# A New Theory of the Characteristic Impedance of General Printed Transmission Lines Applicable When Power Leakage Exists

Nirod K. Das, *Member, IEEE*

***Abstract*—Conventional definitions of the characteristic impedance, such as the voltage–current, power–current, and power–voltage methods, which have been commonly used for standard nonleaky transmission lines, become invalid when power leakage occurs. In this paper, we present a new theory of the characteristic impedance for printed transmission lines, applicable under the general conditions with or without power leakage. The theory is founded on dual field and circuit theories of transmission lines, formulated in the spectral domain, and uses a new approach called "the wavenumber perturbation approach." In order to correctly compute the complex characteristic impedance under leakage conditions, the new theory requires to carefully "extract out" the surface-wave or parallel plate-wave poles on the complex $k$-plane. In obvious difference to this, it is well known that the poles must be "included" for a correct solution of the complex propagation constant of the leaky line. Incidentally, unlike the conventional methods, the new theory derives the complex characteristic impedance together with the solution of the phase and attenuation constants, in a single unified procedure. This avoids additional efforts in computational or analytical/formulational complexity. Results for selected cases of interest are presented, which demonstrate the validity and simplicity/elegance of the new theory.**

***Index Terms*—Characteristic impedance, leaky waves, planar transmission line, stripline, slotline.**

## I. Introduction

It is well known that, under appropriate conditions, printed transmission lines can leak power transversely to the characteristic surface-wave mode(s) of its surrounding structure [1]–[4]. However, early research on this subject mainly concentrated on the propagation behavior. It is equally important to model the characteristic impedance of the leaky lines for use in circuit design purposes. The standard definitions of the characteristic impedance for TEM or quasi-TEM transmission lines, such as the current–voltage, current–power, and voltage–power definitions [5]–[7], which have been commonly used for nonleaky transmission lines, do not apply when leakage exists. This is due to the strong non-TEM nature of the transverse fields excited by the leaky lines, having a nonstandard exponential growth in the transverse directions. Further, any analysis of a printed transmission line, which uses a surrounding boxed structure [8], [9] for computational or analytical convenience, would fail to work. The boxed structure will not allow any leakage power to escape out in transverse directions and, therefore, would not fundamentally support the leaky mode. An equivalent characteristic impedance, as seen by a particular input excitation, may be extracted from a three-dimensional (3-D) electromagnetic modeling of a section of a leaky line [10], [11], excited by a suitable source. However, it is always desirable to define a characteristic impedance, based on a simple two-dimensional (2-D) analysis of an infinite-length leaky line, which is independent of the specific source of excitation, or of the specific circuit configuration the line is used in. Some attempts have recently been made to model the characteristic impedance in such a manner [11], [12], with reasonable results.

In this paper, we present a new theory of the characteristic impedance of a general printed line, applicable when the line leaks to a guided mode of the surrounding structure. The derivation is well founded on dual circuit and field theories of transmission lines, employs a new technique called the "wavenumber perturbation technique," and applies to leaky as well as nonleaky lines, with or without any material loss. The analysis starts with a general transmission line with distributed power-leakage elements, in addition to the conventional storage and/or material loss elements. The wavenumber (in general complex) of the transmission line is then perturbed to see various changes in the total field and circuit behavior. The resulting information is used in order to obtain an equivalent characteristic impedance of the line. Using this approach, the information already available in a standard moment-method solution of the propagation constant [1], [13] can be reused to derive the characteristic impedance of the line, without any additional formulation or computation. This results in a theoretically as well as computationally efficient/elegant approach.

The basic theory is developed in Section II, first for leaky strip-type lines, which is then generalized for slot-type lines as well. Results for selected cases of interest are presented in Section III in order to demonstrate the accuracy and validity of the new theory.

## II. Theory

### A. Strip-Type Leaky Lines

Fig. 1 shows the cross-sectional geometry of a leaky stripline that leaks power to its parallel-plate mode. This geometry represents leakage from general strip-type transmission lines. The

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The author is with the Weber Research Institute, Department of Electrical Engineering, Polytechnic University, Farmingdale, NY 11735 USA.

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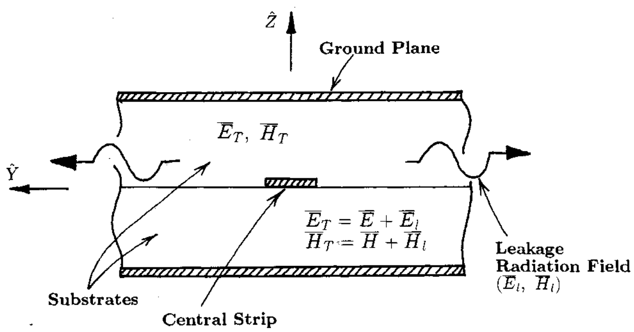


Fig. 1. Cross-sectional geometry of an example leaky stripline, showing power leaking from the central strip through the sidewalls. Due to the presence of the leakage fields, which spread in the transverse directions indefinitely, the transmission line is electrically unbounded in transverse dimension.

total electric and magnetic fields, i.e., $\overline{E}_T$ and $\overline{H}_T$, of the line can be decomposed into two parts [1]

$$\overline{E}_T = \overline{E} + \overline{E}_l; \ \overline{H}_T = \overline{H} + \overline{H}_l. \quad (1)$$

$\overline{E}_l$ and $\overline{H}_l$ are the parts contributed due to the leakage radiation, having nonstandard exponential growth in transverse ($\hat{y}$) directions. These leakage fields may be referred to as the "growing fields." If the growing fields are removed from the total field, the remaining fields $\overline{E}$ and $\overline{H}$ in (1) are confined or bound to the central region, and are called the "bound fields." A spectral-domain analysis of the transmission line can be used to derive the "growing fields" $\overline{E}_l$ and $\overline{H}_l$ from the total fields $\overline{E}_T$ and $\overline{H}_T$. This is done by careful extraction of the singular parts, contributed due to surface-wave poles on the transverse spectral $k_y$-plane [1]. The surface-wave poles (that are responsible for the power leakage) in the spectral plane of the total fields $\overline{E}_T$ and $\overline{H}_T$ are to be carefully removed using proper "extraction functions." The remaining parts are the bound fields $\overline{E}$ and $\overline{H}$. The singular parts that were extracted out constitute the unbounded growing leakage fields $\overline{E}_l$ and $\overline{H}_l$

We first discuss the moment-method analysis of the propagation constant of a leaky strip line, as relevant to the rest of the derivation of the characteristic impedance. The surface current $\overline{J}$ on the central strip is expanded using a set of basis functions with unknown coefficients $a_i$

$$\overline{J} = \overline{f}(y)e^{-jk_e x} = \sum_{i=1}^{N} a_i \overline{J}_i = \sum_{i=1}^{N} a_i \overline{f}_i(y) e^{-jk_e x} \quad (2)$$

$$\overline{f}(y) = \sum_{i=1}^{N} a_i \overline{f}_i(y) \quad (3)$$

$$I = \int_{\text{strip}} \overline{f}(y) \cdot \hat{x}\, dy = 1. \quad (4)$$

Just for simplicity of formulation, we have normalized such that the total current $I$ along the longitudinal ($\hat{x}$)-direction is unity. The normalization should not affect our final results. The tangential component of the electric field should be zero everywhere on the central strip. This boundary condition is enforced using a Galerkin-type testing procedure, resulting in a moment-matrix equation

