Electric Current and Electric Field Induced in the Human Body When Exposed to an Incident Electric Field Near the Resonant Frequency

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***Abstract*—The electric field and current density induced in the human body when this is exposed to electric fields near the resonant frequency, 53 MHz, are determined analytically. Since this frequency range includes an important amateur radio band of 50–60 MHz and exposure to electric fields at this frequency has been shown to be hazardous, the study has a specific motivation. A cylindrical model of the body is used to derive formulas for the total axial current and current density induced in the body subject to skin effect. Tabulations and graphical representations illuminate the results.**

***Index Terms*—Amateur radio electric fields, megahertz electric fields induced in human body, resonant electric fields in human body.**

I. Introduction

IN THE MODERN environment, people are exposed to electromagnetic fields from many sources over a wide range of frequencies. A systematic study is in progress to derive biologically useful formulas for the electric fields and currents induced in the human body (region 1, wavenumber k_1) as a whole, and in its organs and cells, when it is exposed to an incident electromagnetic field in the air (region 2, wavenumber k_2). This has been carried out for the important low-frequency ranges, *viz.* 50–60 Hz for high-voltage power lines and 10–30 kHz for high-power transmitters [1]–[7]. It has also been completed for the 1–30-MHz range for shipboard exposure to the fields of transmitters communicating with ships and the shore [8]. This paper continues the study with a determination of the total induced axial current $I_{1z}(z)$, the current density $J_{1z}(\rho, z)$, and the electric field $E_{1z}(\rho, z)$ when the body is exposed to an axial electric field $E_{2z}^{\rm inc}$ in the frequency range 50–200 MHz. This range includes the principal television frequencies, the FM broadcast frequencies 88–108 MHz, and most of the frequencies assigned to radio amateurs. This frequency range is of particular interest and importance because it contains the resonant frequency of the human body. As will be shown, this coincides with a principal amateur-radio frequency. Noteworthy is the fact that Milham [9] has reported that amateur radio operator licensees have a statistically determined, significantly increased mortality due to lymphatic and hematopoietic malignancies.

Possible effects due to exposure to high-power radar transmitters have been studied in rats and mice by scaling to electromagnetic fields at microwave frequencies. These studies indicate tumor promotion [10], [11], an increased incidence of leukemia [12], and the promotion of cell transformation *in vitro* [13]. If scaling is assumed to be valid according to the formulas $f_r h_r = f_m h_m$, $f_r a_r = f_m a_m$, where f is the frequency, h the half-length, and a the mean radius, these results could be significant for humans. Specifically, with $f_m = 100$ MHz, h_m, and a_m for a man, and f_r, h_r, and a_r for a rat or mouse, the frequency $f_r = 2450$ MHz gives $2h_r = 2h_m f_m / f_r = 100 \times 1.75/2450 = 0.071$ m or 7.1 cm. This is a reasonable length for a rat or mouse. The validity of such scaling is examined as part of this study.

II. Background

The biological effects of exposure to radio-frequency electromagnetic radiation have been a subject of study and concern for many years. In 1971, a special issue of the IEEE Transactions on Microwave Theory and Techniques [14] was published on the biological effects of microwaves. This contains four invited papers and 11 contributed papers with long lists of references to earlier work. Most of this early work was concerned with power absorption by the whole body or parts of the body when irradiated by plane-wave electromagnetic beams from high-power radar antennas and FM antennas. Absorption was measured by the temperature increase in a human body modeled by a tissue sphere [15]–[19], and a prolate spheroid [20]. Extensive studies by Gandhi [21] using both parallel-plate waveguide exposure of small models and full-size free-space irradiation determined the power deposition as a function of the electrical length of the body. He determined the resonant frequency of the human body with the length 1.75 m from the observed maximum power deposition. Decreased depositions occurred at lower and higher frequencies. He also observed that this maximum occurs when the incident electromagnetic field is polarized with the electric vector parallel to the length of the body. The measure for these observations was the heating of the body, including the convulsion time for animals. Using scaled biological-phantom figurines, Gandhi also observed that the power absorption coefficient for the neck was over 17 times the average value for the entire body. It is the purpose of the following analysis to provide clear explanations for these observations in terms of the total axial current, current density, and electric field induced throughout the human body. The localized power deposition is readily found from the current density with the use of the available conductivities of the different organs in the body.

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Very few studies have been directed specifically to the biological effects of exposure of the human body to electromagnetic fields in the frequency range from 50 to 200 MHz. Extensive work has been directed to the exposure of the human head to frequencies in the 900-MHz range. Studies that have possible application to the 50–200-MHz range of frequencies include papers by Sullivan [22], Chen *et al.* [23], and Chuang [24]. These are concerned with improving the accuracy and usefulness of the FDTD and other numerical methods. Also of interest are papers in a special issue of this TRANSACTIONS [25].

