Efficient Description of Impedance and Radiation Features in Printed-Circuit Leaky-Wave Structures—An Unconventional Scattering-Matrix Approach

Paolo Burghignoli, *Student Member, IEEE*, Carlo Di Nallo, *Associate Member, IEEE*, Fabrizio Frezza, *Senior Member, IEEE*, Alessandro Galli, *Member, IEEE*, and Paolo Lampariello, *Fellow, IEEE*

***Abstract*—This paper presents an original circuit model that furnishes an efficient description of the impedance and of the radiation performance for typical printed-circuit leaky-wave structures. In particular, referring to a junction between slot-coupled feeding and radiating microstrips, we have developed an unconventional equivalent transmission-line formulation, involving the propagation of the dominant mode for the feeding line and of the first higher mode for the radiating line, which is leaky. The quantification of the relevant scattering matrix, achieved with spectral-domain techniques, suitably summarizes the coupling effects and, consequently, the radiative features. Various antenna configurations have been tested, and the relevant results have been validated through comparisons with heavier full-wave numerical approaches. In addition to substantial computational advantages, this innovative approach gives the possibility of treating printed leaky-wave structures in a convenient fashion by means of a network formalism similar to that used in standard microwave circuits.**

***Index Terms*—Equivalent circuits, feeds, leaky-wave antennas, microstrip antennas, planar transmission lines.**

I. Introduction

The use of leaky-wave antennas as a possible alternative to conventional radiating systems has been proposed for many years also in microwave integrated-circuit (MIC) applications [1]–[3] even though, in this case, various practical aspects still need careful investigation. Among the crucial aspects in this type of structures, we mention, in particular, the problems related to the simple design and characterization of the feeding network, and also to an efficient description and prediction of the radiation phenomena [4]–[6]. The development of numerical codes and/or commercial software based on full-wave techniques, such as the finite-element method (FEM), method of moments (MoM), etc. [7] has given, in recent times, undoubted improvements for the analysis of a large variety of microwave structures also involving antennas. However, if compared to simpler models appropriately conceived to solve more specific problems, these general-purpose techniques present serious drawbacks that can make the design procedures rather difficult (e.g., reduction of computational efficiency, lack of an intuitive mathematical and physical description of the electromagnetic phenomena, etc.).

In this paper, the attention is focused on the usual types of printed-circuit leaky-wave antennas for which the development of a simple, but accurate, model is searched with the aim of efficiently describing the relevant coupling and radiation phenomena. The approach is primarily based on the fact that, in practical situations, the radiative features of these structures can be described in a convergent way through only a leaky mode supported by the line [2], [6], [8] (e.g., a sufficiently long section of a microstrip line). On this ground, here we investigate the excitation of such a leaky mode by developing an equivalent network also involving a properly coupled feeding line (e.g., another section of microstrip line with a coupling slot). This model is unconventional because a transmission line has to be associated suitably to the bound mode of the feeding line and also to the leaky mode of the radiating line, which, as is known, is improper or nonspectral (it does not satisfy the radiation conditions since it diverges exponentially on the transverse plane) [2].

We will show that, with appropriate conditions, a particular scattering matrix can nevertheless be found, which quantifies the coupling effect between the feeding and radiating lines. As a consequence, it is thus possible to reach all the basic information for an efficient and accurate description of the radiation features (given in terms of admittance, radiation patterns, etc.), as occurs in the usual nonleaky structures.

The main aspects of the proposed theoretical formulation will be described in Section II, while Section III will provide quantitative results for different printed leaky-wave antenna topologies. Suitable comparisons will be presented to validate the model and emphasize the advantages with respect to full-wave numerical techniques.

II. Formulation of the Circuit Model

A. Reference Structure: Slot-Coupled Guiding and Radiating Microstrips

The proposed approach is suitable for the characterization of general classes of radiative structures derivable by open printed waveguiding lines with appropriate feeders. In order to present all the main qualitative and quantitative aspects of our approach,

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The authors are with the Department of Electronic Engineering, “La Sapienza” University of Rome, 00184 Rome, Italy (e-mail: plamp@die.ing.uniroma1.it).