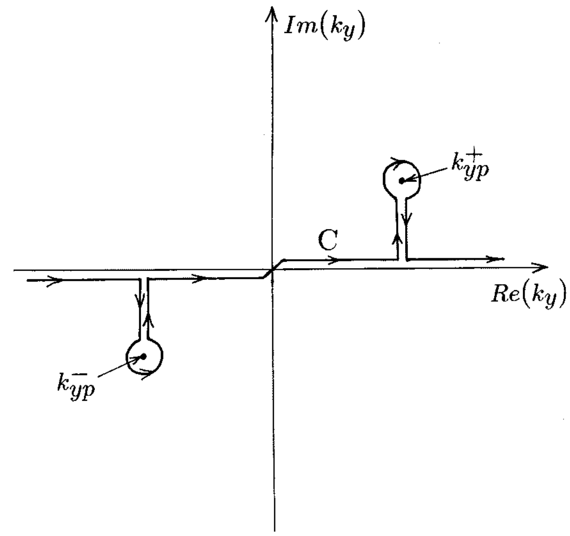


Fig. 2. Contour $C$ on the complex $k_y$ spectral plane, properly deformed around the poles at $\pm k_{yp}$ due to surface-wave-type modes. Such a contour must be used in spectral integrals in order to include the contribution due to leakage fields. The residue contributions around the singularities can be attributed to the power leakage to the surface-wave mode. The singularities at $\pm k_{yp}$ may be properly extracted in order to exclude the leakage fields. The remaining integral is attributed only to the "bound fields," which then can be integrated along the real axis.

$$\sum_{i=1}^{N} a_i Z_{Tij}(k_x = k_e) = 0, \qquad j = 1, \cdots, N \quad (5)$$

$$\det\left[Z_{Tij}(k_x = k_e)\right]_{N\times N} = 0 \quad (6)$$

$$Z_{Tij}(k_x) = \int_{\text{strip, x=0}} \overline{E}_{Ti}(y;\ k_x) \cdot \overline{J}_j\, dy = \int_C \tilde{\overline{f}}_i(k_y) \cdot \tilde{\overline{\overline{G}}}_{EJ}(-k_x,\, k_y) \cdot \tilde{\overline{f}}_j(-k_y)\, dk_y. \quad (7)$$

The quantities $Z_{Tij}$ in (7) are the moments of the total field produced by the $i$th basis current with the $j$th basis current, and are computed using a Fourier-plane integration. $\tilde{\overline{f}}_{i,\,j}(k_y)$ are the Fourier transforms of the space functions $\overline{f}_{i,\,j}(y)$, $\tilde{\overline{\overline{G}}}_{EJ}$ is the spectral-domain dyadic Green's function for the electric field, produced due to electric currents [5]. When leakage exists, because of the exponential growth of the leakage field, the required spectral-integration contour should be properly deformed along $C$ around surface-wave singularities, as shown in Fig. 2. After $Z_{Tij}$'s are computed, the determinant equation (6) can be solved for the unknown propagation constant, $k_x = k_e$.

If the total current $\overline{f}(y)$ is known, at least approximately, then the solution of the matrix equation in (7) can be simplified to solving for the zero of a single moment function

$$Z_T(k_x = k_e) = \int_{\text{strip, }x=0} \overline{E}_T(y;\ k_x = k_e) \cdot \overline{J}(y)\, dy = 0 \quad (8)$$

and

$$Z_T(k_x) = \int_{\text{strip},\, x=0} \overline{E}_T(y;\, k_x) \cdot \overline{J}\, dy = \int_C \tilde{\overline{f}}(k_y) \cdot \tilde{\overline{\overline{G}}}_{EJ}(-k_x,\, k_y) \cdot \tilde{\overline{f}}(-k_y)\, dk_y. \tag{9}$$

The above procedure only finds the propagation characteristics of the line. In the following, we establish a new theory for the complex characteristic impedance of the leaky line. The new theory relates the characteristic impedance of the line to the above moment computations, based on a dual field-circuit formalism. The procedure will allow derivation of the characteristic impedance, together with the propagation constant, without additional computation.

Under an ideal situation, when a transmission line is infinitely long in the direction of propagation $\hat{x}$, the growing fields $\overline{E}_l$ and $\overline{H}_l$ increase indefinitely in transverse $(\hat{y})$-directions, to infinity at large $(y = \pm\infty)$ distances. This makes the line electrically unbounded in the transverse plane. One can transform the electrically unbounded structure of Fig. 1 to an equivalent transmission line with a bounded transverse section. This is shown in Fig. 3(a), consisting of only the bounded fields $\overline{E}$ and $\overline{H}$. The leakage from the central strip is a distributed radiation process, which carries power away from the line, and is responsible for the "unbounded" leakage fields $\overline{E}_l$ and $\overline{H}_l$. This distributed radiation process may be equivalently modeled using a distributed radiation impedance, $Z_l$ per unit length, along the strip. One may visualize this as having distributed antennas loaded along the transmission line, which produce the leakage radiation fields. Since we assume a total current of a unit amplitude, $Z_l$ is the negative of the complex reaction per unit length due to the leakage field. Therefore, we may express $Z_l$ using the following reaction integral:

$$Z_l = -\int_{\text{strip},\, x=0} \overline{E}_l \cdot \overline{J}\, dy. \tag{10}$$

The central strip of Fig. 3(a) is electrically different from the metal strip in the original transmission line. This may be seen by comparing the required boundary conditions of the electric fields in the two cases. In the original transmission line, the tangential component of the total electric field $\overline{E}_T$ is zero everywhere on the strip (assume for now a perfectly conducting strip). Whereas, the equivalent transmission line of Fig. 3(a) consists of only the bounded electric fields $\overline{E}$, the tangential component of which is no longer zero on the central strip.

$$\overline{E}_T \cdot \hat{p} = 0, \qquad \overline{E} \cdot \hat{p} = -\overline{E}_l \cdot \hat{p} \neq 0 \tag{11}$$

where $\hat{p}$ is any unit vector along the surface of the strip, which may be either $\hat{x}$ or $\hat{y}$.

Besides the boundary condition on the strip, it can be shown that the bound fields $\overline{E}$ and $\overline{H}$ independently satisfy the Maxwell's equations everywhere inside the enclosure of Fig. 3(a), and also have zero tangential electric and/or magnetic fields on the enclosure surface $S_3$. Under this condition, we

