Port Reduction Methods for Scattering Matrix Measurement of an n-Port Network

Hsin-Chia Lu, *Member, IEEE,* and Tah-Hsiung Chu, *Member, IEEE*

***Abstract*—The port reduction method (PRM) is a method to acquire the scattering matrix of an n-port network from the scattering matrix measured at a reduced port order by terminating certain ports. This then relaxes the instrumentation requirement and calibration procedure. As the port order is reduced to two, the scattering matrix of an n-port network can be obtained from the measurement using a conventional two-port vector network analyzer. In this paper, we describe two novel PRM's, which can reduce the order of measured ports to two. The experiment results show good accuracy. These two PRM's can provide a simpler calibration procedure and instrumentation than those directly using an n-port network analyzer. In addition, they give more accurate results than those measured by a two-port network analyzer with the assumption of using ideal terminators.**

***Index Terms*—Multiport network, scattering matrix measurement.**

I. Introduction

THE instruments and calibration methods of using a two-port vector network analyzer (VNA) for two-port scattering matrix measurement are well developed. However, the scattering matrix measurement of an n-port network may require the following two approaches.

In one approach, it may need a special multiport VNA [1] or special calibration standards, as in [2], to include the effects of leakage between each port. Sharma and Gupta [3] proposed a method to deembed the n-port error matrix. In their method, instead of using a large error network to connect all the ports of a multiport network, the effects of leakage are ignored by assuming a number of independent two-port error networks at each port of the device-under-test. In [4], Ferrero proposed a generalized multiport VNA using commercially available hardware by ignoring the leakage effects between different ports in the calibration. A general formulation of multiport VNA was later given in [5]. The associated calibration method considering leakage effect to use one- and two-port calibration standards was described in [6]. Multiport network analyzer using a six-port circuit is an alternate design in hardware [7], [8]. However, for this direct measurement approach, the multiport network analyzer may not be commercially available in the near future.

Another measurement approach is to use a conventional two-port VNA by terminating all other $(n-2)$-ports with perfect terminators based on the definition of the scattering matrix. In practice, the perfect termination cannot be achieved with sufficient accuracy, especially in the higher frequency range. Rigorous methods for solving the scattering matrix of a multiport network using a two-port VNA with imperfect terminators were described in [9]–[11]. In this approach, one requires the connection and disconnection of the VNA and terminators for each permutation of the two-port connection. Based on this approach, a four-port measurement system using a two-port network analyzer and switches is given in [12].

Lin and Ruan [13] proposed a method to solve the measurement problem of an n-port scattering matrix from a different point-of-view. As a terminator is connected to an n-port network, it becomes an $(n-1)$-port network. With their method, the n-port scattering matrix can be reconstructed from n sets of the reduced $(n-1)$-port scattering matrix by connecting known terminators. This port reduction process is continued until it fails or reaches a port order at which a VNA is available. In [13], the minimum port order is three; hence, the n-port scattering matrix can be obtained from the measurement using a three-port VNA. This approach will then relax the requirement of a perfect terminator or multiport VNA.

Instead of connecting one terminator to each port sequentially for n times, as in [13], one can connect several different known terminators at a certain port; therefore, it can greatly reduce the number of connection and disconnection for the VNA and terminators. For example, in [14], a method to reduce from a three- to two-port was proposed for a reciprocal device by connecting three different known terminators at one port only.

In this paper, we develop two novel generalized port reduction methods (PRM's) for the scattering matrix measurement of an n-port network. They are denoted as the type-I PRM and type-II PRM. For the type-I PRM, one of the n-ports is connected to three different known terminators to acquire three sets of the $(n-1)$-port scattering matrix. We will show that the n-port scattering matrix of a reciprocal device can be calculated from these three $(n-1)$-port measurements, except for a sign ambiguity. An additional $(n-1)$-port measurement with an unknown terminator connected at a different port will then not only solve this sign ambiguity problem, but also solve the case of a nonreciprocal network. This port reduction process can be continued until the order of two. In other words, one can use a two-port VNA to measure the scattering matrix of an n-port network by this type-I PRM.

For the type-II PRM, it uses two different terminators connected to one port and the third terminator connected to the other port to acquire three sets of the $(n-1)$-port scattering matrix. We will derive the formulation to reconstruct the n-port

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The authors are with the Department of Electrical Engineering, National Taiwan University, Taipei, Taiwan, R.O.C. (e-mail: leonardo@emwave.ee.ntu.edu.tw; thc@emwave.ee.ntu.edu.tw).

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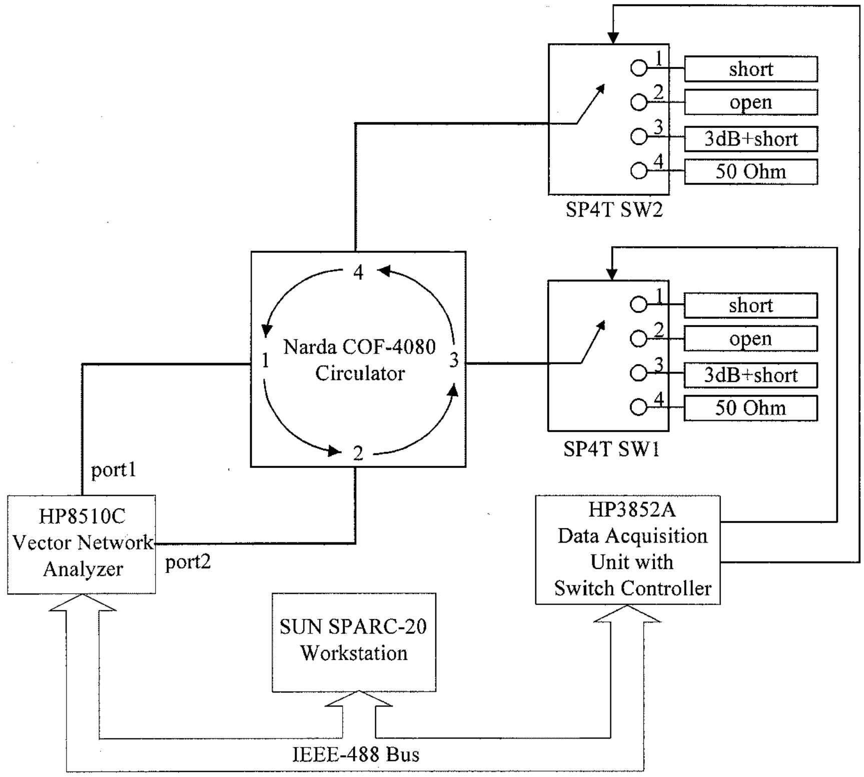


Fig. 1. Measurement arrangement of a four-port circulator.

scattering matrix explicitly from these three sets of measured $(n-1)$-port scattering matrix. Similarly, the minimum order of measured ports can also be reduced to two.