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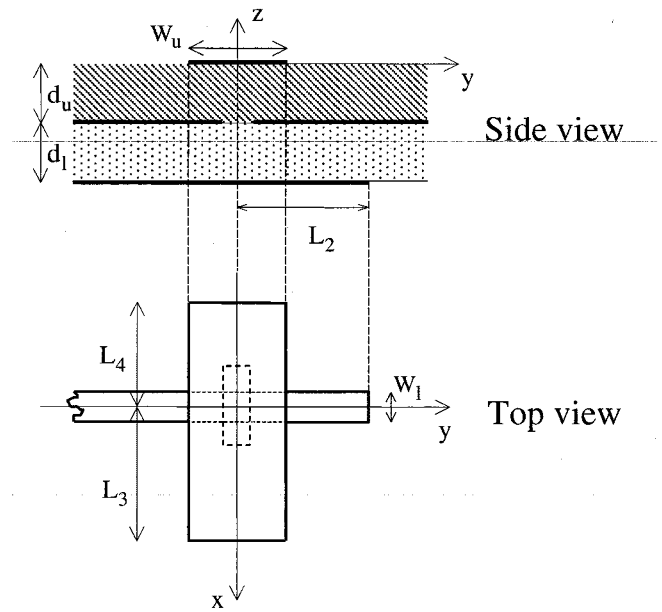


Fig. 1. Reference structure with coordinate system and geometrical parameters for the derivation of a circuit model describing the excitation and coupling in printed-circuit leaky-wave antennas: a length of radiating microstrip (upper line) coupled to a length of feeding microstrip (lower line) by means of a slot etched on the common ground plane.

it is convenient to consider a reference topology that is commonly employed in printed-circuit leaky-wave antennas. The structure under specific investigation is shown in Fig. 1, with the coordinate system and geometrical parameters involved: it consists of a junction between two orthogonal microstrips placed on opposite sides with respect to their common ground plane, on which a narrow rectangular slot is etched. In this configuration, the lower strip feeds (through the slot) the upper one, which acts as a leaky-wave radiating line.

In the feeding microstrip (the lower one, with related quantities identified by the letter l), it is, therefore, supposed that the dominant quasi-TEM (EH_0) mode propagates [it has an even symmetry with respect to the yz-plane, which is a perfect magnetic conductor (PMC)].

As concerns the upper microstrip (with related quantities identified by the letter u), we recall that, by properly choosing the relevant physical parameters involved (permittivity and height of the dielectric, strip width, frequency), it is possible to find a range where the propagation constant of the first higher order mode of the microstrip (EH_1) becomes complex [2]. A representative example of the EH_1 dispersion behavior of the propagation complex wavenumber for a microstrip is illustrated in Fig. 2, with a fixed choice of the parameters for a microwave application. Fig. 2(a) gives the phase constant and Fig. 2(b) the attenuation or leakage constant (both normalized to the free-space wavenumber k_o) as a function of frequency in a "useful" leaky range. (We have obtained these results by means of a spectral-domain MoM analysis [7], with proper deformation of the integration path in the complex plane for the transverse spectral variable [9], [10].)

We, therefore, suppose that, in our case, the physical parameters are chosen to have a condition of spatial leakage [8]–[10],