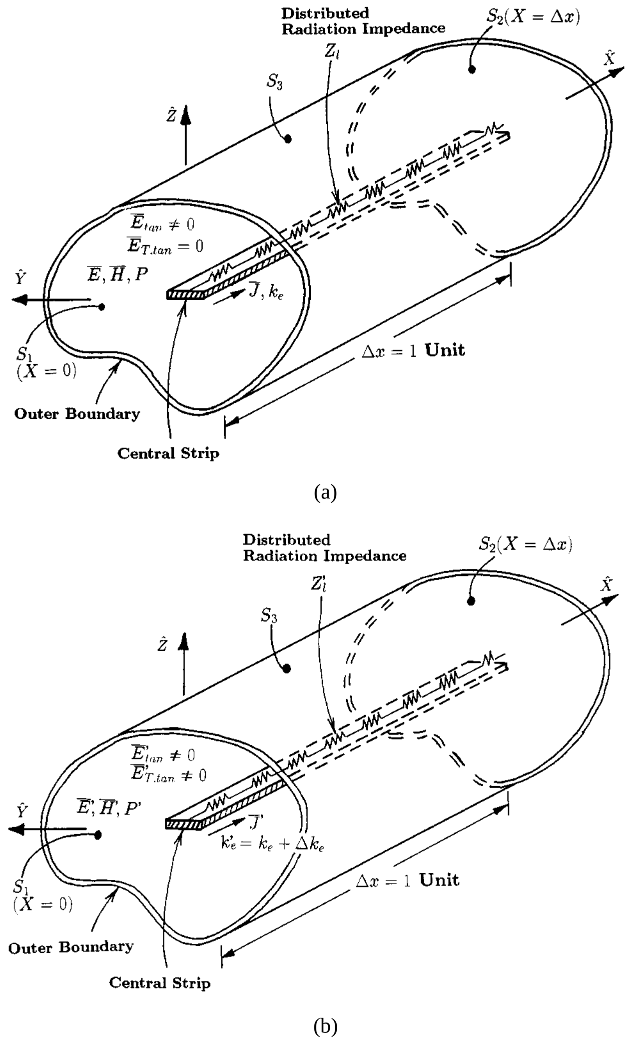


Fig. 3. (a) Equivalent bounded geometry of a general strip-type leaky line with distributed radiation impedance $Z_l$ to account for the leakage radiation. The outer boundary $S_3$ is chosen on a metal wall (when there is a ground plane on top and/or bottom) or sufficiently far away where open (sides and top/bottom, depending if there is a metal wall or not) such that the electric and magnetic fields $\overline{E}, \overline{H} \simeq 0$. (b) The modified bounded geometry of (a), when the strip current $\overline{J}$ is perturbed to $\overline{J}'$ with a slightly different propagation constant $k_e + \Delta k_e$. The corresponding fields $\overline{E}, \overline{H}$, power $P$, and distributed leakage impedance $Z_l$ are perturbed to $\overline{E}'$, $\overline{H}'$, $P'$, and $Z_l'$, respectively.

can apply the following reaction (not power) integral to the equivalent transmission line of Fig. 3(a) [14]:

$$\begin{aligned}\iint_{S_1+S_2} \overline{E} \times \overline{H} \cdot \overline{dS} &= -j\omega \iiint_V \hat{\epsilon}\overline{E} \cdot \overline{E}\, dV - j\omega \iiint_V \hat{\mu}\overline{H} \cdot \overline{H}\, dV - \iint_{\text{strip}} \overline{E} \cdot \overline{J}\, dS \\ &= -j\omega \iiint_V \hat{\epsilon}\overline{E} \cdot \overline{E}\, dV - j\omega \iiint_V \hat{\mu}\overline{H} \cdot \overline{H}\, dV + \iint_{\text{strip}} \overline{E}_l \cdot \overline{J}\, dS - \iint_{\text{strip}} \overline{E}_T \cdot \overline{J}\, dS.\end{aligned} \tag{12}$$

Equation (1) is used to deduce the last step of (12). In the right-hand side of (12), the first two terms may be interpreted, respectively, as the negative of the complex electric reaction $P_e$ and the complex magnetic reaction $P_m$ stored in the enclosed element volume $V$ of Fig. 3(a). $\hat{\mu}$ and $\hat{\epsilon}$ are the magnetic permeability and electric permittivity of the medium, respectively, which may be complex quantities in order to handle any magnetic or electric loss in the medium. If the top or/and bottom layer of a transmission line is a lossy ground plane, then the resulting metal loss can be modeled in (12) by treating the ground plane(s) as a lossy dielectric material.

The third term in the right-hand side of (12) involving $\overline{E}_l$ may be identified as the negative of the power radiated per unit length $P_l$ from the transmission line in the form of the leakage field. $P_l$ is equal to the distributed radiation impedance $Z_l$ per unit length, as defined earlier in (10). Let us call the last term in (12) a driving term, i.e., $P_d$, which for now is zero, because the total electric field $\overline{E}_T$ is zero on the central strip

$$\iint_{S_1+S_2} \overline{E}\times\overline{H}\cdot\overline{dS} = -P_e - P_m - P_l + P_d = -P_e - P_m - Z_l \tag{13}$$

$$P_l = -\iint_{\text{strip}} \overline{E}_l\cdot\overline{J}\,dS = -\int_{\text{strip},\,x=0} \overline{E}_l\cdot\overline{J}\,dy = Z_l. \tag{14}$$

Since the total strip current is initially assumed to be unity, the integration of $\overline{E}\times\overline{H}$ over the transverse cross section $S_1$ in Fig. 3(a) is equal to the negative of the complex characteristic impedance $Z_c$ [11]. This is based on a power-current (or alternately reaction-current) definition of the characteristic impedance, where the reaction is defined as $VI = P = I^2 Z_c = Z_c$ (because $I = 1$), and $P$ is cross-sectional power (= reaction, $I = 1$) due to the bound fields [11]

$$\iint_{S_1} \overline{E}\times\overline{H}\cdot\overline{dS} = -Z_c. \tag{15}$$

The contribution to the reaction integral from the surface $S_2$ can be derived from that of $S_1$ by including the incremental phase shifts of the field quantities over the element length $\Delta x = 1$

$$\iint_{S_2} \overline{E}\times\overline{H}\cdot\overline{dS} = Z_c e^{-2jk_e\Delta x} \simeq Z_c(1-2jk_e). \tag{16}$$

Adding (15) to (16), and then using the results in (13), we get

$$-2jZ_ck_e = -P_e - P_m - Z_l. \tag{17}$$

Consider a perturbation in the equivalent geometry of Fig. 3(a), where the propagation constant $k_x = k_e$ is changed to $k_x = k_e + \Delta k_e$. The perturbed situation is shown in Fig. 3(b). Let us represent all new quantities with primed variables, with the corresponding change with a prefix $\Delta$. The perturbed field components are in general different from the corresponding original fields. However, like the unperturbed case, the bound fields in the perturbed case also independently satisfy Maxwell's equations inside the enclosed volume $V$, and the same boundary conditions everywhere on the surfaces $(S_1 + S_2 + S_3)$. However, the perturbed boundary conditions on the strip will change. The total electric field $\overline{E}_T$ in the perturbed problem will no longer be zero on the strip. The leakage radiation fields $\overline{E}_l$ and $\overline{H}_l$ and, hence, $Z_l$, are also strongly affected by the change in $k_e$. As the total field changes, as well as the leakage field (which is only a part of the total field), the remaining bound field $\overline{E}$ will also change accordingly. On the strip, we have

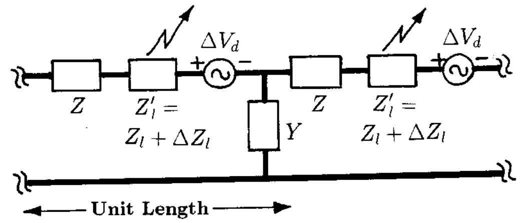


Fig. 4. Circuit equivalent of the perturbed stripline geometry of Fig. 3(b) (with propagation constant $k_e + \Delta k_e$). For the unperturbed case of Fig. 3(a) (with propagation constant $k_e$), the changes necessary in the equivalent circuit are $\Delta V_d = 0$ and $\Delta Z_l = 0$. $Z$ and $Y$ ideally remain unchanged with the perturbation. Whereas $Z_l$ is the distributed series impedance due to the leakage radiation, which is strongly changed by the perturbation.

$$\Delta\overline{E}_T\cdot\hat{p} = (\Delta\overline{E}_l + \Delta\overline{E})\cdot\hat{p} \neq 0 \tag{18}$$

$$\Delta\overline{E}\cdot\hat{p}\neq 0,\qquad -\Delta\overline{E}_l\cdot\hat{p}\neq 0,$$
$$\Delta\overline{E}\cdot\hat{p}\neq -\Delta\overline{E}_l\cdot\hat{p},\qquad \hat{p}=\hat{x},\,\hat{y}. \tag{19}$$

Under these conditions, we can reapply the reaction equations (12)–(17) to the new perturbed situation of Fig. 3(b). The final results may be expressed as follows:

$$-2j(Z_c+\Delta Z_c)(k_e+\Delta k_e) = -(P_e+\Delta P_e) - (P_m+\Delta P_m) - (Z_l+\Delta Z_l) + \Delta P_d \tag{20}$$

$$P_d + \Delta P_d = \Delta P_d = -\iint_{\text{strip}} (\overline{E}_T + \Delta\overline{E}_T)\cdot\overline{J}\,dS = -\int_{\text{strip},\,x=0} \Delta\overline{E}_T\cdot\overline{J}\,dy = \Delta V_d. \tag{21}$$