In comparison with the method described in [13], the type-I or type-II PRM requires only four or three sets of the $(n-1)$-port scattering matrix in the measurement. This will then reduce the measurement time and repeatability difficulty. In addition, the developed PRM's can use a two-port VNA, while the method in [13] requires a three-port VNA. The method in [14] is a special case for measuring a three-port reciprocal network. The PRM's in this paper are presented in a generalized formulation to solve not only the sign ambiguity problem in [14], but also the nonreciprocal n-port network.

In the following sections, the basic formulation of type-I and type-II PRM's is first presented. The measurement results of a four-port circulator are then described in Section III. These measured results are verified and compared with those using the assumption of perfect terminators. Finally, the conclusion is given in Section IV.

II. Basic Formulation

In this section, we will describe the basic formulation of type-I and type-II PRM's, by which one can reconstruct the n-port scattering matrix from the measured scattering matrix of an $(n-1)$-port network. Therefore, one can apply this port reduction process continuously until the formulation fails or the measurement can be conducted using available VNA.

As a terminator with reflection coefficient of Γ_k is connected at the port k of an n-port network, the relationship between $S_{ij}^{(k)}$ of this $(n-1)$-port network and S_{ij} of the original n-port network is given as

$$S_{ij}^{(k)} = S_{ij} + \frac{S_{ik}S_{kj}\Gamma_k}{1 - S_{kk}\Gamma_k}. \tag{1}$$

A. Type-I PRM

In the type-I PRM, one connects three different terminators to one port to acquire three sets of $(n-1)$-port scattering matrix. Assume the terminator is connected at the nth port and the reflection coefficients of three terminators are given as Γ_{n_1}, Γ_{n_2} and Γ_{n_3}, respectively. For each terminator, (1) becomes

$$S_{ij}^{(n_1)} = S_{ij} + \frac{S_{in}S_{nj}\Gamma_{n_1}}{1 - S_{nn}\Gamma_{n_1}} \tag{2}$$

$$S_{ij}^{(n_2)} = S_{ij} + \frac{S_{in}S_{nj}\Gamma_{n_2}}{1 - S_{nn}\Gamma_{n_2}} \tag{3}$$

$$S_{ij}^{(n_3)} = S_{ij} + \frac{S_{in}S_{nj}\Gamma_{n_3}}{1 - S_{nn}\Gamma_{n_3}} \tag{4}$$

where $1 \le i, j \le (n-1)$.

By equating $S_{in}S_{nj}$ at the right-hand side in (2)–(4), one can obtain two linear equations of S_{ij} and S_{nn} as

$$\left(\frac{1}{\Gamma_{n_1}} - \frac{1}{\Gamma_{n_2}}\right) S_{ij} + \left(S_{ij}^{(n_1)} - S_{ij}^{(n_2)}\right) S_{nn} = \frac{S_{ij}^{(n_1)}}{\Gamma_{n_1}} - \frac{S_{ij}^{(n_2)}}{\Gamma_{n_2}} \tag{5}$$

$$\left(\frac{1}{\Gamma_{n_1}} - \frac{1}{\Gamma_{n_3}}\right) S_{ij} + \left(S_{ij}^{(n_1)} - S_{ij}^{(n_3)}\right) S_{nn} = \frac{S_{ij}^{(n_1)}}{\Gamma_{n_1}} - \frac{S_{ij}^{(n_3)}}{\Gamma_{n_3}}. \tag{6}$$

TABLE I
PORT DESCRIPTION OF THE SCATTERING MATRIX FOR TYPE-I PRM AND TYPE-II PRM ("$*$" DENOTES THAT NOT USED IN THE TYPE-II PRM)

Ports of resulting four-port S-matrix	Ports of intermediate three-port S-matrix	Ports of measured two-port S-matrix	Ports of actually measured two-port S-matrix
1234	123_1	13_11	13_11
		13_21	13_21
		13_31	13_31
		12_11	12_11
	123_2	13_12	13_12
		13_22	13_22
		13_32	13_32
		12_12	12_12
	123_3*	13_13*	13_13*
		13_23*	13_23*
		13_33*	13_33*
		12_13*	---
	124_1	12_11	---
		12_12	---
		12_13	12_13
		24_11	24_11

From (5) and (6), S_{ij} and S_{nn} can be solved for $1 \leq i, j \leq (n-1)$. In addition, $S_{in}S_{nj}$ can be calculated by substituting the results of S_{ij} and S_{nn} into (2)–(4).

In the following, we will derive the formulation to solve S_{in} and S_{nj} for a reciprocal or nonreciprocal network. Let $Z_{ij} = S_{in}S_{nj}$. For a reciprocal network, i.e., $S_{in} = S_{ni}$, $Z_{ii} = S_{in}S_{ni} = S_{in}^2$ or

$$S_{in} = \pm\sqrt{Z_{ii}} \tag{7}$$

which has a sign ambiguity problem to be solved. It then requires a fourth terminator to be connected to give an additional set of $(n-1)$-port scattering matrix. However, this terminator should be connected to a different port, e.g., the port l ($l \neq n$), to acquire

$$S_{ij}^{(l)} = S_{ij} + \frac{S_{il}S_{lj}\Gamma_l}{1 - S_{ll}\Gamma_l} \tag{8}$$

where $1 \leq i, j \leq n$, $i, j \neq l$, and $l \neq n$. Note in (8), S_{ij}, S_{il}, S_{lj}, and S_{ll} can be solved for $i, j \neq n$ from (5) and (6), and $S_{ij}^{(l)}$ is the measured value. Therefore, Γ_l can be solved from (8). In other words, this fourth terminator can be unknown.