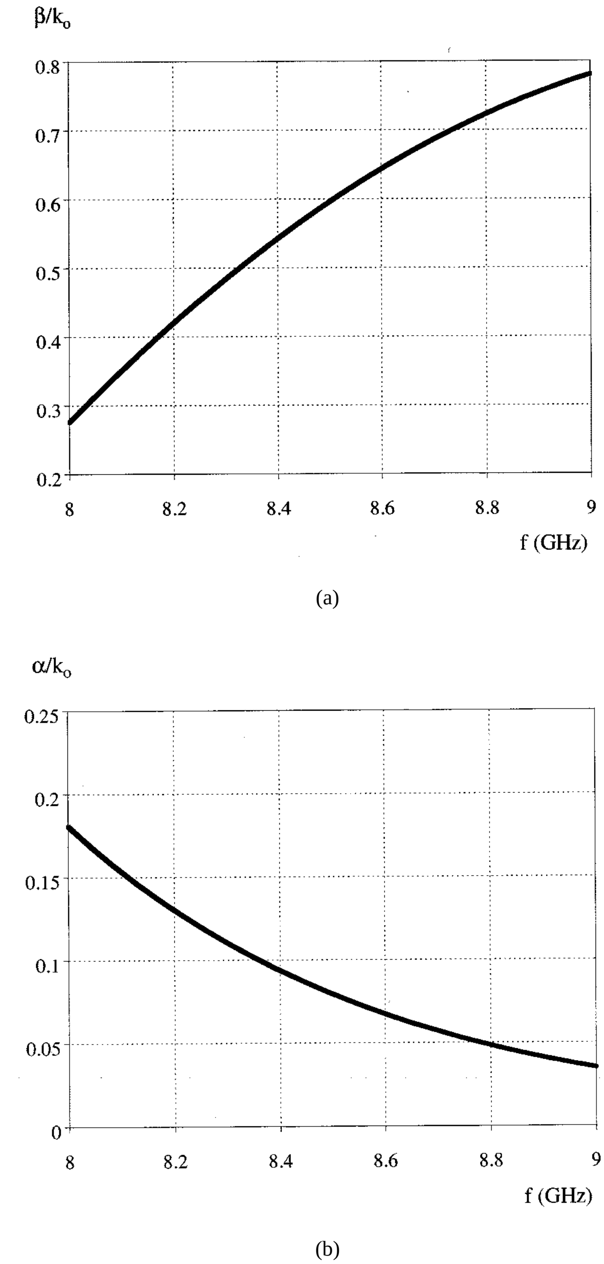
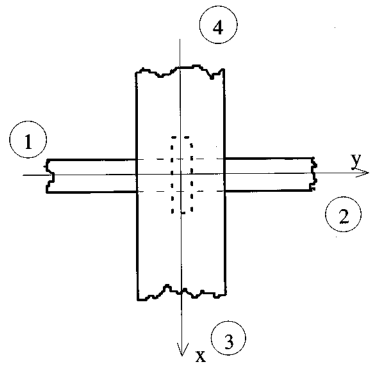


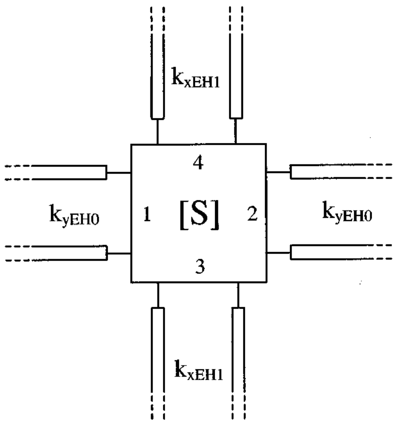
Fig. 2. Dispersion characteristics of the first higher mode EH_1 of the microstrip line: (a) Normalized phase constant versus f. (b) Normalized attenuation or leakage constant versus f. Parameters: strip width: 1 cm, substrate height: 0.508 mm, substrate relative permittivity: 3.05.

for which the line can effectively act as a leaky-wave radiator. As is known, the frequency region for a convenient use of such a structure as a leaky-wave antenna is where the normalized phase constant decreases below the unit value and the normalized leakage constant assumes rather "low"' values [2] (see the results shown in Fig. 2). In this case, it is seen that, if properly excited, such a leaky mode can describe in a highly convergent way the radiation features of the line [2], [6], [8].