III. ANALYTICAL FORMULATION

The total axial current induced in the human body when this is approximated by a cylinder with half-length h and mean radius a is governed by an integral equation. This is [26, eqs. (28) and (30)[1]]

$$\int_{-h}^{h} I_{1z}(s)\frac{e^{-jk_2 r}}{r}\,ds = -\frac{j4\pi}{\zeta_0}[C\cos k_2 z + U^{\rm inc} - z^i P_z]. \tag{1}$$

Here, $\zeta_0 = 120\pi\ \Omega$, $U^{\rm inc} = E_{2z}^{\rm inc}/k_2$, $r = \sqrt{(z-s)^2 + a^2}$ and

$$\begin{aligned} P_{-z} = P_z &= \int_0^z I_{1z}(s)\sin k_2(z-s)\,ds \\ &= I_{1z}(0)\int_0^z \frac{I_{1z}(s)}{I_{1z}(0)}\sin k_2(z-s)\,ds. \end{aligned} \tag{2}$$

The impedance per unit length is [27, eq. (3.9-46a)]

$$\begin{aligned} z^i &= \frac{1}{\sigma_1 \pi a^2}\left(\frac{k_1 a}{2}\right)\frac{J_0(j^{-1/2}k_1 a)}{J_1(j^{-1/2}k_1 a)} \\ k_1 &= \omega[\mu_0(\epsilon_0\epsilon_{1r} - j\sigma_1/\omega)]^{1/2}. \end{aligned} \tag{3}$$

Here, σ_1 is the conductivity and ϵ_{1r} the relative permittivity of the human body.

The integral equation (1) has no exact solution. An approximate analytical solution expresses the current distribution in the cylinder by the well-known value for a parasitic antenna, *viz.*

$$\frac{I_{1z}(z')}{I_{1z}(z)} = \frac{\cos k_2 z' - \cos k_2 h}{\cos k_2 z - \cos k_2 h} \tag{4}$$

so that

$$\begin{aligned} &\int_{-h}^{h} I_{1z}(z')\frac{e^{-jk_2 r}}{r}\,dz' \\ &\quad= \frac{I_{1z}(z)}{\cos k_2 z - \cos k_2 h} \\ &\qquad\times \int_{-h}^{h}(\cos k_2 z' - \cos k_2 h)\frac{e^{-jk_2 r}}{r}\,dz' \\ &\quad= I_{1z}(z)\Psi_u(z) \sim I_{1z}(z)\Psi_u \end{aligned} \tag{5}$$

where

$$\begin{aligned} \Psi_u(z) = &(\cos k_2 z - \cos k_2 h)^{-1} \\ &\times \int_{-h}^{h}(\cos k_2 z' - \cos k_2 h)\frac{e^{-jk_2 r}}{r}\,dz'. \end{aligned} \tag{6}$$

[1] with $\delta = 0, \Theta_2 = 0, C_{2s} = 0$.

With

$$C_a(h,z) = \int_0^h \cos k_2 s\left[\frac{e^{-jk_2 r_1}}{r_1} + \frac{e^{-jk_2 r_2}}{r_2}\right] ds \tag{7}$$

$$E_a(h,z) = \int_0^h \left[\frac{e^{-jk_2 r_1}}{r_1} + \frac{e^{-jk_2 r_2}}{r_2}\right] ds \tag{8}$$

and

$$r_1 = [(z-s)^2 + a^2]^{1/2}, \quad r_2 = [(z+s)^2 + a^2]^{1/2} \tag{9}$$

it follows that

$$\begin{aligned} \Psi_u = \Psi_u(0) = &(1-\cos k_2 h)^{-1}\{C_a(h,0) - C_a(h,h) \\ &- [E_a(h,0) - E_a(h,h)]\cos k_2 h\}. \end{aligned} \tag{10}$$

With (5), (1) becomes

$$I_{1z}(z) = -\frac{j4\pi}{\zeta_0 \Psi_u}[C\cos k_2 z + U^{\rm inc} - z^i P_z]. \tag{11}$$

The constant C can be determined at $z = h$, $I_{1z}(h) = 0$. This gives

$$C = -\frac{U^{\rm inc} - z^i P_h}{\cos k_2 h} \tag{12}$$

so that

$$\begin{aligned} I_{1z}(z) = &\frac{j4\pi}{\zeta_0 \Psi_u}\frac{1}{\cos k_2 h}[(U^{\rm inc} - z^i P_h)\cos k_2 z \\ &- (U^{\rm inc} - z^i P_z)\cos k_2 h]. \end{aligned} \tag{13}$$

This reduces to

$$\begin{aligned} I_{1z}(z) = &\frac{j4\pi}{\zeta_0 \Psi_u \cos k_2 h}[U^{\rm inc}(\cos k_2 z - \cos k_2 h) \\ &- z^i(P_h\cos k_2 z - P_z\cos k_2 h)]. \end{aligned} \tag{14}$$