Now, by arbitrarily taking $S_{1n} = k$, S_{in} and S_{nj} can be expressed as $S_{in} = (Z_{i1}/Z_{11})k$ and $S_{nj} = (Z_{1j}/k)$. From (8), for $j = n$, one can write

$$S_{in}^{(l)} = S_{in} + \frac{S_{il}S_{ln}\Gamma_l}{1 - S_{ll}\Gamma_l}. \tag{9}$$

By substituting S_{in} and S_{ln} into (9), it becomes

$$S_{in}^{(l)} = \frac{Z_{i1}}{Z_{11}}k + \frac{S_{il}\dfrac{Z_{l1}}{Z_{11}}k\Gamma_l}{1 - S_{ll}\Gamma_l}. \tag{10}$$

Since $S_{in}^{(l)}$ are measured values and all the elements in the right-hand side of (10) are solvable from (2)–(6), one can solve k. Therefore, S_{in} and S_{nj} can be calculated.

Note in the above derivation, the n-port network is, in general, a nonreciprocal network. In the special case of reciprocal network, one can use the same formulation to solve S_{in}, or substituting (7) into (9) to solve the sign ambiguity problem by comparing the resulting value to the measured value of $S_{in}^{(l)}$.

Using the above formulation, one can reduce the order of the measured port for an n-port network from n to $(n-1)$. It can be shown that the formulation of port reduction from three ports to two ports has the same form. This then leads to the scattering matrix of an n-port network being properly measured using a two-port VNA with the described type-I PRM.

B. Type-II PRM

For the type-II PRM, one connects two different terminators at one port and a third terminator to another port to acquire three sets of $(n-1)$-port scattering matrix.

Assume the third terminator Γ_{n-1} is connected to the $(n-1)$th port and other two terminators Γ_{n_1} and Γ_{n_2} are connected to the nth port, respectively. Using (1) to relate the three sets of scattering matrix of the terminated $(n-1)$-port network and the original n-port network, they are given as

$$S_{n-1\ n-1}^{(n_1)} = S_{n-1\ n-1} + \frac{S_{n-1\ n}S_{n\ n-1}\Gamma_{n_1}}{1 - S_{nn}\Gamma_{n_1}} \tag{11}$$

$$S_{n-1\ n-1}^{(n_2)} = S_{n-1\ n-1} + \frac{S_{n-1\ n}S_{n\ n-1}\Gamma_{n_2}}{1 - S_{nn}\Gamma_{n_2}} \tag{12}$$

$$S_{nn}^{(n-1)} = S_{nn} + \frac{S_{n\ n-1}S_{n-1\ n}\Gamma_{n-1}}{1 - S_{n-1\ n-1}\Gamma_{n-1}}. \tag{13}$$

Note that (11) and (12) are the same as (2) and (3), while (13) is different from (4). By equating $S_{n-1\ n}S_{n\ n-1}$ at the right-hand side of (11)–(13), one can obtain two linear equations as

$$\left(S_{nn}^{(n-1)} - \frac{1}{\Gamma_{n_1}}\right)S_{n-1\ n-1} + \left(\frac{1}{\Gamma_{n-1}} - S_{n-1\ n-1}^{(n_1)}\right)S_{nn} = S_{nn}^{(n-1)}\frac{1}{\Gamma_{n-1}} - S_{n-1\ n-1}^{(n_1)}\frac{1}{\Gamma_{n_1}} \tag{14}$$

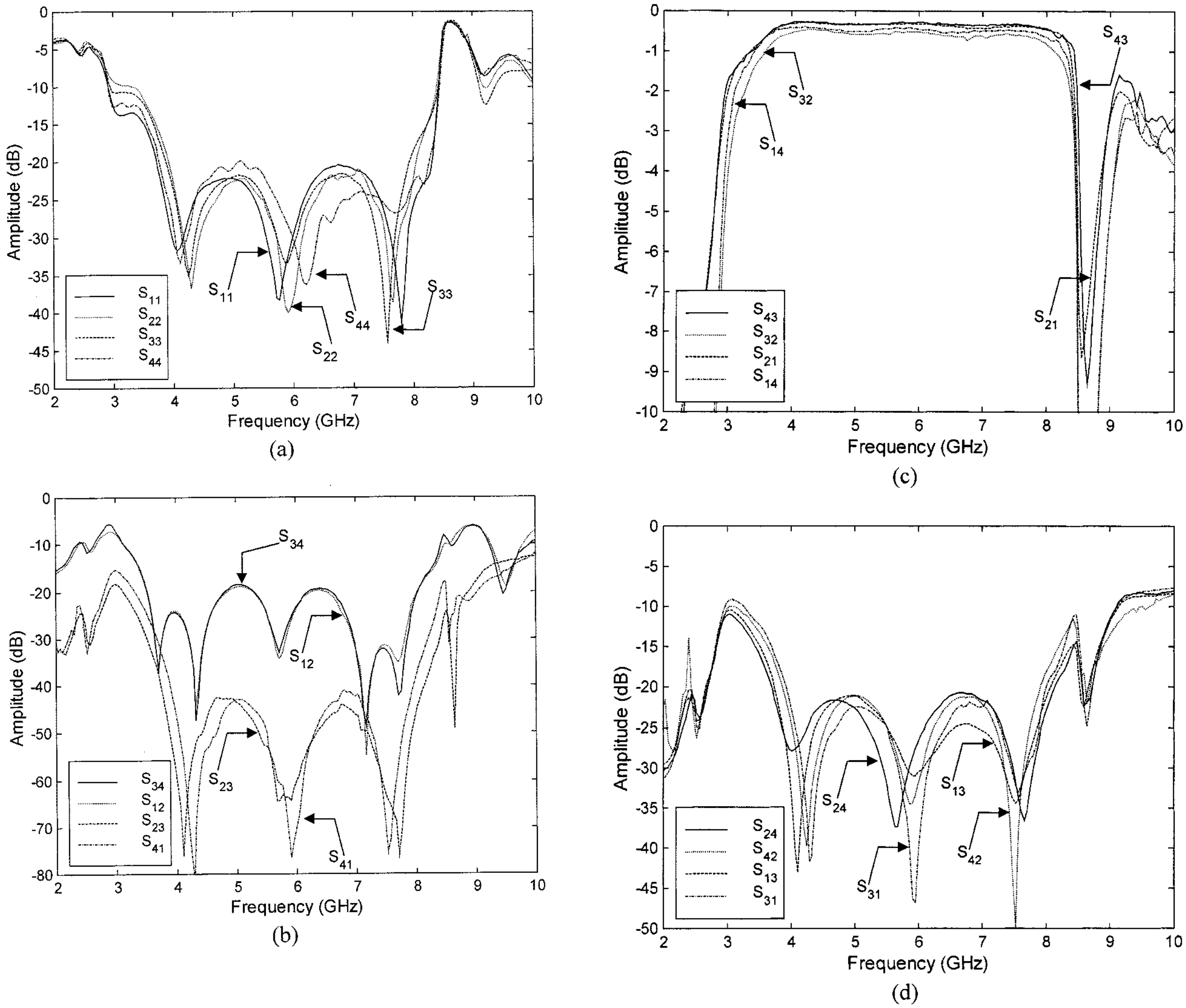


Fig. 2. Reconstructed results of: (a) input, (b) isolation, (c) transmission, and (d) leakage terms of the circulator using type-I PRM.