The slot is then placed to mainly excite such a leaky mode that has on odd symmetry with respect to the xz-plane [which is a perfect electric conductor (PEC)] so that the excitation of the guided dominant mode EH_0 in the upper strip is avoided. All the other higher modes can be seen as below cutoff or giving



(a)



(b)

Fig. 3. Generalization of the junction of Fig. 1. (a) Numbering of the four ports for the two microstrip lines. (b) Equivalent four-port circuit model of the slot junction.

a negligible leakage effect, mostly contributing to reactive phenomena. Under these conditions, it is justifiable to suppose that, in particular, the currents excited along the radiating microstrip can be well approximated by the currents of the only EH_1 leaky mode on a significant length of the strip (we recall that the linear dimensions of practical traveling-wave antennas are typically of several wavelengths). Thus, the radiative features can reasonably be associated to the contribution of the leaky-mode currents.

Since under these conditions the pair of microstrips can be considered in a unimodal range, we intend to describe the slot-coupled junction with a 4 × 4 scattering matrix, after introducing a suitable equivalent transmission line for each mode. In Fig. 3, we show a generalization of the structure of Fig. 1 with the numbering of ports for the junction [see Fig. 3(a)] and the relevant network scheme [see Fig. 3(b)].

The proposed approach, therefore, follows the traditional characterization of microwave junctions through the scattering matrix, but a fundamental difference is that, as noted above, in the upper line, the mode diverges in the transverse plane yz, and it could not be treated with conventional transmission-line methods (for instance, the assumption that a leaky mode is flowing along the strip makes it difficult to define a characteristic impedance [11]). Actually, we will see how the problem of defining a characteristic impedance can be overcome here by simply linking the scattering parameters to the amplitude of the modal leaky-wave current excited on the strip.

In order to evaluate the scattering-matrix coefficients of the junction under investigation, we have employed suitable formulations based on the spectral domain and solved by means of MoM method. Due to the symmetries of the structure, it is evidently sufficient to solve the electromagnetic problem in two different situations only: one case of excitation in the lower strip (e.g., considering the presence of the incident mode EH_0 at port 1), and another case of excitation in the upper strip (e.g., considering the presence of the incident mode EH_1 at port 4). However, distinct approaches have to be used in connection with the different nature of the problem in the two cases since, as already said, in the lower structure, the incident mode has a "conventional" behavior (since it is spectral), while in the upper structure, the incident mode has an "unconventional" behavior (since it is nonspectral).

In Sections II-B and II-C, we will furnish a description of these two different approaches, which lead to the derivation of the basic scattering coefficients of the junction. Hence, we will show in Section II-D how to derive the complete scattering matrix and the related quantities (input impedance, current distributions, radiation patterns) that are useful for a practical design of the circuit.

B. Excitation from the Lower Structure (Port 1)

As said, in this case, the incident field is the quasi-TEM EH_0 mode, having a current distribution on the strip that can be approximated as (refer to Fig. 1 for the parameters of the structure)

$$\begin{aligned}\mathbf{J}(x,y) &= I_o^+ T(x) e^{-jk_{y0}y}\mathbf{y}_0\\ T(x) &= \frac{2}{\pi W_l}\frac{1}{\sqrt{1-\left(\frac{2x}{W_l}\right)^2}}\end{aligned}\tag{1}$$

where k_{y0} is the real propagation constant of the dominant mode of the lower microstrip line, easily known from the dispersion behavior of the guiding structure, and I_o^+ is the amplitude of the incident current wave flowing along the strip (which will be assumed equal to unity and omitted in the following developments). The quasi-TEM incident field, therefore, has the one-dimensional spectral representation of the following type (the "tilde" describes spectral quantities)