The evaluation of P_z and P_h is straightforward. With (4), (2) becomes

$$P_z = \frac{I_{1z}(0)}{1-\cos k_2 h}\int_0^z (\cos k_2 s - \cos k_2 h)\sin k_2(z-s)\,ds. \tag{15}$$

By expanding the sine, four trigonometric integrals are obtained that give

$$\begin{aligned} P_z &= I_{1z}(0)p_z \\ p_z &= \frac{1}{k_2}\left[\frac{\frac{1}{2}k_2 z\sin k_2 z - \cos k_2 h(1-\cos k_2 z)}{1-\cos k_2 h}\right] \end{aligned} \tag{16}$$

$$\begin{aligned} P_h &= I_{1z}(0)p_h \\ p_h &= \frac{1}{k_2}\left[\frac{\frac{1}{2}k_2 h\sin k_2 h}{1-\cos k_2 h} - \cos k_2 h\right]. \end{aligned} \tag{17}$$

With these values

$$\begin{aligned} I_{1z}(z) = &\frac{j4\pi}{\zeta_0 \Psi_u \cos k_2 h}[U^{\rm inc}(\cos k_2 z - \cos k_2 h) \\ &- I_{1z}(0)z^i(p_h\cos k_2 z - p_z\cos k_2 h)]. \end{aligned} \tag{18}$$

At $z = 0$

$$I_{1z}(0) = \frac{j4\pi}{\zeta_0 \Psi_u \cos k_2 h}[U^{\rm inc}(1 - \cos k_2 h) - I_{1z}(0) z^i p_h] \tag{19}$$

or

$$I_{1z}(0)\left(1 + \frac{j4\pi z^i p_h}{\zeta_0 \Psi_u \cos k_2 h}\right) = \frac{j4\pi U^{\rm inc}}{\zeta_0 \Psi_u \cos k_2 h}(1 - \cos k_2 h) \tag{20}$$

so that

$$\begin{aligned} I_{1z}(0) &= \frac{j4\pi U^{\rm inc}}{\zeta_0 \Psi_u \cos k_2 h}\left(\frac{1 - \cos k_2 h}{1 + \frac{j4\pi z^i p_h}{\zeta_0 \Psi_u \cos k_2 h}}\right) \\ &= \frac{U^{\rm inc}(1 - \cos k_2 h)}{(\zeta_0 \Psi_u \cos k_2 h / j4\pi) + z^i p_h}. \end{aligned} \tag{21}$$

When (21) is substituted in (18), this becomes

$$\begin{aligned} I_{1z}(z) = \frac{j4\pi U^{\rm inc}}{\zeta_0 \Psi_u \cos k_2 h}\bigg[&\cos k_2 z - \cos k_2 h \\ &- \frac{(1 - \cos k_2 h) z^i (p_h \cos k_2 z - p_z \cos k_2 h)}{(\zeta_0 \Psi_u \cos k_2 h / j4\pi) + z^i p_h}\bigg] \end{aligned} \tag{22}$$

or

$$\begin{aligned} I_{1z}(z) = \frac{j4\pi U^{\rm inc}}{\zeta_0 \Psi_u \cos k_2 h}\bigg\{ &\cos k_2 z - \cos k_2 h - \frac{j4\pi z^i}{\zeta_0} \\ &\times \left[\frac{(1 - \cos k_2 h)(p_h \cos k_2 z - p_z \cos k_2 h)}{\Psi_u \cos k_2 h + (j4\pi z^i / \zeta_0 p_h)}\right]\bigg\}. \end{aligned} \tag{23}$$

This is the final formula. However, it is of zero order and $I_{1z}(z) \to \infty$ when $k_2 h \to \pi/2$ at the resonant frequency. As discussed in [28] and defined in [29, eqs. (11b) and (12)], a more accurate first-order formula is obtained from (23) by substituting $\cos k_2 h + F(h)/\Psi_u$ for $\cos k_2 h$. Here

$$F(h) = \frac{j4\pi z^i}{\zeta_0} p_h - C_a(h,h) + E_a(h,h)\cos k_2 h. \tag{24}$$

With this substitution, the final first-order formula is

$$\begin{aligned} I_{1z}(z) = &\frac{j4\pi U^{\rm inc}}{\zeta_0[\Psi_u \cos k_2 h + F(h)]} \\ &\times \bigg\{\cos k_2 z - \cos k_2 h - \frac{j4\pi z^i}{\zeta_0} \\ &\times \left[\frac{(1 - \cos k_2 h)(p_h \cos k_2 z - p_z \cos k_2 h)}{\Psi_u \cos k_2 h + F(h) + (j4\pi z^i/\zeta_0) p_h}\right]\bigg\}. \end{aligned} \tag{25}$$

Note that the leading term in this distribution of current is $(\cos k_2 z - \cos k_2 h)$. This was used in evaluating the integral in (5).