$$\left(S_{nn}^{(n-1)} - \frac{1}{\Gamma_{n_2}}\right) S_{n-1\ n-1} + \left(\frac{1}{\Gamma_{n-1}} - S_{n-1\ n-1}^{(n_2)}\right) S_{nn} = S_{nn}^{(n-1)} \frac{1}{\Gamma_{n-1}} - S_{n-1\ n-1}^{(n_2)} \frac{1}{\Gamma_{n_2}}. \tag{15}$$

After solving S_{nn} and $S_{n-1\ n-1}$, S_{ij} can be solved from (5) for $i, j \leq (n-1)$. In addition, one can calculate $S_{in}S_{nj}$ by substituting the results of S_{ij} and S_{nn} into (2).

Note in the type-II PRM, the fourth terminator is not required. S_{in} and S_{nj} can be solved as follows. Let $S_{1n} = k$ and $S_{in}S_{nj} = Z_{ij}$, then $S_{nj} = (Z_{1j}/k)$ and $S_{in} = (Z_{ij}/Z_{1j})k$. Since from (1)

$$S_{in}^{(n-1)} = S_{in} + \frac{S_{i\ n-1} S_{n-1\ n} \Gamma_{n-1}}{1 - S_{n-1\ n-1}\Gamma_{n-1}} \tag{16}$$

by taking $i = 1$, it becomes

$$S_{1n}^{(n-1)} = S_{1n} + \frac{S_{1\ n-1} S_{n-1\ n} \Gamma_{n-1}}{1 - S_{n-1\ n-1}\Gamma_{n-1}} = k + \frac{S_{1\ n-1} \dfrac{Z_{n-1\ j}}{Z_{1j}} k \Gamma_{n-1}}{1 - S_{n-1\ n-1}\Gamma_{n-1}}. \tag{17}$$

One can then solve k from (17). By repeating this process for all the ith ports and using $S_{nj} = (Z_{1j}/k)$, all the values of S_{in} and S_{nj} can be calculated.

Based on the above derivation, one can reduce the order of the measured port for an n-port network from n to $(n-1)$. Similarly, this port reduction process can be continued until the order reaches two. This type-II PRM is then also suitable for n-port scattering matrix reconstruction from the measurement using a two-port VNA.

III. Experimental Results and Verification

To illustrate these two novel PRM's, we use a four-port circulator (Narda COF-4080) as the device-under-test. The circulator operation frequency range is from 4 to 8 GHz. The measurement arrangement with the use of an HP8510C two-port VNA is shown in Fig. 1. The measurement system is linked with a Sun SPARC-20 workstation for the VNA and switches controlling, data recording, and processing. Two SP4T switches are used to select the proper terminators, including a short load, an open load, and a 3-dB attenuator with a short load. Discussion on the selection of terminators used in type-I and II PRM's are