$$\begin{aligned}&\mathbf{E}^{\mathrm{inc}}(x,y,z)\\ &\quad=\frac{1}{2\pi}e^{-jk_{y0}y}\int_{-\infty}^{+\infty}\underline{\tilde{\mathbf{G}}}_t^{ee}\left(k_x,k_{y0},z;-(d_l+d_u)\right)\\ &\qquad\cdot\mathbf{y}_0\tilde{T}(k_x)e^{-jk_x x}\,dk_x\\ &\mathbf{H}^{\mathrm{inc}}(x,y,z)\\ &\quad=\frac{1}{2\pi}e^{-jk_{y0}y}\int_{-\infty}^{+\infty}\underline{\tilde{\mathbf{G}}}_t^{he}\left(k_x,k_{y0},z;-(d_l+d_u)\right)\\ &\qquad\cdot\mathbf{y}_0\tilde{T}(k_x)e^{-jk_x x}\,dk_x\end{aligned}\tag{2}$$

where, as is typical, the superscript in the spectral dyadic Green's function describes the contribution to the electric or magnetic (e or h) field due to the electric source (e), and the subscript represents the transverse (t) components (x, z) with respect to the symmetry direction y.

The presence of the slot aperture (of width w_a, length L_a, and area S_a), etched in the ground plane, is schematized with an equivalent magnetic current on a PEC, given in terms of the electric field on the aperture $\mathbf{E}^a$ according to simple expressions [12]

$$\begin{aligned} \mathbf{M}_u &= -z_0 \times \mathbf{E}^a, \qquad \text{for } z = -d_u^+ \\ \mathbf{M}_l &= z_0 \times \mathbf{E}^a = -\mathbf{M}_u, \qquad \text{for } z = -d_u^- \end{aligned} \tag{3a}$$

with

$$\begin{aligned} \mathbf{E}^a(x,y) &= V_0 e_y^a(x,y)\mathbf{y}_0 \\ &= V_0 \frac{\sin[k_e(L_a/2 - |x|)]}{\sin\left(k_e \frac{L_a}{2}\right)} \mathbf{y}_0 \quad \text{with } \begin{cases} |x| \le L_a/2 \\ |y| \le w_a/2 \end{cases} \end{aligned} \tag{3b}$$

and the effective wavenumber k_e can be calculated as

$$\begin{aligned} k_e &= \frac{k_e^u + k_e^l}{2} \\ k_e^{u/l} &= \sqrt{\frac{\varepsilon_{u/l} + 1}{2} + \frac{\varepsilon_{u/l} - 1}{2} \Bigg/ \sqrt{1 + 10 \frac{d_{u/l}}{W_{u/l}}}} \end{aligned} \tag{3c}$$

where V_0 is an unknown amplitude coefficient. The approximations for the analytical behavior of the slot field are valid since, as is typical, the coupling aperture is assumed electrically small, i.e., the slot is “narrow”' (with respect to the guided wavelength $w_a \ll \lambda$) and relatively “short” (with respect to the effective wavelength $\lambda_e = 2\pi/k_e : L_a \le \lambda_e/2$).

In the presence of the equivalent impressed magnetic current on the slot, the field in the space below the ground plane can now be expressed as

$$\begin{aligned} \mathbf{E}^{\text{tot}}(x,y,z) &= \begin{cases} \mathbf{e}(x,z)(e^{-jk_{y0}y} + R_1 e^{jk_{y0}y}), & y \le 0 \\ T_1 \mathbf{e}(x,z) e^{-jk_{y0}y}, & y \ge 0 \end{cases} \\ \mathbf{H}^{\text{tot}}(x,y,z) &= \begin{cases} \mathbf{h}(x,z)(e^{-jk_{y0}y} - R_1 e^{jk_{y0}y}), & y \le 0 \\ T_1 \mathbf{h}(x,z) e^{-jk_{y0}y}, & y \ge 0 \end{cases} \end{aligned} \tag{4}$$

where R_1 and T_1 are the reflection and transmission coefficients, respectively, referred to the feeding at port 1, and $\mathbf{e}(x,z)$, $\mathbf{h}(x,z)$ are the electric and magnetic modal components of the incident quasi-TEM field. In order to determine this field, it is possible to use an approach such as the one described in [13], which is based on the reciprocity theorem. Rephrasing such a procedure, it is possible to express the reflection coefficient R_1 in terms of V_0

$$\begin{aligned} R_1 &= -\frac{V_0}{2A^2} \int_{S_a} h_x e_y^a \, dx \, dy \\ &= -\frac{V_0}{2A^2} \Delta V \\ \Delta V &= \int_{S_a} h_x e_y^a \, dx \, dy. \end{aligned} \tag{5}$$