Since the formula (25) is general, it reduces to the simple form for $k_2 h \ll 1$. Thus, $\cos k_2 z - \cos k_2 h \to (1/2) k_2^2 h^2 (1 - z^2/h^2)$, $z^i \to 1/\pi a^2 \sigma_1$, $p_z \to -k_2^{-1}(1 - z^2/h^2)$, $p_h \to 0$, and $F(h) \sim 0$. The integral in (5) is $\Psi_u = 2\ln(2h/a) - 3$. Hence, with $U^{\rm inc} = E_{2z}^{\rm inc}/k_2$

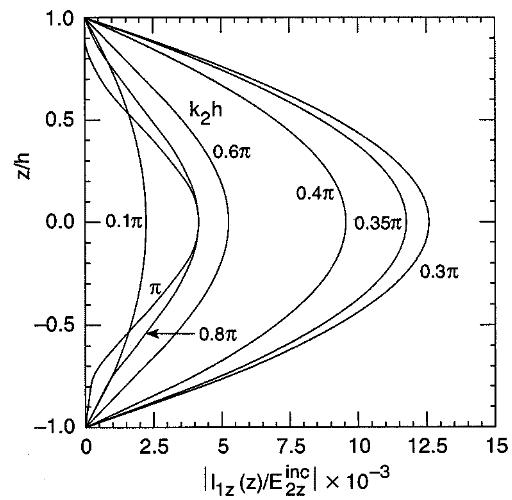


Fig. 1. Normalized total axial current $|I_{1z}(z)/E_{2z}^{\rm inc}|$ as a function of z/h for a cylinder with the radius $a = 0.14$ m and half-length $h = 0.875$ m; $k_2 h = 0.1\pi, 0.3\pi, 0.35\pi, 0.4\pi, 0.6\pi, 0.8\pi$, and π.

$$\begin{aligned} I_{1z}(z) &= \frac{j4\pi E_{2z}^{\rm inc}}{\zeta_0 k_2 \Psi_u}\left[\frac{k_2^2 h^2}{2}\left(1 - \frac{z^2}{h^2}\right) - \frac{j4\pi z^i}{\zeta_0 \Psi_u}\frac{k_2 h^2}{2}\left(1 - \frac{z^2}{h^2}\right)\right] \\ &= \frac{j2\pi k_2 h^2 E_{2z}^{\rm inc}}{\zeta_0 \Psi_u}\left(1 - \frac{z^2}{h^2}\right)\left(1 - \frac{j4\pi z^i}{\zeta_0 k_2 \Psi_u}\right) \\ &\sim \frac{j2\pi k_2 h^2 E_{2z}^{\rm inc}}{\zeta_0 \Psi_u}\left(1 - \frac{z^2}{h^2}\right) \end{aligned} \tag{26}$$

since

$$\left|1 - \frac{j4\pi z^i}{\zeta_0 k_2 \Psi_u}\right| \sim 1.$$

Formula (26) is precisely that previously derived [1].

Also of interest are the current density $J_{1z}(\rho, z)$ and the induced axial electric field $E_{1z}(\rho, z)$. These are represented as follows [30, eq. (13)]:

$$J_{1z}(\rho, z) = \frac{I_{1z}(z)}{\pi a^2}\left(\frac{k_1 a}{2}\right)\frac{J_0(j^{-1/2} k_1 \rho)}{J_1(j^{-1/2} k_1 a)}. \tag{27}$$

Here, J_0 and J_1 are Bessel functions. The axial component of the electric field is

$$E_{1z}(\rho, z) = \frac{J_{1z}(\rho, z)}{\sigma_1 - j\omega\epsilon_0\epsilon_{1r}} \tag{28}$$

where $\sigma_1 = 0.5$ S/m is the conductivity of the saline fluid that permeates the human body and $\epsilon_{1r} \sim 60$ is the relative permittivity in the 50–200-MHz range of frequencies. At 60 MHz, $\sigma_1 - j\omega\epsilon_0\epsilon_{1r} = 0.5 - j0.20 = 0.54e^{-j0.38}$. The energy transformed into heat per unit volume (power deposition) is $P = \sigma_1 E_{1z}^2(\rho, z)$.