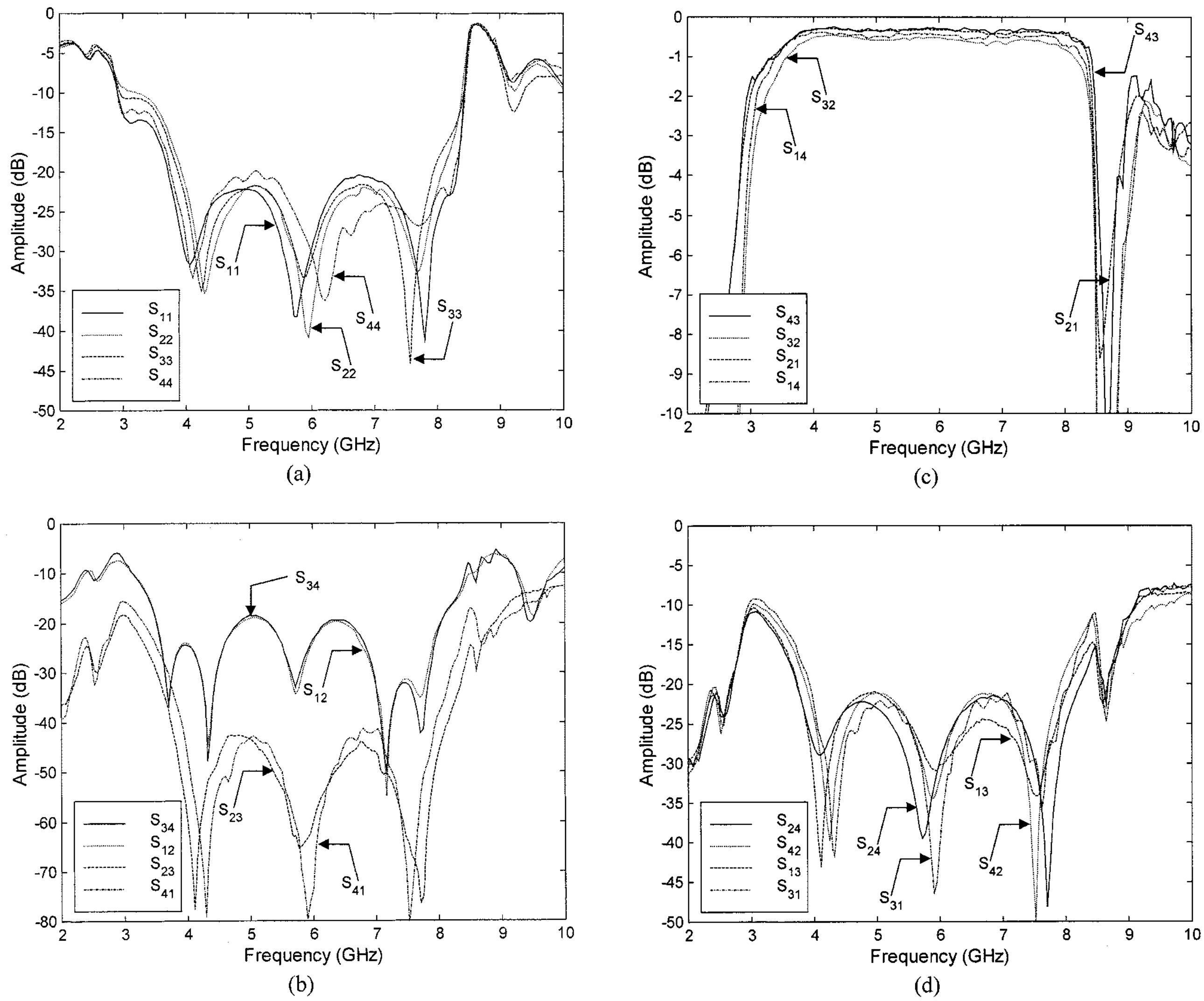


Fig. 3. Reconstructed results of: (a) input, (b) isolation, (c) transmission, and (d) leakage terms of the circulator using a type-II PRM.

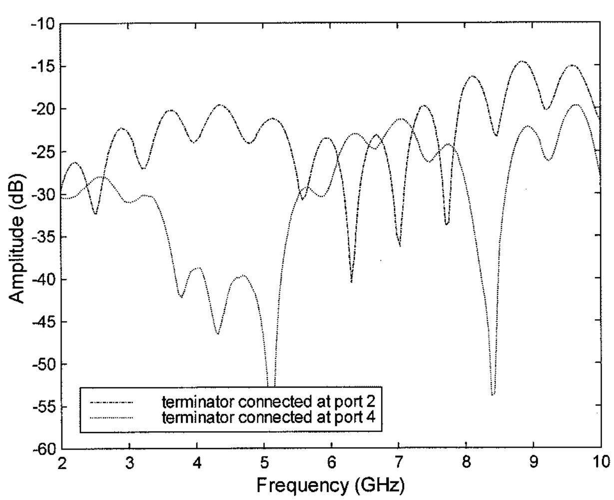
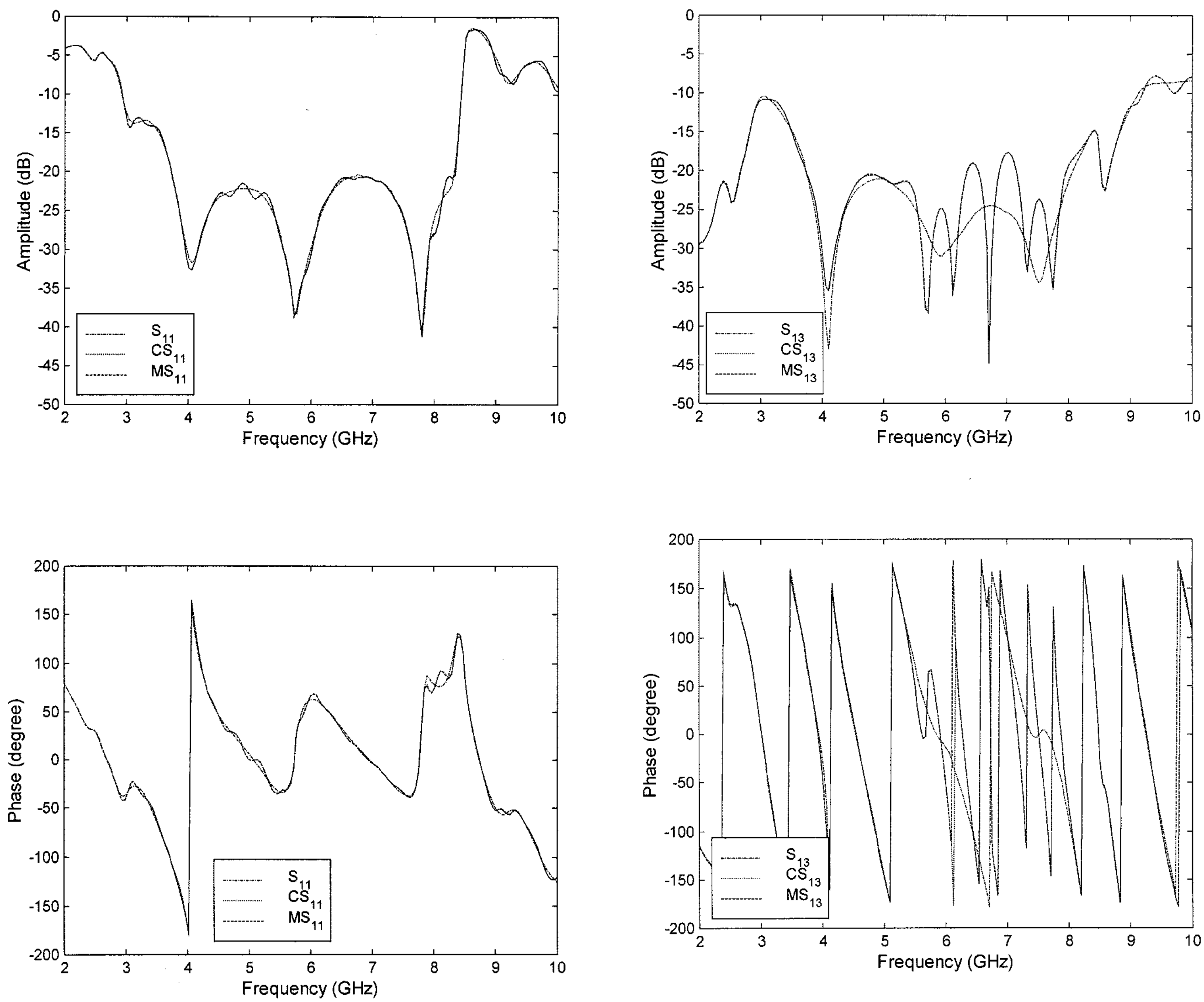


Fig. 4. Measured reflection coefficients of the two 50-Ω terminators used for measurement verification.

given in Appendixes A and B, respectively. The 50-Ω load is for the measurement verification. The reflection coefficients of all these terminators, including the effect of cables and switches, are recorded in the workstation. In the measurement, two of the circulator ports are connected to the HP8510C and two of the terminators are properly selected for the type-I or type-II PRM's.

