz using (50), one obtains $r_1 = 0.3\ \Omega$ and $r_2 = 5\ \Omega$. Using (47), one obtains $K = 10^4$.

## VII. Conclusion

Different suggestions to characterize switching microwave devices can be unified by introducing the CQF. A simple universal formula is derived for determination of the CQF. The invariance of the CQF with respect to lossless reciprocal transformation is shown. The CQF does not depend on the physical nature of the device. The CQF is a working tool for a selection of switching devices to be used in electronically controlled microwave components and subsystems. The CQF allows one to determine the available minimum of insertion loss of a switching microwave component. Thus, the introduced CQF can be used for optimization of the switching microwave component designed.

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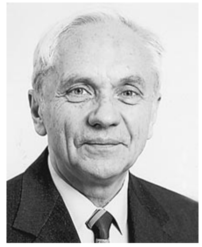

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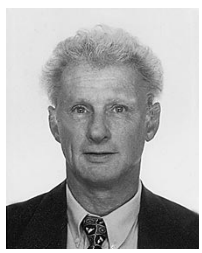

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