The equation (17) may be subtracted from (20), keeping only the first-order differential terms

$$-2j(Z_c\Delta k_e + k_e\Delta Z_c) = -\Delta P_e - \Delta P_m - \Delta Z_l + \Delta V_d. \tag{22}$$

From the above field analysis, it may be useful to derive a dual circuit representation for the bound-mode transmission line of Fig. 3(a). This is shown in Fig. 4, where the four terms in (13), (17), (20), (22), i.e., $P_e$, $P_m$, $P_l$ and $P_d$, respectively, correspond to the four circuit elements $Y$, $Z$, $Z_l$, and $\Delta V_d$ of Fig. 4. The analytical relationships between the corresponding elements can be established via simple circuit theory [15]. Careful attention may be given to the signs of different field components associated with the four parts. Some specific points may be observed, which are of particular significance to the new perturbation field-circuit treatment. The distributed driving voltage $\Delta V_d$ in Fig. 4 represents $\Delta P_d$ in (20), which, as shown in (21), is related to the tangential total electric field (not bound electric field) $\overline{E}_T$ on the strip. Consistent with the conditions of tangential $\overline{E}_T$ on the strip, as discussed, $\Delta V_d$ is zero without the perturbation, but nonzero under the perturbation. Next, the leakage-radiation impedance $Z_l$ relates to the leakage field $\overline{E}_l$, and is strongly affected by the perturbation in $k_e$. This is because the power associated with a leakage radiation is strongly dependent on the phasing of the source of radiation or, equivalently, $k_e$. In contrast to $Z_l$, the distributed series impedance $Z$ and shunt admittance $Y$ are determined only by the geometrical structure and physical medium, not by the mode of excitation of the strip current or, equivalently, by the phasing due to $k_e$. Strictly speaking, the last condition is valid for TEM-type fields, but can be treated as a good approximations for quasi-TEM lines, or for leaky transmission lines where the "bound fields" closely resemble TEM or quasi-TEM fields. Fortunately, this is the case for many practical geometries. Under these constraints, we can make approximations in the dual field-circuit equations of (22)

$$P_m = I^2 Z = Z, \qquad \Delta P_m = \Delta Z \simeq 0 \tag{23}$$

$$\begin{aligned} P_e &= V^2 Y = Z_c^2 Y \\ \Delta P_e &= Y \Delta(Z_c^2) \simeq 2(Z_c Y)\Delta Z_c = 2jk_e \Delta Z_c. \end{aligned} \tag{24}$$

Equations (23) and (24) may be used to simplify (22)

$$\begin{aligned} -2jZ_c\Delta k_e &= -\Delta Z_l + \Delta V_d = \int_{\text{strip},\, x=0} \Delta\overline{E}_l \cdot \overline{J}\, dy \\ &\quad - \int_{\text{strip},\, x=0} \Delta\overline{E}_T \cdot \overline{J}\, dy \\ &= -\int_{\text{strip},\, x=0} \Delta(\overline{E}_T - \overline{E}_l) \cdot \overline{J}\, dy \\ &= -\int_{\text{strip},\, x=0} \Delta\overline{E} \cdot \overline{J}\, dy = -\Delta Z_b \end{aligned} \tag{25}$$

$$Z_c = \frac{1}{2j}\frac{\Delta Z_b}{\Delta k_e} = \frac{1}{2j}\frac{\partial Z_b(k_x)}{\partial k_x}\bigg|_{k_x=k_e}. \tag{26}$$

The expression of $\Delta Z_b$ in (25) may be compared with that of $Z_T$ in (8) and (9). It should be recognized that $Z_b(k_x)$ is equal to the part of $Z_T(k_x)$ after excluding the contribution from the leakage field. In other words, $Z_b$ is the moment function of the bound electric field $\overline{E}$ (instead of $\overline{E}_T$ for $Z_T$) on the source $J$ at $x = 0$.

$$\begin{aligned} Z_b(k_x) &= \int_{\text{trip},\, x=0} \overline{E}(y;\, k_x) \cdot \overline{J}(y)\, dy \\ &= \int_{\text{strip},\, x=0} \overline{E}(y;\, k_x) \cdot \overline{f}(y)\, dy. \end{aligned} \tag{27}$$

When the transverse variation of the strip current $\overline{f}(y)$ is known *a priori*, at least approximately, $Z_b$ can be derived from the total moment function $Z_T$ of (9) by properly excluding the contribution due to the leakage field via residue theory or, in other words, by properly extracting the part only due to the bound fields. A simple method of pole extraction can be used in the expression of $Z_T$ in (9) to derive $Z_b$

$$\begin{aligned} Z_b(k_x) &= Z_T(k_x) - \frac{1}{2\pi}\int_C \tilde{\overline{f}}(k_y) \\ &\quad \cdot \Bigg[\text{Res}\Big\{\tilde{\overline{G}}_E(-k_x,\, k_y)\Big\}_{k_y=k_{yp}^+} \frac{1}{(k_y - k_{yp}^+)} \\ &\qquad + \text{Res}\Big\{\tilde{\overline{G}}_E(-k_x,\, k_y)\Big\}_{k_y=k_{yp}^-} \frac{1}{(k_y - k_{yp}^-)}\Bigg] \\ &\quad \cdot \tilde{\overline{f}}(-k_y)\, dk_y \end{aligned} \tag{28}$$

$$k_{yp}^{\pm} = \pm\sqrt{k_{\text{sw}}^2 - k_x^2}, \qquad \text{Im}(k_{yp}^+) > 0 \tag{29}$$

where $k_{\text{sw}}$ is the propagation constant of the characteristic wave (surface-wave or parallel-plate wave) of the surrounding structure to which power leaks from the transmission line. When the strip width is electrically small (most practical cases,) it can be shown that the second part of (28) may be approximated by residues of the integrand of $Z_T$ in (9)

$$\begin{aligned} Z_b(k_x) \simeq Z_T(k_x) + \pi j\Big[&\text{Res}\big\{Z_T(k_x,\, k_y = k_{yp}^+)\big\} \\ &- \text{Res}\big\{Z_T(k_x,\, k_y = k_{yp}^-)\big\}\Big] \end{aligned} \tag{30}$$

where $\text{Res}\{Z_T(k_x,\, k_y = k_{yp}^{\pm})\}$ means the residue of (9) at $k_y = k_{yp}^{\pm}$. It is of mathematical significance that in the above expression of $Z_b$, we have $\pi j$, and not $2\pi j$. This means, in order to obtain the "bound mode" moment function $Z_b$, one needs to subtract from the total moment function $Z_T$ only half of its residue contribution around the poles at $k_y = k_{yp}^{\pm}$.