The R_1 parameter is expressed as a function of a “coupling” quantity ΔV (represented by an integral of fixed field expressions extended to the slot aperture) and of the factor A^2, linked to the incident power, which equals the characteristic impedance of the dominant mode of the lower microstrip line when a unit-strength incident current is considered.

An additional link between R_1 and V_0 is derived by imposing the continuity of the tangential magnetic field on the slot aperture. The fields in the upper half-space can be calculated once the electric current on the radiating strip excited by the equivalent magnetic current on the slot in the case of unit strength $V_0 = 1$ is known. To this purpose, we have assumed that such a current $\mathbf{J}^M$ can be expressed in the spectral domain as a linear combination of k_y-dependent basis functions, with amplitude coefficients that are functions of the longitudinal spectral variable k_x according to the procedure described in [6], [8]

$$\begin{aligned} \tilde{J}_{ux}^M(k_x,k_y) &= \sum_{m=1}^{M} A_m(k_x) \tilde{T}_m^x(k_y) \\ \tilde{J}_{uy}^M(k_x,k_y) &= \sum_{n=1}^{N} B_n(k_x) \tilde{T}_n^y(k_y). \end{aligned} \tag{6}$$

After some analytical arrangements, we can obtain the following relationships, which lead to determine s_{11}

$$\begin{aligned} s_{11} &= R_1 \\ R_1 &= \frac{\Delta V^2}{\Delta V^2 - 2A^2(Y_J + Y_M)}. \end{aligned} \tag{7}$$

The quantities Y_M and Y_J (describing the reactions of the equivalent magnetic current on the aperture with itself and with the electric current on the upper line, respectively) are given by spectral expressions as follows:

$$\begin{aligned} Y_M &= \frac{1}{(2\pi)^2} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \tilde{e}_y^a(-k_x,-k_y) \\ &\quad \times \left(\tilde{G}_{u,xx}^{hh} + \tilde{G}_{l,xx}^{hh}\right) \tilde{e}_y^a(k_x,k_y) \, dk_x \, dk_y \\ Y_J &= \frac{1}{(2\pi)^2} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \tilde{e}_y^a(-k_x,-k_y) \\ &\quad \times \left(\tilde{G}_{u,xx}^{he} \tilde{J}_{ux}^M + \tilde{G}_{u,xy}^{he} \tilde{J}_{uy}^M\right) dk_x \, dk_y \end{aligned} \tag{8}$$

where the spectral Green's functions related to the upper/lower regions have been introduced.

Thus, it is possible to calculate the total current present on the upper strip when fed by port 1 once the amplitude coefficient V_0 is obtained

$$V_0 = \frac{2A^2 \Delta V}{\Delta V^2 - 2A^2(Y_J + Y_M)} \tag{9a}$$

and

$$\tilde{\mathbf{J}}_u(k_x,k_y) = V_0 \tilde{\mathbf{J}}_u^M(k_x,k_y). \tag{9b}$$

For a suitable characterization of the coupling with the upper radiating microstrip due to the slot, it is necessary to find a convenient relationship that appropriately describes the excitation of the fundamental contribution for the leaky mode EH_1.

Since in our situation the EH_1 leaky mode is supposed to be dominant and its propagation constant is a solution of the transverse problem for the upper microstrip line, its value has to correspond to a pole for the Fourier transforms of the amplitudes A_m, B_n of (6). To evaluate such amplitudes, a recently developed technique for the calculation of the modal-excitation coefficients in planar structures has been used [6], [8]. Fixing the spectral variable k_x at the propagation wavenumber $k_{x\,\mathrm{LW}}$ of the leaky-wave pole of the EH_1 mode, from the residue theorem, we have (10), shown at the bottom of this page.