Since only two ports are measured ports, Table I illustrates the port arrangement of the scattering matrix at a different order in the calculation using type-I and type-II PRM's. The first column gives the ports of the resulting four-port scattering matrix, i.e., "1234" means a four-port scattering matrix of ports 1–4. The ports of intermediate three-port scattering matrices required to reconstruct this four-port scattering matrix are shown in the second column. The terminators for reducing one port are also given. For example, "123_2" of the second element means a three-port scattering matrix of ports 1–3 with terminator 2 connected at the port 4. Terminators 1–3 correspond to a short load, an open load, and a 3-dB attenuator with a short load. In this column, the third element "123_3∗" with an asterisk is the port arrangement not used in the type-II PRM.

The third column describes the ports and terminators for the two-port scattering matrix measurement. Similarly, the first



(a) (b)

Fig. 5. Comparison of: (a) S_{11} and (b) S_{13} of the circulator using type-I PRM from reconstructed four-port scattering matrix, MS_{ij} from measured two-port scattering matrix, and CS_{ij} from calculated two-port scattering matrix.

two digits denote the measured ports connected to HP8510C and the next two digits are the terminators used. For example, "13_31" of the third element means ports 1 and 3 are connected to HP8510C and ports 2 and 4 are connected to terminators 3 and 1, respectively. The actual measured two-port scattering matrices are listed in the last column.

In the use of type-I or type-II PRM's, the intermediate three-port scattering matrices are first reconstructed from the measured two-port scattering matrices. The four-port scattering matrix of circulator is then reconstructed from these three-port scattering matrices. Figs. 2 and 3 are results for each type of PRM, and they are shown closely identical.

The reconstructed scattering matrix is verified as follows. Ports 2 and 4 of the test circulator are terminated with two 50-Ω loads, which are not used in the calculation of the PRM, and then measured by HP8510C. In addition, its two-port scattering matrix is calculated using the reconstructed scattering matrix and the measured reflection coefficients of two 50-Ω loads, as shown in Fig. 4. Figs. 5 and 6 are the typical results of the measured and calculated two-port scattering matrix of the terminated circulator, denoted by MS_{ij} and CS_{ij}. It shows they are closely identical. This means that the reconstructed scattering matrix from a type-I or II type-PRM is quite accurate. A quantitative discussion on the accuracy of reconstructed S_{ij} is given in Appendix C.

Note that MS_{13} is quite different from S_{13}, as shown in Figs. 5 and 6. This indicates that the reflections from the 50-Ω loads at ports 2 and 4 contaminate the original S_{13}. In other words, if the 50-Ω loads used are assumed to be perfectly matched, one may get erroneous results as the measured MS_{13} instead of the correct S_{13}.

IV. Conclusion

In this paper, we have developed two novel PRM's. With these two methods, one can accurately measure the scattering

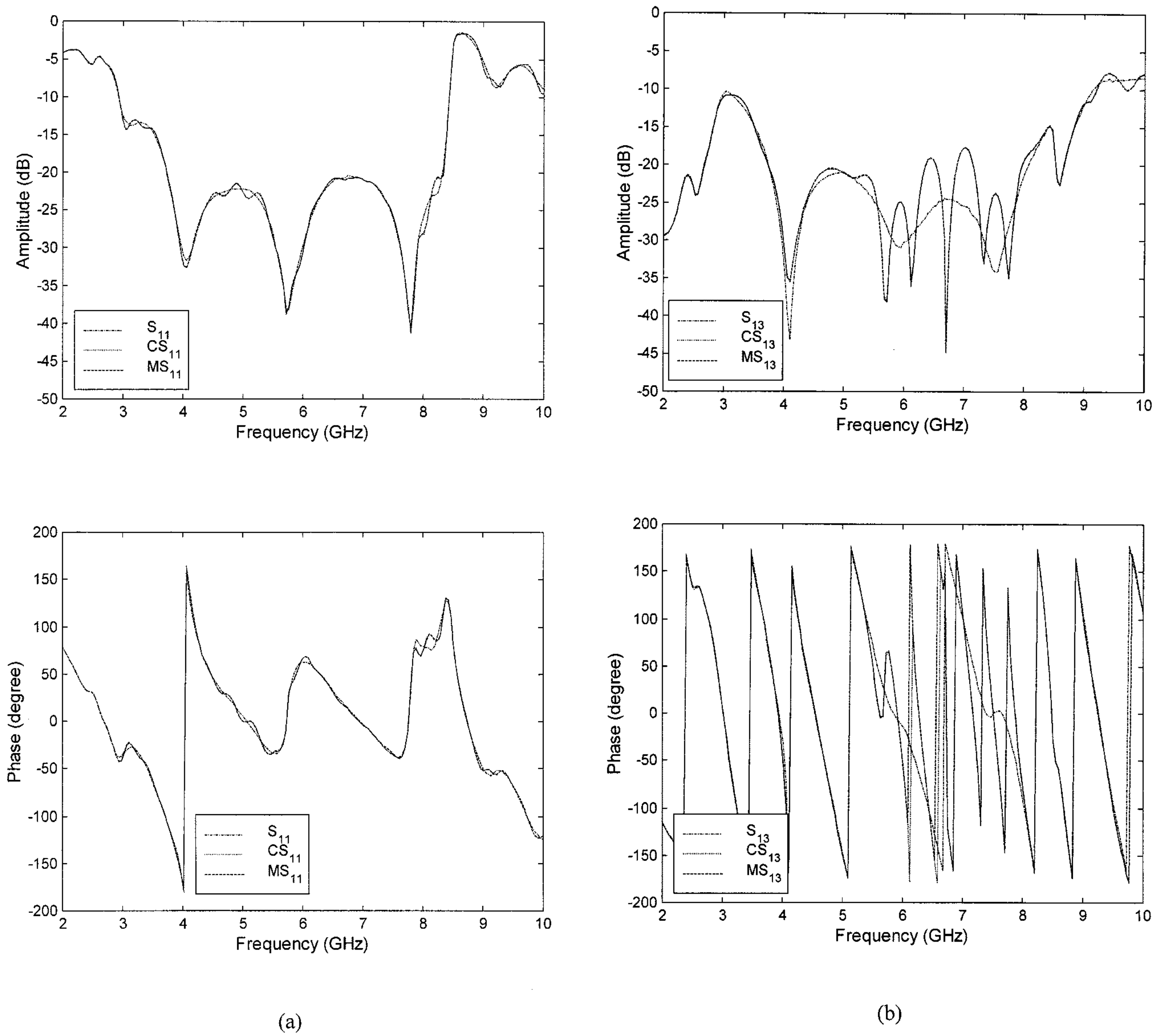


Fig. 6. Comparison of: (a) S_{11} and (b) S_{13} of the circulator using type-II PRM with S_{ij} from reconstructed four-port scattering matrix, MS_{ij} from measured two-port scattering matrix, and CS_{ij} from calculated two-port scattering matrix.