If the transverse variation of the strip current is expanded using a basis set, as in (2), $Z_b$ can be expressed as a superposition of moment testing functions $[Z_{bij}]_{(1\times N)}$. The moment testing functions $[Z_{bij}]_{(1\times N)}$ can be derived/extracted from the corresponding total testing functions $[Z_{Tij}]_{(1\times N)}$ of (7)

$$Z_b(k_x) = \sum_{j=1}^{N} a_j \sum_{i=1}^{N} a_i Z_{bij}(k_x) = [a_i]^T \big[Z_{bij}(k_x)\big][a_i] \tag{31}$$

$$\begin{aligned} Z_{bij}(k_x) = Z_{Tij}(k_x) + \pi j\Big[&\text{Res}\big\{Z_{Tij}(k_x,\, k_y = k_{yp}^+)\big\} \\ &- \text{Res}\big\{Z_{Tij}(k_x,\, k_y = k_{yp}^-)\big\}\Big]. \end{aligned} \tag{32}$$

The results of (26)–(32) may now be summarized as follows. The characteristic impedance of a strip-type transmission line is $1/2j$ times the derivative of the moment (reaction) function $Z_b(k_x)$, computed at $k_x = k_e$. The $Z_b(k_x)$ needed for this computation can be derived from the total moment function $Z_T$ [or, alternately from moment testing functions $Z_{Tij}(k_x)$] by just extracting out the singular contribution at the surface-wave poles. It is important to note that the same moment function $Z_T(k_x)$ is also used to search for the complex propagation constant $k_e$ of the line, where $Z_T(k_x = k_e) = 0$. In other words, the zero of

$Z_T$ gives the complex propagation constant, while the derivative of $Z_T$ (after simple removal of the singular contribution) provides the characteristic impedance of the line without any additional computation or analytical formulation. The derivative may be numerically computed at $k_x = k_e$ by incrementing the $k_x$ along any convenient direction on the complex plane, which is already performed anyway in the process of searching for the propagation constant [1].

It is also important to observe that, in a spectral-domain solution of the propagation constant of a leaky line, one must *include* the pole contribution [1] in the computation. Whereas for the computation of the characteristic impedance of the leaky line, the present process requires to *exclude* the pole contribution for a correct result. This conclusion follows from the theoretical derivation we have presented, the validity of which will be demonstrated through independent comparisons in the following section.

### *B. Slot-Type Leaky Lines*

The results are similar for a slot-type line, except that in the final equations, the characteristic impedance $Z_c$ should be replaced by characteristic admittance $Y_c$ of the slotline, and the various electric-field moment functions $Z_b$, $Z_T$, $Z_{bij}$, and $Z_{Tij}$ be replaced by appropriate magnetic-field moment functions $Y_b$, $Y_T$, $Y_{bij}$, and $Y_{Tij}$, respectively. Fig. 5 shows the geometry of a slot-type leaky transmission line, indicating changes required when the complex propagation constant $k_e$ is perturbed. The corresponding circuit model is shown in Fig. 6. In contrast to the electric currents $\overline{J}$ used to model the central strip of a strip-type leaky line, here, two equivalent magnetic surface currents $\overline{M} = \overline{E_s} \times \hat{z}$ and $-\overline{M}$ are placed, respectively, slightly above and below the slot plane in order to model the slot-electric field $\overline{E_s}$. In the above equivalent magnetic-current formulation, one needs to fill in the slot region by continuation of the surrounding ground plane. In addition, the boundary condition that needs to be enforced now is the continuity of the magnetic fields across the two sides of the slot (or $\Delta\overline{H}$ across the $\mathrm{slot} = 0$). Based on the above changes, and following similar steps as for strip-type leaky lines in Section II-A, we present here only the key expressions for the characteristic admittance, $Y_c$ of a slot-type leaky line

$$Y_c = \frac{1}{2j}\frac{\Delta Y_b}{\Delta k_e} = \frac{1}{2j}\left.\frac{\partial Y_b(k_x)}{\partial k_x}\right|_{k_x=k_e} \tag{33}$$

$$Y_b(k_x) = \sum_{j=1}^{N} a_j \sum_{i=1}^{N} a_i Y_{bij}(k_x) = [a_i]^T\left[Y_{bij}(k_x)\right][a_i] \tag{34}$$

$$Y_b(k_x) = Y_T(k_x) + \pi j\Big[\mathrm{Res}\{Y_T(k_y = k_{yp}^+)\} - \mathrm{Res}\{Y_T(k_y = k_{yp}^-)\}\Big] \tag{35}$$

$$Y_{bij}(k_x) = Y_{Tij}(k_x) + \pi j\Big[\mathrm{Res}\{Y_{Tij}(k_y = k_{yp}^+)\} - \mathrm{Res}\{Y_{Tij}(k_y = k_{yp}^-)\}\Big] \tag{36}$$

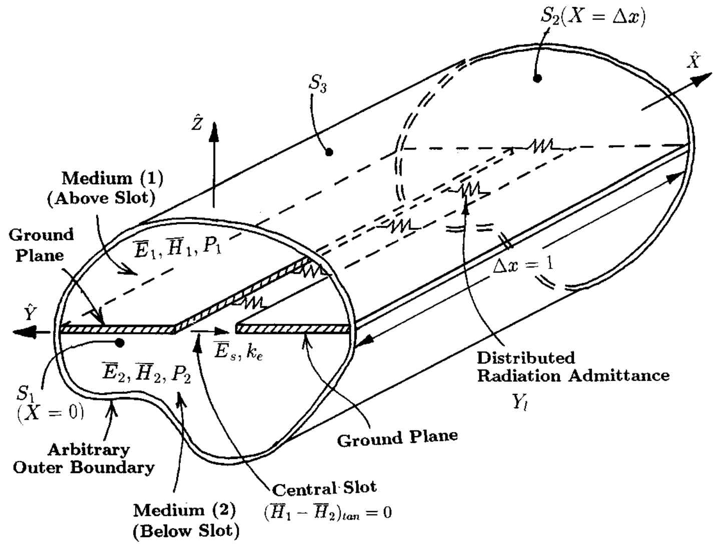


Fig. 5. Equivalent bound geometry of a general slot-type leaky line, with distributed radiation admittance $Y_l$ to account for the leakage radiation. The outer arbitrary boundary is chosen on a outer metal wall (when there is a ground plane) or sufficiently far away (on sides or on top/bottom when open) such that $\overline{E}, \overline{H} \simeq 0$. Only one slot is shown here. For coplanar waveguides, which are also treated as slot-type lines, the single slot in this figure can be replaced with two slots, each of width $W$, with center-to-center separation $= S$. As in Fig. 3(b), the perturbed case here for the slotline would require the following changes: $\overline{E}_s \rightarrow \overline{E}_s'$, $\overline{E}_1 \rightarrow \overline{E}_1'$, $\overline{H}_1 \rightarrow \overline{H}_1'$, $\overline{E}_2 \rightarrow \overline{E}_2'$, $\overline{H}_2 \rightarrow \overline{H}_2'$, $k_e \rightarrow k_e' = k_e + \Delta k_e$, $P_1 \rightarrow P_1'$, $P_2 \rightarrow P_2'$, $Y_l \rightarrow Y_l'$, and $(\overline{H}_1' \rightarrow \overline{H}_2')_{\tan} \neq 0$ across the slot.

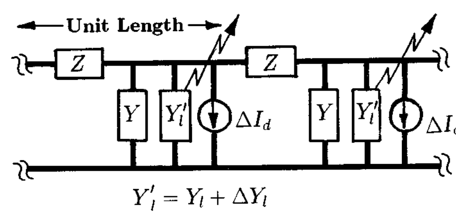


Fig. 6. Circuit equivalent of Fig. 5 for the perturbed case. For the corresponding unperturbed case, $\Delta I_d = 0$ and $\Delta Y_l = 0$. As in the stripline equivalent circuit of Fig. 4, $Z$ and $Y$ are ideally unchanged with perturbation. Whereas the distributed shunt admittance $Y_l$ due to leakage radiation is strongly affected by the perturbation.

$$Y_{Tij}(k_x) = \frac{1}{2\pi}\int_C \tilde{\tilde{f}}_i(k_y)\cdot\Big[\overset{\cong}{G}_{H1M}(-k_x, k_y) + \overset{\cong}{G}_{H2M}(-k_x, k_y)\Big]\cdot\tilde{\tilde{f}}_j(-k_y)\, dk_y \tag{37}$$

$$Y_T(k_x) = \frac{1}{2\pi},\int_C \tilde{\tilde{f}}(k_y)\cdot\Big[\overset{\cong}{G}_{H1M}(-k_x, k_y) + \overset{\cong}{G}_{H2M}(-k_x, k_y)\Big]\cdot\tilde{\tilde{f}}(-k_y)\, dk_y. \tag{38}$$