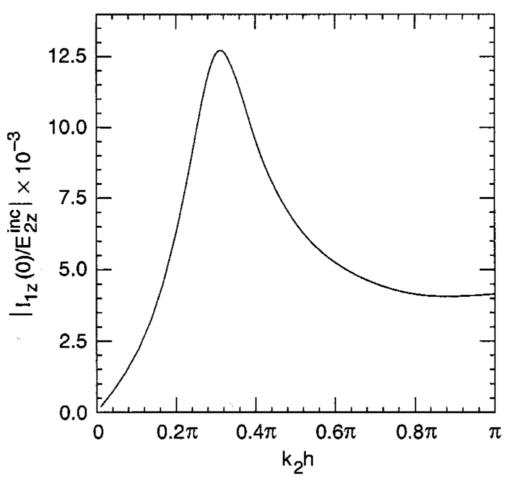


Fig. 2. Normalized total axial current $|I_{1z}(0)/E_{2z}^{\rm inc}|$ as a function of the electrical length k_2h of the cylinder with the radius $a = 0.14$ m and half-length $h = 0.875$ m.

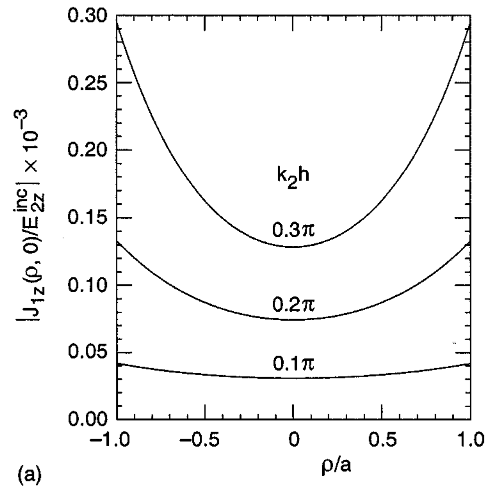
IV. Results

The total axial current $I_{1z}(z)$ is shown graphically in Fig. 1 as a function of z/h for selected values of k_2h from 0.1π to π. The resonant value with maximum $I_{1z}(z)$ is $k_2h = 0.31\pi$. Here, $h = 0.875$ m is the half-length of the body and $a = 0.14$ m is the mean radius. Note that the current distribution in Fig. 1 has the typical shifted cosine form, $\cos k_2z - \cos k_2h$, for a parasitic antenna. Near resonance where $k_2h = \pi/2$, the distribution is $\cos k_2z$. When $k_2h \ll 1$, the distribution is parabolic.

It is interesting to compare the resonant electrical length $k_2h = 0.31\pi$ (or $2h/\lambda_2 = 0.31$) for a cylinder with experimentally determined values for prolate spheroidal models obtained by Gandhi [21]. Gandhi used figurines ranging in length from 7.6 to 25.4 cm at frequencies for which the ratio of axial length L to central diameter $2b$ was $L/2b = 6$. Note that for the cylinder, $2h/2a = 1.75/0.14 = 6.25$. Resonance was determined by Gandhi from the temperature increase in the model. From Fig. 2 in [21, p. 1023], the maximum temperature was obtained when $L/\lambda = 0.36$. Since the shape of a spheroid is significantly different from a cylinder, the respective resonant lengths $L/\lambda = 0.36$ and $2h/\lambda = 0.31$ are in good agreement.

A graph of $I_{1z}(0)$ as a function of k_2h is shown in Fig. 2. Resonance at $k_2h = 0.31\pi$ yields a maximum axial current across the midsection of the body at $z = 0$ of $I_{1z}(0)/E_{2z}^{\rm inc} = 12.6$ mA/V/m. Note that with $h = 0.875$ m and $k_2 = 2\pi f/c$, the resonant frequency is $f = 53 \times 10^6$ Hz. This is very close to the amateur radio frequency band 56–60 MHz.

The current density $J_{1z}(\rho, z)/E_{2z}^{\rm inc}$ is shown graphically in Fig. 3(a) and (b) as a function of ρ/a in the central cross section of the body, i.e., at $z = 0$; k_2h is the parameter. It is seen that the maximum occurs at the resonant value, $k_2h = 0.31\pi$. As at all values of k_2h, the maximum density is at the surface $\rho = a$, the minimum at the center $\rho = 0$. This is a consequence of skin effect. As seen from Fig. 3(a) and (b), skin effect is greatest at the highest frequency, *viz.* with $k_2h = \pi$, and smallest at the lowest frequency where $k_2h = 0.1\pi$. In this last case, the radial distribution is almost uniform, as in the



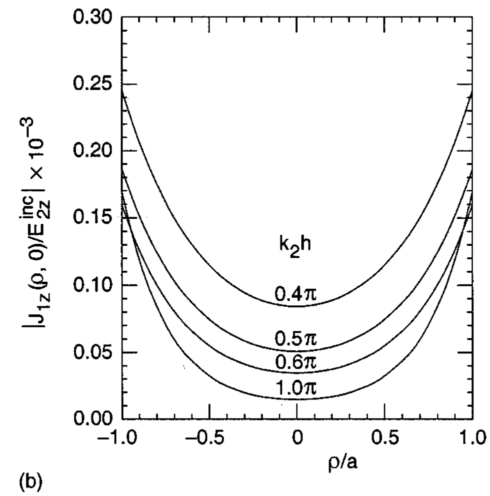


Fig. 3. Normalized current density $|J_{1z}(\rho, 0)/E_{2z}^{\rm inc}|$ at the central cross section $z = 0$ of the cylinder with the radius $a = 0.14$ m and half-length $h = 0.875$ m as a function of ρ/a. (a) $k_2h = 0.1\pi$, 0.2π, and 0.3π. (b) $k_2h = 0.4\pi$, 0.5π, 0.6π, and 1.0π.