matrix of an n-port network with the use a conventional two-port VNA. It then eliminates the needs of a special n-port VNA and calibration procedure. In addition, there are two advantages by using the developed PRM's in comparison with the conventional two-port measurement approach. Firstly, the operation of connecting and disconnecting of the VNA to the test device can be reduced. Secondly, the constraint to use perfect terminators for accurate scattering matrix measurement is relaxed.

APPENDIX A
SELECTION OF TERMINATORS FOR THE TYPE-I PRM

In the type-I PRM, (5) and (6) can be written in the matrix form as

$$Ax = b \tag{A.1}$$

where

$$A = \begin{bmatrix} \dfrac{1}{\Gamma_{n_1}} - \dfrac{1}{\Gamma_{n_2}} & S_{ij}^{(n_1)} - S_{ij}^{(n_2)} \\ \dfrac{1}{\Gamma_{n_1}} - \dfrac{1}{\Gamma_{n_3}} & S_{ij}^{(n_1)} - S_{ij}^{(n_3)} \end{bmatrix}$$

$$x = \begin{bmatrix} S_{ij} \\ S_{nn} \end{bmatrix}$$

$$b = \begin{bmatrix} \dfrac{S_{ij}^{(n_1)}}{\Gamma_{n_1}} - \dfrac{S_{ij}^{(n_2)}}{\Gamma_{n_2}} \\ \dfrac{S_{ij}^{(n_1)}}{\Gamma_{n_1}} - \dfrac{S_{ij}^{(n_3)}}{\Gamma_{n_3}} \end{bmatrix}.$$

By substituting (2)–(4) into matrix A, it becomes

$$A = \begin{bmatrix} t_1 - t_2 & S_{in}S_{nj}\left(\dfrac{1}{t_1 - S_{nn}} - \dfrac{1}{t_2 - S_{nn}}\right) \\ t_1 - t_3 & S_{in}S_{nj}\left(\dfrac{1}{t_1 - S_{nn}} - \dfrac{1}{t_3 - S_{nn}}\right) \end{bmatrix} \tag{A.2}$$

TABLE II
CALCULATED RESULTS OF THE ABSOLUTE MEAN VALUE $|\mu|$, STANDARD DEVIATION σ, MAGNITUDE ERROR AND PHASE ERROR FOR TYPE-I AND TYPE-II PRM'S

S_{ij}		S_{11}	S_{22}	S_{33}	S_{44}
Type I PRM	$\|\mu\|$	0.09×10^{-3}	0.98×10^{-3}	0.05×10^{-3}	0.27×10^{-3}
	σ	0.37×10^{-3}	7.80×10^{-3}	0.19×10^{-3}	4.20×10^{-3}
	mag. error (dB)	0.032	0.652	0.016	0.357
	phase error (degree)	0.212	4.460	0.109	2.405
Type II PRM	$\|\mu\|$	0.08×10^{-3}	0.74×10^{-3}	0.06×10^{-3}	0.92×10^{-3}
	σ	0.54×10^{-3}	2.60×10^{-3}	0.34×10^{-3}	4.90×10^{-3}
	mag. error (dB)	0.047	0.223	0.029	0.416
	phase error (degree)	0.309	1.489	0.195	2.805
S_{ij}		S_{12}	S_{21}	S_{13}	S_{31}
Type I PRM	$\|\mu\|$	0.15×10^{-3}	0.19×10^{-3}	0.06×10^{-3}	0.04×10^{-3}
	σ	1.03×10^{-3}	3.10×10^{-3}	0.81×10^{-3}	2.44×10^{-3}
	mag. error (dB)	0.086	0.027	0.070	0.0209
	phase error (degree)	0.573	0.178	0.464	1.398
Type II PRM	$\|\mu\|$	0.10×10^{-3}	0.20×10^{-3}	0.11×10^{-3}	0.16×10^{-3}
	σ	0.77×10^{-3}	3.20×10^{-3}	1.46×10^{-3}	5.10×10^{-3}
	mag. error (dB)	0.067	0.028	0.126	0.432
	phase error (degree)	0.441	0.183	0.836	2.920

where $t_1 = (1/\Gamma_{n_1})$, $t_2 = (1/\Gamma_{n_2})$, and $t_3 = (1/\Gamma_{n_3})$.

The condition number of matrix A is then used as a measure of the sensitivity of solution x to the measurement error. The condition number of A is defined as the ratio of its largest singular value to its smallest singular value. Therefore, a smaller condition number indicates the value of x is less sensitive to the measurement error.

In the calculation of the condition number of matrix A for various types of terminators at a different port n, it shows that a larger value of $S_{in}S_{nj}$ yields a smaller condition number. Therefore, one should choose the port n to connect terminators to have the largest $S_{in}S_{nj}$. In addition, we found the condition number is as quite low as 1.91, if the reflection coefficients of three terminators are located at a unit circle and with 120° apart. However, these terminators are not easily implemented for a wide operation bandwidth. The three terminators we used in the measurement are a short, open, and 3-dB attenuator with a short load. The condition number is 2.91. As the third terminator is close to a 50-Ω load with $t_3 = 100$, the condition number increases to be 54. This means that short, open, and 50-Ω load is not a good set of terminators for a type-I PRM. This result is quite different from the conventional selection of terminators, as in [14]. The reason is that, in a type-I PRM, a smaller reflection coefficient of the terminator will cause the contribution of $S_{in}S_{nj}$ and S_{nn} to the measured scattering parameters be less prominent; hence, the reconstructed result of S_{nn} becomes less accurate.