While the fields related to the leaky mode are improper (since they diverge transversely), the relevant current is instead limited everywhere and allows for a characterization of the structure as regards the radiation features. Therefore, to quantify the amount of the wave outgoing from port 3 when port 1 is fed, we make the choice of taking the most significant part of the current density on the upper strip, represented by the first coefficient of the series (10) for the transverse component. Therefore, s_{31} can be defined as

$$s_{31} \equiv -jV_0\,\mathrm{Res}[B_1(k_x); k_{x\,\mathrm{LW}}]. \tag{11}$$

It is worth noting that such a component has the same symmetry of the transverse electric field and then of a usual equivalent voltage.

C. Excitation from the Upper Structure (Port 4)

The aim of analyzing a leaky-wave radiator with a circuit model based on transmission lines should require the presence of an incident leaky mode in the upper structure. Such an improper mode, however, cannot be considered as incident at an ideally infinite distance (e.g., at $x = -\infty$ for port 4) since the field should diverge both in the longitudinal and transverse directions, and also the use of Fourier transforms would not be possible. On the other side, the leaky mode is practically useful to represent, in a convergent way, the continuous spectrum of the open structure in a suitable space region that is related to the location of a source at a certain finite distance. In our case, we consider on the upper strip the leaky mode EH_1 that is excited in a certain section at a finite distance, let us say, $x = -D$.

The determination of the field in the lower region is achieved again with an approach based on the reciprocity theorem. The field on the slot has the already-shown form with a different amplitude coefficient V_1 instead of V_0 in (3a)–(3c). We can express the s_{14} parameter as a function of V_1 and, by applying the continuity of the magnetic field on the slot, it is now possible to obtain the additional relation between s_{14} and V_1. In this case, in the upper part, there is an additional contribution related to the current due to the field incident from port 4. The relationships become

$$\begin{aligned} V_1 &= -\frac{2A^2 I}{\Delta V^2 - 2A^2(Y_J + Y_M)} \\ s_{14} &= -\frac{I\Delta V}{\Delta V^2 - 2A^2(Y_J + Y_M)} \end{aligned} \tag{12}$$

where the additional contribution I is given by

$$\begin{aligned} I = \frac{1}{(2\pi)^2}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} &\tilde{e}_y^a(-k_x, -k_y) \\ \times \Big[&\tilde{G}^{he}_{u,xx}(k_x, k_y, -d_u; 0)\tilde{J}^{\mathrm{inc}}_{ux}(k_x, k_y) \\ &+ \tilde{G}^{he}_{u,xy}(k_x, k_y, -d_u; 0)\tilde{J}^{\mathrm{inc}}_{uy}(k_x, k_y)\Big]\, dk_x\, dk_y. \end{aligned} \tag{13}$$

The evaluation of I has been reached as an asymptotic value for increasing distance D of the reference plane of the incident wave (see Section III-A for further discussion on the evaluation of this term).

Once V_1 is calculated, it is possible to determine the current on the radiating strip again through (9b) with V_1 instead of V_0. As already seen, the currents can be expanded in the spectral domain and by operating the inverse transform; with the residue theorem, it is possible to extract the leaky-pole contribution for $k_x = k_{x\,\mathrm{LW}}$. In connection with the choice of the first term of the expansion of the transverse current on the upper strip, it is possible to calculate the scattering parameter s_{44} with the relationship

$$s_{44} \equiv jV_1\,\mathrm{Res}\big[B_1(k_x); -k_{x\,\mathrm{LW}}\big] = -jV_1\,\mathrm{Res}\big[B_1(k_x); k_{x\,\mathrm{LW}}\big]. \tag{14}$$

Also, s_{44} is suitably related to the first coefficient in the expansion of the transverse component of the upper strip current. The basic parameters required for the complete circuit description of the junction are thus determined.