low-frequency ranges. Since the induced electric field is given by $E_{1z}(\rho, z) = J_{1z}(\rho, z)/(\sigma_1 - j\omega\epsilon_0\epsilon_{1r})$ and the power density by $P(\rho, z) = \sigma_1|E_{1z}^2(\rho, z)|$, the maximum power deposition occurs when $J_{1z}(\rho, z)$ is greatest, i.e., at the resonant frequency. This is in agreement with the observations of Gandhi [21] and is here quantitatively expressed in terms of the induced electric field.

An examination of (3) for z^i and (27) for $J_{1z}(\rho, z)$ shows that the radius a occurs in a^2 and in k_1a, not only as k_2a as required for frequency scaling. It follows that scaling as described in the introduction and as used by Gandhi [21] is not quantitatively valid in the frequency range 50–200 MHz. As shown by King [6], scaling is valid in the low-frequency range.

V. Discussion

An analytical solution has been derived for the total axial current $I_{1z}(z)$ induced in a cylindrical model of the human body when it is exposed to an incident axial electric field $E_{2z}^{\rm inc}$. The frequency range includes the frequency for which the body is

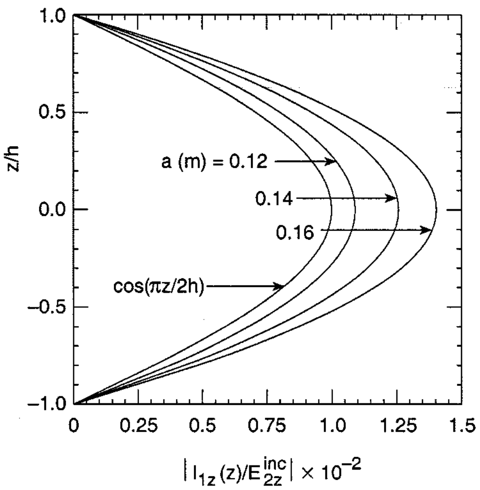


Fig. 4. Normalized total axial current $|I_{1z}(z)/E_{2z}^{\rm inc}|$ for cylinders with half-length $h = 0.875$ m and radii $a = 0.12, 0.14, 0.16$ m, as a function of z/h for $k_2h = 0.35\pi$. Also shown is $0.01\cos(\pi z/2h)$.

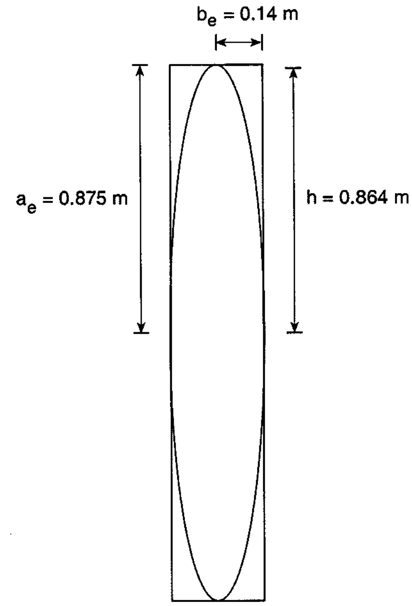


Fig. 5. The elliptical cross section of the spheroid $a_e^2 - b_e^2 = h^2$ with $a_e = 0.875$ m, $b_e = 0.14$ m, $h = 0.864$ m, and the rectangular cross section of the cylinder with half-length 0.875 m and radius 0.14 m.

resonant and $I_{1z}(z)$ has its maximum. Also evaluated are the current density $J_{1z}(\rho, z)$ and the induced axial electric field $E_{1z}(\rho, z)$. The formulation is quite accurate for the cylindrical model of the body but is only approximate for an actual body, which is roughly cylindrical but with varying cross-sectional shapes and radii. The cylindrical approximation applies only when the arms are in contact with the sides.