APPENDIX B
SELECTION OF TERMINATORS FOR THE TYPE-II PRM

The matrix A for a type-II PRM is given in (B.1), shown at the bottom of this page, where $t_3 = (1/\Gamma_{n-1})$. We found

$$A = \begin{bmatrix} S_{nn} + \dfrac{S_{n\,n-1}S_{n-1\,n}}{t_3 - S_{n-1\,n-1}} - t_1 & t_3 - S_{n-1\,n-1} + \dfrac{S_{n\,n-1}S_{n-1\,n}}{t_1 - S_{nn}} \\ S_{nn} + \dfrac{S_{n\,n-1}S_{n-1\,n}}{t_3 - S_{n-1\,n-1}} - t_2 & t_3 - S_{n-1\,n-1} + \dfrac{S_{n\,n-1}S_{n-1\,n}}{t_2 - S_{nn}} \end{bmatrix} \tag{B.1}$$

that if t_1, t_2, and t_3 are set to -1, 1, and -1, the condition number of A in (B.1) is too low to be 1.13. As $|t_3|$ increases, the condition number increases. However, the number only slightly increases as $S_{n-1\;n}S_{n\;n-1}$ increases. This indicates that one should choose the port n and port $n-1$ for connecting terminators to have a small value of $S_{n-1\;n}S_{n\;n-1}$. In addition, the condition number is smaller for a type-II PRM than for a type-I PRM, as the same terminators are connected. This is due to the fact that S_{nn}'s are not directly measured by the VNA for a type-I PRM, while for a type-II PRM, they are directly measured by the VNA.

APPENDIX C
ACCURACY OF RECONSTRUCTED S_{ij}

The accuracy of reconstructed four-port scattering matrix is discussed below by expressing CS_{ij} and MS_{ij} as $CS_{ij} = S_{ij} + ST_C$ and $MS_{ij} = S^a_{ij} + ST_M$, where S^a_{ij} is the actual scattering parameter of the circulator. ST_C and ST_M denote the spurious terms due to the reflection from nonideal 50-Ω loads to CS_{ij} and MS_{ij}, respectively. They are both related to the characteristics of 50-Ω loads and circulator used. One can then use the difference of CS_{ij} and MS_{ij} to estimate the reconstructed S_{ij} accuracy given by

$$S_{ij} - S^a_{ij} = (CS_{ij} - MS_{ij}) - (ST_C - ST_M). \qquad \text{(C.1)}$$

In (C.1), the mean and standard deviation of $CS_{ij} - MS_{ij}$ are first calculated. Since ST_C and ST_M are at least -20 dB below S_{ij} for the input, transmission, and isolation terms, one can assume the mean and standard deviation of these terms for $S_{ij} - S^a_{ij}$ are equal to those for $CS_{ij} - MS_{ij}$.

For the leakage terms, the signal from the reflection of the imperfect terminator is about the same level as the original signal. Taking S_{13}, for example, $S_{13} - S^a_{13}$ can be expressed as

$$S_{13} - S^a_{13} \cong (CS_{13} - MS_{13}) - \frac{1}{\Gamma_4}\left(S_{14}S_{43} - S^a_{14}S^a_{43}\right) \quad \text{(C.2)}$$

where Γ_4 is the reflection coefficient of the 50-Ω load connected at port 4. $(1/\Gamma_4)S_{14}S_{43}$ and $(1/\Gamma_4)S^a_{14}S^a_{43}$ are the dominant terms in ST_C and ST_M, respectively. In (C.2), $S_{14}S_{43} - S^a_{14}S^a_{43}$ can be rewritten as

$$S_{14}S_{43} - S^a_{14}S^a_{43} = S_{14}p_1 + S_{43}p_2 - p_1p_2 \qquad \text{(C.3)}$$

where $p_1 = S_{43} - S^a_{43}$ and $p_2 = S_{14} - S^a_{14}$. Since p_1 and p_2 are both the transmission terms, their estimated mean and variance are known. The mean and variance of $S_{14}S_{43} - S^a_{14}S^a_{43}$ can then be calculated.

The similar process can be applied by using the estimated values for $S_{ij} - {}^a_{ij}$ to validate the input, transmission, and isolation terms. The calculated results show that the mean and variance differences between $S_{ij} - S^a_{ij}$ and $CS_{ij} - MS_{ij}$ are quite small, as expected. Results of the estimated values of the mean and standard deviation for $S_{ij} - S^a_{ij}$ are listed in Table II.

Since the standard deviation gives the root mean square distance between S^a_{ij} and S_{ij}, one can use the typical value of S^a_{ij} and calculated standard deviation to estimate the magnitude and phase errors. The typical values of S^a_{ij} are assumed to be equal to S_{ij}. They are given to be "0.1" for the input, isolation, and leakage terms and "1" for transmission terms. The calculated magnitude and phase errors are listed in Table II. It shows that both methods have comparable accuracy, with magnitude and phase errors less than 0.1 dB and 1°, except for S_{22}, S_{44}, and S_{31}. S_{22} and S_{44} have larger error values because the measured ports given in Table I are mostly ports 1 and 3. Whereas S_{31} is strongly influenced by the terminators at port 2, the accuracy is then not as good as the other terms.

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Hsin-Chia Lu (S'93–M'00) was born in Taiwan, R.O.C., in 1969. He received the Ph.D. degree in electrical engineering from the National Taiwan University, Taipei, Taiwan, R.O.C. in 1999.

He is currently with the Department of Electrical Engineering, National Taiwan University, as a Post-Doctoral Researcher. His research interests include microwave measurement, calibration techniques, six-port reflectometers, microwave circuits, and microwave imaging.

Tah-Hsiung Chu (M'87) received the B.S. degree from the National Taiwan University, Taipei, Taiwan, R.O.C., in 1976, and the M.S. and Ph.D. degrees from the University of Pennsylvania, Philadelphia, in 1980 and 1983, respectively, all in electrical engineering.

From 1983 to 1986, he was a Member of the Technical Staff at the Microwave Technology Center, RCA David Sarnoff Research Center, Princeton, NJ. Since 1986, he has been on the Faculty of the Department of Electrical Engineering, National Taiwan University, where he is currently a Professor of electrical engineering. His research interests include microwave imaging systems and techniques, microwave circuit and subsystem design, electromagnetic theory, microwave measurement, and calibration techniques.