In (37) and (38), $\overset{\cong}{G}_{H1M}(k_x, k_y)$ is the spectral dyadic Green's function for magnetic field observed above the slot plane, produced due to a magnetic current placed on top of the slot plane, whereas $\tilde{\tilde{G}}_{H2M}(k_x, k_y)$ is the same, but observed below the slot plane, produced due to a magnetic current placed below the slot plane. $\tilde{\tilde{G}}_{H1M}$ and $\tilde{\tilde{G}}_{H2M}$ may be derived as in [5], [16], and [17]. $\tilde{\overline{f}}(y)$ is the transform of the transverse variation $\overline{f}(y)$ of the equivalent magnetic current placed on top of the slot (negative for below the slot). $\overline{f}(y)$ is expanded using $N$ basis functions with unknown coefficients $a_i$ and normalized such that the total slot voltage is unity.

### C. General Comments

As described in the derivation, the new theory models the leakage loss, in addition to any material loss that may be present [dielectric or magnetic loss, and metal loss in the ground plane(s)]. The same general procedure will also apply when there is metal loss in the strip. In this case, an appropriate impedance boundary condition is to be used [1] on the strip, which can be shown to result in an additional reaction term in (17) and (20). The additional term can be treated as an extra distributed impedance in the circuit model. Assuming that this new strip-loss element is independent of any perturbation in $k_e$ (similar to the case for $Z$ and $Y$), the final expression for the characteristic impedance in (26) will remain unchanged.

Clearly, as a simpler special case, the method will also apply when there is no leakage. In this case, the treatment is actually simplified by having $Z_T = Z_b$ and $Y_T = Y_b$ without the need for any special pole extraction. This is, however, significant, considering that unlike the conventional methods (power-current or voltage-current) [5], [18], [19] used in the past, no additional theoretical or computational effort will be needed in order to obtain the characteristic impedance. It may also be mentioned that, though the theory is explicitly developed for printed-type transmission lines, it is applicable in principle to nonplanar lines as well. Except, for nonplanar lines the formulation only in the spatial domain would be meaningful, while the spectral $(k_y)$ formulation will no longer be relevant.

## III. Results

### A. Strip-Type Lines

We first check the correctness of the theory for the special case of a nonleaky strip-type line, for which data for the characteristic impedance are well established. We chose a standard microstrip line without any material loss. Here, the characteristic impedance is a real number because there is no leakage or material loss. The characteristic impedance using the new wavenumber perturbation theory is compared in Fig. 7 with that from a commonly used power-current definition [5], [18], [19], [20]. Fig. 7 shows practically no difference in the two sets of results. As we have discussed in Section II, in the new method, we obtain the $Z_C$ from information already available in a moment-method solution of the propagation constant of the line. Whereas in the power-current method, one needs a separate reformulation in order to find the cross-sectional power $P_c$, from which the characteristic impedance is derived as $Z_c = P_c/I^2$. For a standard single-layer microstrip line, the additional effort needed may not be that significant. However, the effort could significantly increase for complex multilayering arrangements, in which case, the present method may be analytically and/or computationally superior.

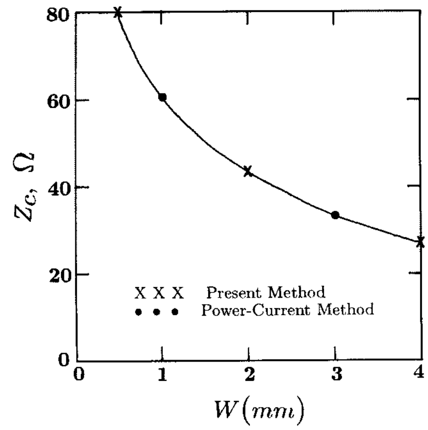


Fig. 7. Characteristic impedance of a standard nonleaky microstrip line computed using the new wavenumber perturbation method, as compared with that computed using a power-current method. Strip width $= W$. Substrate: relative dielectric constant $= \epsilon_r = 10.2$, thickness $= d = 0.127$ cm, no material loss. Frequency $= 10$ GHz.

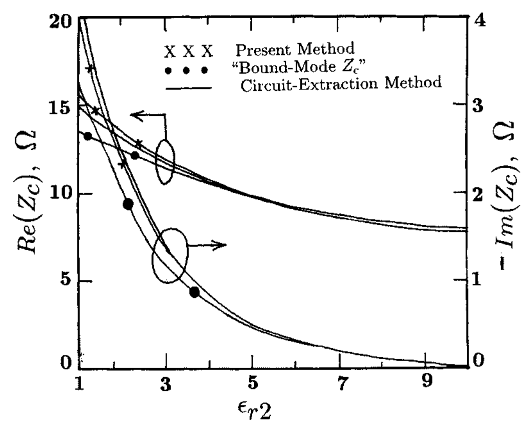


Fig. 8. (a) Real and (b) imaginary parts of the complex characteristic impedance of a leaky two-layer stripline calculated using the new method and compared with the "bound $Z_c$" method [11] and the "circuit extraction" method [12]. Strip width $= W = 0.3$ cm, frequency $= 10$ GHz. Lower substrate: relative dielectric constant $\epsilon_{r1} = 10.2$, thickness $= d_1 = 0.127$ cm. Upper substrate: relative dielectric constant $= \epsilon_{r2}$, thickness $= d_2 = 0.0254$ cm.

We then apply the theory to a leaky stripline, consisting of two different substrates above and below the central strip. In a standard stripline, the same substrate is used above and below the strip, which results in having the central strip placed in a uniform medium, placed symmetrically between the top and bottom conducting planes. In contrast to the standard stripline, for the stripline we consider here the top substrate is thinner than the bottom substrate, and the dielectric constant of the top substrate $(\epsilon_{r2})$ is lower than that $(\epsilon_{r1})$ of the lower substrate. Under this condition, it is known that the stripline can leak power to the parallel-plate mode [1], [3], [21]. The characteristic impedance as well as the propagation constant of the line becomes a complex number, with both real and imaginary parts. Fig. 8 shows the real and imaginary parts of the characteristic impedance of one such leaky stripline as a function of the dielectric constant $\epsilon_{r2}$ of the upper substrate. The results are compared with two

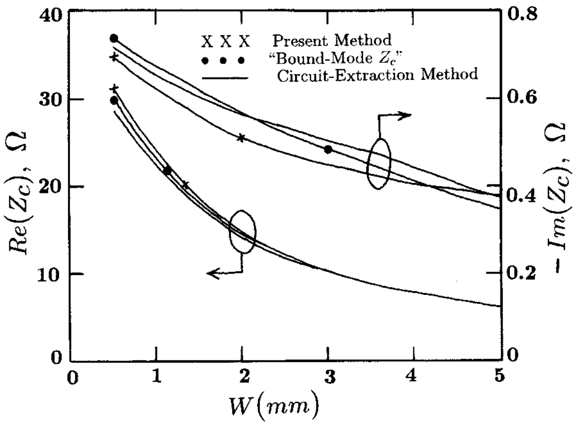


Fig. 9. Results for Fig. 8 as a function of strip width $W$ with $\epsilon_{r2} = 5.0$.