How good is the cylindrical approximation of the human body? For the 50–200-MHz range of frequencies, the total axial current is not independent of the transverse dimensions and shape as at ELF and VLF when $k_2h \ll 1$. As shown in Fig. 4, the amplitude $|I_{1z}(0)|/E_{2z}^{\rm inc}$ increases moderately with increasing radius of the cylinder. This figure shows the axial distribution of current for cylinders with radii $a = 0.12$ m, 0.14 m, and 0.16 m, together with $\cos(\pi z/2h)$. Of particular interest is the fact that for all three values of a, the current distribution is cosinusoidal. However, the cosinusoidal distribution is accurate for a cylinder only when its radius approaches zero, i.e., $a \to 0$. When a is not nearly zero, the cosinusoidal distribution of current is only approximate for the cylinder. What must be the shape of a resonant antenna of finite cross-sectional area when the axial distribution of current is cosinusoidal? The primary condition is that its surface be a surface of constant phase in the outward-traveling electromagnetic field that emanates from its surface. The rigorous electric field tangent to a spheroidal conductor has the following form in the spheroidal coordinates k_e and k_h [31, eq. (8.5–37)]:

$$E_e(t) = -\frac{\zeta_0 I_0}{2\pi h}\frac{\cos(\pi k_h/2)\sin(\omega t - \pi k_e/2)}{[(k_e^2 - k_h^2)(1 - k_h^2)]^{1/2}}. \tag{29}$$

As the time passes, a surface of constant phase defined by $(\omega t - \pi k_e/2) =$ a constant expands, i.e., k_e increases from a value slightly greater than 1. This value is readily determined for the human body since $k_e = a_e/h$, where a_e is the semi-major axis of the spheroid defined by $a_e^2 - b_e^2 = h^2$ and b_e is the semi-minor axis. Note that $2h$ is the distance between the foci. With $a_e = 0.875$ m and $b_e = 0.14$ m, $h = (a_e^2 - b_e^2)^{1/2} = 0.8637$ m. It follows that $k_e = 0.875/0.8637 = 1.013$. The elliptical cross section of a spheroid with the length $2a_e = 1.75$ m and the radius $b_e = 0.14$ m at the center $z = 0$ is shown in Fig. 5, together with the rectangular cross section of a cylinder with the length $2a_e = 1.75$ m and radius $b_e = 0.14$ m.

The cosinusoidal current shown in Fig. 4 and calculated from formula (25) is an accurate distribution for the spheroid shown in cross section in Fig. 5 and only an approximate distribution for the cylinder also shown in cross section in Fig. 5. Clearly, the human body is better represented by the spheroid than by the cylinder. It is reasonable to assume that the total axial current will remain substantially the same when the cross section of the spheroid is changed from a circle with central cross-sectional area $A = \pi b_e^2$ to an ellipse with the same central cross-sectional area. This change is appropriate for those human bodies that are approximately elliptical at their central cross-sections rather than circular.

VI. Incident Electric Field When $f = 60$ MHz

The total axial current, the current density, and the electric field shown in the several figures are normalized values. To determine the actual values, it is necessary to know the incident field $E_{2z}^{\rm inc}$. This can be very large near high-power radar or television antennas, but relatively few people are exposed to these. Of interest in this study is the exposure of radio amateurs to the electric fields near their transmitting antennas. As mentioned in the introduction, the currents and electric fields induced in the bodies of amateur radio operators have been shown to be hazardous. How large is the electric field near a simple half-wave dipole antenna at $f = 60$ MHz? Both horizontal and vertical dipole antennas are in use. Consider a vertical dipole supported on a mast at a distance $\rho = 10$ m from the operator's house. For a power output of 1 kW, the current in the central driving-point of the antenna is obtained from $P = I^2 R = 10^3$ W. For instance, with $R = 73\ \Omega$, $I = 3.7$ A. The vertical electric field of a half-wave dipole with a sinusoidally distributed current is [32, eq. (A-VI-35)]

$$E_{2z}(\rho, z) = -\frac{j\omega\mu_0 I_z(0)}{4\pi k_2}\left(\frac{e^{-jk_2 r_{1L}}}{r_{1L}} + \frac{e^{-jk_2 r_{2L}}}{r_{2L}}\right) \quad (30)$$

$$r_{1L} = [(L-z)^2 + \rho^2]^{1/2}$$
$$r_{2L} = [(L+z)^2 + \rho^2]^{1/2}. \quad (31)$$

The half-length of the antenna is L and $k_2 L = \pi/2$. The distances r_{1L} and r_{2L} extend from the point of observation to the ends of the dipole. The center of the dipole is at $z = 0$. The vertically directed field $E_{2z}(\rho, z)$ at $\rho = 10$ m acts on a standing or sitting person in the operator's house.[2] Let the center of the dipole be at a height 5-m above the earth and let this also be the height at which the operator sits or stands in his transmitter room. The direct field from the antenna at this location is obtained from (30) with $z = 0$, so that $r_{1L} = r_{2L} = r_0 = (L^2 + \rho^2)^{1/2}$. Hence, with $\zeta_0 = 120\pi = \omega\mu_0/k_2 \equiv c\mu_0 = \sqrt{\mu_0/\epsilon_0}$,

$$E_{2z}(\rho, 0) = -j\frac{\zeta_0 I_{1z}(0)}{2\pi r_0} e^{-jk_2 r_0}. \quad (32)$$

With $\rho = 10$ m and $L = 1.25$ m, $r_0 = 10.08$ m. At $f = 60 \times 10^6$ Hz, $k_2 = 2\pi f/c = 0.4\pi$, $k_2 r_0 = 12.67$, and

$$E_{2z}(10, 0) = -j\frac{60 \times 3.7}{10.08} e^{-j12.67} = -j22.0 e^{-j12.67}$$
$$= -2.27 - j21.89 \text{ V/m}. \quad (33)$$

The image field $E^i_{2z}(\rho, z)$ with the earth approximated by a perfect conductor is given by (30) with $r_{1L} = (8.75^2 + 10^2)^{1/2} = 13.29$ m and $r_{2L} = (11.25^2 + 10^2)^{1/2} = 15.05$ m. Also, $k_2 r_{1L} = 16.7$ and $k_2 r_{2L} = 18.9$. Hence

$$E^i_{2z}(10, 0) = -j30 \times 3.7\left(\frac{e^{-j16.7}}{13.29} + \frac{e^{-j18.9}}{15.05}\right)$$
$$= -j111(0.075 e^{-j16.7} + 0.066 e^{-j18.9})$$
$$= -j8.325(-0.55 + j0.84)$$
$$\quad - j7.326(0.999 - j0.050)$$
$$= 6.993 + j4.579 - 0.366 - j7.319$$
$$= 6.627 - j2.740 \text{ V/m}. \quad (34)$$

Combining (33) and (34) gives

$$E_{2z}(10, 0) + E^i_{2z}(10, 0)$$
$$= -2.27 - j21.89 + 6.63 - j2.74$$
$$= 4.36 - j24.63 = 25.01 e^{-j1.40} \text{ V/m}. \quad (35)$$

Thus, the electric field $|E^{\text{inc}}_{2z}| = 25.0$ V/m is the field acting on a person in the amateur radio operator's house.

Fig. 1 gives the normalized induced axial current $|I_{1z}(z)/E^{\text{inc}}_{2z}|$ calculated from (25) with $U^{\text{inc}} = E^{\text{inc}}_{2z}/k_2$, for $k_2 h = 0.1\pi$ to 1.0π and z/h from -1.0 to 1.0. With $f = 60$ MHz and $h = 0.875$ m (the half-length of a man), $k_2 = 2\pi f/(3 \times 10^8) = 0.4\pi$ so that $k_2 h = 0.35\pi$. With this value for $k_2 h$ and $|E^{\text{inc}}_{2z}| = 25.0$ V/m, the maximum current at $z = 0$ is

$$I_{1z}(0) = 11.76 \times 10^{-3}\left|E^{\text{inc}}_{2z}\right| = 0.294 \text{ A}. \quad (36)$$

The current density and electric field near the surface $\rho \sim a$ of the body are

$$|J_{1z}(a)| = 0.2886\left|E^{\text{inc}}_{2z}\right| = 7.21 \text{ A/m}^2$$
$$= 0.721 \text{ mA/cm}^2 \quad (37)$$
$$|E_{1z}(a)| = |J_{1z}(a)/(\sigma_1 - j\omega\epsilon_0\epsilon_{1r})|$$
$$= 7.21/0.54 = 13.35 \text{ V/m}. \quad (38)$$

Near the center of the body

$$|J_{1z}(0)| = 0.1115\left|E^{\text{inc}}_{2z}\right| = 2.79 \text{ A/m}^2 \quad (39)$$
$$= 0.279 \text{ mA/cm}^2$$
$$|E_{1z}(0)| = 5.17 \text{ V/m}. \quad (40)$$

These values are significant and provide a quantitative basis for the statistically observed increases in malignancies in amateur radio operators.

VII. Conclusion

The total axial current $I_{1z}(z)$ induced in a cylindrical model of the human body has been calculated quite accurately. When the body is resonant, the axial distribution of current turns out to be cosinusoidal. This is known to be only approximate for a cylinder that is not extremely thin. However, the cosinusoidal current is the accurate distribution for a spheroidal conductor, so that it is concluded that the spheroid more accurately models the body than the cylinder. Unfortunately, the current density $J_{1z}(\rho, z)$ and electric field $E_{1z}(\rho, z)$ calculated accurately for the interior of the cylinder cannot be transferred to the spheroid except close to the central cross section, $z = 0$. The large skin effect that characterizes the transverse distribution of current in the 50–200-MHz range of frequencies can be calculated readily for the entire length of a cylinder, but not for a spheroid.

[2]It is assumed that the walls and roof of the house are made of dielectric materials like wood, brick, plaster, and asphalt which provide no significant shielding.

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