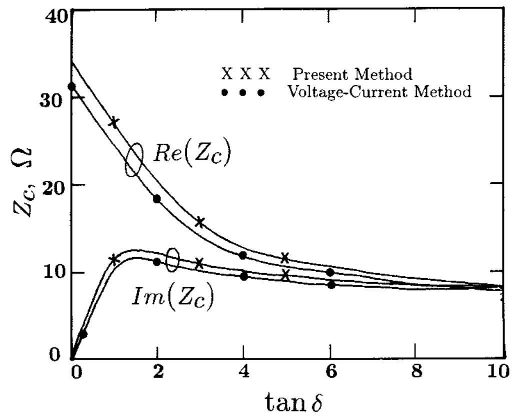


Fig. 10. Example of a standard stripline case with large substrate material loss (dielectric only) for which the characteristic impedance is calculated as a function of the substrate loss tangent = $\tan\delta$ using the new wavenumber perturbation method, and compared with that using a well-known voltage-current method. Strip width = $W = 0.12$ cm. Substrate: relative dielectric constant (complex) = $\epsilon_r = 10.2(1 - j\tan\delta)$, thickness = $d = 0.127$ cm (each above and below the strip). Frequency = $10$ GHz. This example validated the new method when material loss exists (small or large loss).

independent set of results computed using: 1) a power–current definition based on the bound-mode power [11] and 2) a circuit-extraction method [12]. All three methods agree well with each other. As can be seen from Fig. 8, when $\epsilon_{r2}$ is equal to $\epsilon_{r1}$ $(= 10.2)$, the characteristic impedance becomes purely real. This is expected because, in this limiting case, the transmission line becomes a purely TEM structure and, hence, nonleaky. In the absence of the leakage, and also in the absence of any material loss as assumed here, the propagation constant and the characteristic impedance of the limiting TEM line are purely real quantities. As $\epsilon_{r2}$ deviates from $\epsilon_{r1}$, the line becomes increasingly leaky with a nonzero value for the imaginary part of the characteristic impedance. The real part of the impedance (and the magnitude) reduces with increase in $\epsilon_{r2}$, as expected from basic impedance principle. It may be noticed that the imaginary part of the characteristic impedance is negative. This is consistent with the circuit model of Fig. 4, where $Z$ and $Y$ and purely imaginary numbers, and $Z_l$ is a series impedance with a positive real part.

The results for the same leaky geometry of Fig. 8, but as a function of the strip width $W$ (while keeping $\epsilon_{r2} = 5.0$), are plotted in Fig. 9, showing similar comparison with the methods of [11] and [12]. The magnitude of impedance is seen to reduce with increasing $W$, which is a normal behavior for strip-type lines.

Next, we need to validate the new theory for cases with material loss, particularly when the loss is high. We compute the characteristic impedance of a uniform stripline, which does not exhibit any leakage, but is now lossy with a relatively large loss tangent of the dielectric substrate. Like a stripline line with leakage, the propagation constant and the characteristic impedance for the lossy line are also complex numbers. However, it may be recognized that the present theory of the characteristic impedance treats the leakage loss differently from the material loss. It is important to see how the method works for the case with lossy materials as opposed to a case with leakage loss. We have independent computations of the characteristic impedance for the lossy stripline using a voltage-current approach. The results are compared in Fig. 10, showing good agreement that further supports the fundamentals of the theory. Notice that the imaginary part of the lossy stripline in Fig. 10 is positive, whereas that of a leaky stripline in Figs. 8 and 9 is negative. These are consistent with the circuit model in Fig. 4, where the leakage introduces a positive real part to the series impedance, while the dielectric loss introduces a positive real part to the shunt admittance of the transmission line.

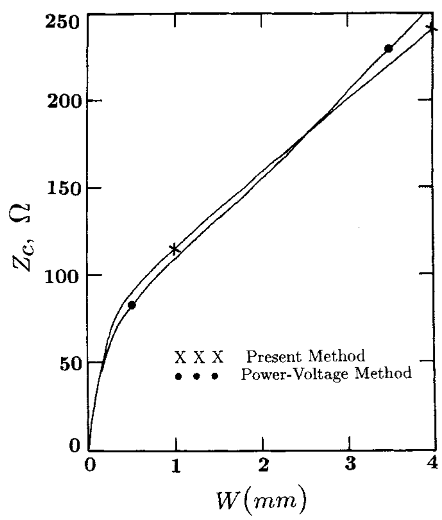


Fig. 11. Comparison of the characteristic impedance of a standard nonleaky slotline computed using the new wavenumber perturbation method, with that using the standard power-voltage method. Slotline width = $W$. Substrate: relative dielectric constant = $\epsilon_r = 10.2$, thickness = $d = 0.127$ cm, no material loss. Frequency = $10$ GHz.

### *B. Slot-Type Lines*

As a first check of the theory for slot-type lines, we apply the new method to a standard nonleaky slotline. Fig. 11 presents the computed results from the new method, as compared with those from a standard power-voltage method $Z_c = V^2/P_c$ [18]–[20]. The comparison of the results continues to be good, though not as good as in the case of a nonleaky microstrip line in Fig. 7.

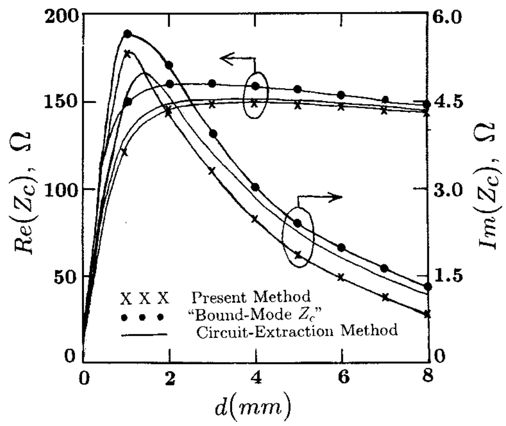


Fig. 12. (a) Real and (b) imaginary parts of the complex characteristic impedance calculated using the new method as compared with the "bound-mode $Z_c$" method of [11] and the "circuit extraction" method of [12] for a leaky CBCPW. Slot width = $W = 0.1$ cm, center-to-center separation = $S = 0.2$ cm, frequency = 10 GHz, substrate thickness = $d$, and substrate $\epsilon_r = 6.0$.

We turn to a slot-type transmission line with power leakage. We selected a conductor-backed coplanar waveguide (CBCPW) for the investigation. The mechanical support provided by the conductor backing of a CBCPW is a particular attraction for integrated-circuits applications. Unfortunately, the geometry is known to suffer from the power leakage problem [2]. The results of the real and imaginary parts of the complex characteristic impedance of a CBCPW, obtained from the present theory, are compared in Fig. 12 with two sets of independent results from [11] and [12]. Good agreement is seen between the independent methods, which demonstrates the applicability of the new method to slot-type leaky lines as well. It should be observed that the imaginary part of the characteristic impedance is positive. This is consistent with the circuit model of Fig. 6, where $Z$ and $Y$ are purely imaginary numbers and $Y_l$ is a shunt impedance with a positive real part. Further, the impedance is zero when the conducting back plane is placed at zero distance from the coplanar waveguide ($d = 0$). This happens due to the short-circuiting effect of the conductor back plane. This is a trivial limiting situation. As the back plane is displaced away from the top plane, the leakage level is expected to sharply increase as $d$ increases, due to strong coupling to the parallel-plate field. However, as $d$ is increased beyond certain limit ($d > S + W$), the leakage should reduce due to eventual weakening of the interaction between the coplanar waveguide fields on top and the conducting back plane at the bottom. The above trend may be seen with the imaginary part of the characteristic impedance in Fig. 12. Accordingly, the imaginary part first exhibits a sharp increase from zero at $d = 0$, and than gradually reduces as $d$ is increased. The real part, on the other hand, increases sharply from a short circuit at $d = 0$, but it quickly saturates due to weak coupling with the back plane. Other studies involving change in the dielectric constant, $S$ and $W$ have also been conducted, showing similar comparison with the independent methods.

## IV. Conclusion

We have presented a new derivation for the characteristic impedance of general printed transmission lines, applicable to conditions when power leaks to a background mode. The theory was numerically implemented for both slot- and strip-type lines that may include dielectric and metal loss. Using this approach, the characteristic impedance is computed together with the propagation constant in a single process, employing information obtained by perturbing the propagation constant of the transmission line. The method applies to nonleaky printed lines as well and, in principle, would also apply to nonplanar transmission lines. When leakage exists, the derivation of the characteristic impedance requires proper removal of the pole contributions (due to background surface-wave or parallel-plate-type modes) from various spectral integrations. As an interesting contrast, the derivation of the propagation constant requires inclusion of the pole contributions in order to properly model the leakage.

We demonstrated the validity and accuracy of the theory through case studies of strip-type as well as slot-type lines. We presented only selected cases to demonstrate the theory for representative situations. However, we have performed other studies for strip- and slot-type lines with different substrate and physical parameters, and with increased number of layers. All observations are generally consistent with those we have presented here, together establishing significant confidence in the validity of the new "wavenumber perturbation" theory to diverse conditions. Together with [1] for the propagation modeling, the present theory of the characteristic impedance is expected to provide a unified analytical/computational framework for a complete modeling of general printed transmission lines.


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## References


[1] N. K. Das and D. M. Pozar, "Full-wave spectral-domain computation of material, radiation and guided wave losses in infinite multilayered printed transmission lines," *IEEE Trans. Microwave Theory Tech.*, vol. 39, pp. 54–63, Jan. 1991.

[2] H. Shigesawa, M. Tsuji, and A. A. Oliner, "Conductor backed slot-line and coplanar waveguide: Dangers and full-wave analyses," in *IEEE MTT-S Int. Microwave Symp. Dig.*, 1988, pp. 199–202.

[3] L. Carin and N. K. Das, "Leaky waves in broadside-coupled microstrips," *IEEE Trans. Microwave Theory Tech.*, vol. 40, pp. 58–66, Jan. 1992.

[4] D. Nghiem, J. T. Williams, and D. R. Jackson, "Leakage of the dominant mode on stripline with a small air gap," *IEEE Trans. Microwave Theory Tech.*, vol. 43, pp. 2549–2556, Nov. 1995.

[5] N. K. Das and D. M. Pozar, "A generalized spectral-domain Green's function for multilayer dielectric substrates with applications to multilayer transmission lines," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-35, pp. 326–335, Mar. 1987.

[6] R. W. Jackson, "Considerations in the use of coplanar waveguide for millimeter-wave integrated circuits," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-34, pp. 1021–1027, Dec. 1986.

[7] E. J. Denlinger, “A frequency dependent solution for microstrip transmission lines,” *IEEE Trans. Microwave Theory Tech.*, vol. MTT-19, pp. 30–39, Jan. 1971.

[8] D. Mirshekar-Syahkal and J. B. Davies, “Accurate solution of microstrip and coplanar structures for dispersion and for dielectric and conductor losses,” *IEEE Trans. Microwave Theory Tech.*, vol. MTT-27, pp. 694–699, July 1979.

[9] F. Arndt and G. U. Paul, “The reflection definition of the characteristic impedance of microstrips,” *IEEE Trans. Microwave Theory Tech.*, vol. MTT-27, pp. 724–731, Aug. 1979.

[10] J. C. Rautio, “A new definition of characteristic impedance,” in *IEEE MTT-S Int. Microwave Symp. Dig.*, vol. 2, June 1991, pp. 761–764.

[11] N. K. Das, “Power leakage, characteristic impedance and mode-coupling behavior of finite-length leaky printed transmission lines,” *IEEE Trans. Microwave Theory Tech.*, vol. 44, pp. 526–536, Apr. 1996.

[12] ——, “Spectral-domain analysis of complex characteristic impedance of a leaky conductor-backed slotline,” in *IEEE MTT-S Int. Microwave Symp. Dig.*, 1996, pp. 1791–1794.

[13] T. Itoh, “Spectral domain immitance approach for dispersion characteristics of generalized printed transmission lines,” *IEEE Trans. Microwave Theory Tech.*, vol. MTT-28, pp. 733–736, July 1980.

[14] R. F. Harrington, *Time Harmonic Electromagnetic Fields*. New York: McGraw-Hill, 1984.

[15] D. M. Pozar, *Microwave Engineering*. Reading, MA: Addison-Wesley, 1990.

[16] N. K. Das, “A study of multilayered printed antenna structures,” Ph.D. dissertation, Dept. Elect. Comput. Eng., Univ. Massachusetts at Amherst, Amherst, MA, 1987.

[17] N. K. Das and D. M. Pozar, “A generalized CAD model for printed antennas and arrays with arbitrary multilayer geometries,” in *Computer Physics Communication, Thematic Issue on Computational Electromagnetics*, L. Safai, Ed. Amsterdam, The Netherlands: Elsevier, 1991, vol. 68, pp. 393–440.

[18] K. C. Gupta, R. Garg, and I. J. Bahl, *Microstrip Lines and Slotlines*. Norwood, MA: Artech House, 1979.

[19] T. Itoh, *Planar Transmission Line Structures, Edited Volume*. New York: IEEE Press, 1987.

[20] N. K. Das and D. M. Pozar, *PCAAMT–Personal Computer Aided Analysis of Multilayer Transmission Lines—Version 1.0*. Leverette, MA: Antenna Design Associates, 1990.

[21] J. T. Williams, N. Nghiem, and D. R. Jackson, “Proper and improper modal solutions for inhomogeneous stripline,” in *IEEE MTT-S Int. Microwave Symp. Dig.*, 1991, pp. 567–570.

**Nirod K. Das** (S’87–M’88) was born in Orissa, India, on February 27, 1963. He received the B.Tech degree in electronics and electrical communication engineering from the Indian Institute of Technology (IIT), Kharagpur, India, in 1985, and the M.S. and Ph.D. degrees in electrical engineering from The University of Massachusetts at Amherst, in 1987 and 1989, respectively.

From 1985 to 1990, he was with the Department of Electrical and Computer Engineering, The University of Massachusetts at Amherst, first as a Graduate Research Assistant and then as a Post-Doctoral Research Associate. In 1990, he joined the Department of Electrical Engineering, Polytechnic University of New York, Farmingdale, where he is currently an Associate Professor since 1997. He co-edited *Next Generation of MMIC Devices and Systems* (New York: Plenum, 1997). He also authored a computer-aided design (CAD)/instructional tool, i.e., PCAAMT, for microwave multilayer printed transmission lines, and another simulation tool, i.e., UNIFY, for unified modeling of multilayer printed antennas and arrays. His research interests have been in the general areas of electromagnetics, antennas, and microwave and millimeter-wave integrated circuits. His recent research activities include numerical–analytical methods for electromagnetics, multilayered integrated circuits, leaky waves and applications, and advanced materials for microwave circuits and antennas.

Dr. Das is a member of the IEEE Antennas and Propagation Society (IEEE AP-S), the IEEE Microwave Theory and Techniques Society (IEEE MTT-S), and the New York Academy of Sciences. He currently serves on the Editorial Board of the IEEE TRANSACTIONS ON MICROWAVE THEORY AND TECHNIQUES, and the Technical Program Committee of the IEEE MTT-S International Symposia. He was the co-chair of the 1996 International WRI Symposium, Brooklyn, NY. For his doctoral research work on multilayer printed antennas, he received a Student Paper Award (Third Prize) in 1990 from the U.S. National Council of the International Scientific Radio Union (URSI), and the R. W. P. King Paper Award (below 35 age group) in 1993 from the IEEE AP-S.