D. Complete Scattering Matrix and Relevant Design Parameters

Based on the assumed hypotheses and symmetries for the junction, the determination of the four parameters (s_{11}, s_{31}, s_{14},

$$J^{\mathrm{LW}}_{ux}(x,y) = \begin{cases} -jV_0 \displaystyle\sum_{m=1}^{M} \mathrm{Res}[A_m(k_x); k_{x\,\mathrm{LW}}] e^{-jk_{x\,\mathrm{LW}}x} T^x_m(y), & x \ge 0 \\ jV_0 \displaystyle\sum_{m=1}^{M} \mathrm{Res}[A_m(k_x); -k_{x\,\mathrm{LW}}] e^{+jk_{x\,\mathrm{LW}}x} T^x_m(y), & x \le 0 \end{cases}$$

$$J^{\mathrm{LW}}_{uy}(x,y) = \begin{cases} -jV_0 \displaystyle\sum_{n=1}^{N} \mathrm{Res}[B_n(k_x); k_{x\,\mathrm{LW}}] e^{-jk_{x\,\mathrm{LW}}x} T^y_n(y), & x \ge 0 \\ jV_0 \displaystyle\sum_{n=1}^{N} \mathrm{Res}[B_n(k_x); -k_{x\,\mathrm{LW}}] e^{+jk_{x\,\mathrm{LW}}x} T^y_n(y), & x \le 0 \end{cases} \tag{10}$$

s_{44}), according to the procedures outlined in the previous subsections, is sufficient to completely derive the scattering matrix.

From symmetry considerations it is easily seen that results

$$s_{22} = s_{11} \quad s_{33} = s_{44}. \tag{15}$$

From symmetry it derives as well: $s_{12} = s_{21}$, $s_{43} = s_{34}$. For the lower structure, from the continuity of the transverse magnetic field on the narrow slot at $y = 0$ [see (3a)–(3c)], it is $T_1 = 1 - R_1$, while for the upper structure, from the continuity of the transverse electric field, it is $s_{34} = 1 + s_{44}$; therefore,

$$s_{21} = 1 - s_{11} = s_{12} \quad s_{34} = 1 + s_{44} = s_{43}. \tag{16}$$

Due to the symmetries of the structure with respect to the $x = 0$ (PMC) and the $y = 0$ (PEC) planes when fed from ports 1 and 4, respectively, it comes out that

$$s_{41} = s_{31} \quad s_{24} = -s_{14}. \tag{17}$$

It is also interesting to note that, due to the combined effects of the PEC/PMC symmetry planes, the transmission coefficients from lower to upper structures (and vice-versa) change their sign when the structure is rotated by 180°; in fact, it can be seen that it results in

$$\begin{aligned} s_{42} &= -s_{31} \\ s_{13} &= -s_{24} = s_{14} \\ s_{23} &= -s_{14} \\ s_{32} &= -s_{41} = -s_{31}. \end{aligned} \tag{18}$$

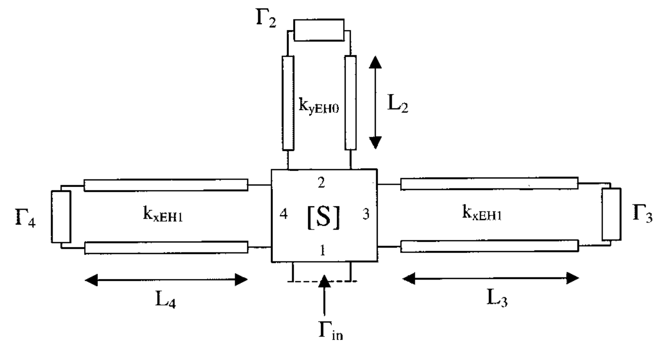
The final expression of the scattering matrix for the junction is, therefore,

$$[S] = \begin{pmatrix} s_{11} & 1 - s_{11} & s_{14} & s_{14} \\ 1 - s_{11} & s_{11} & -s_{14} & -s_{14} \\ s_{31} & -s_{31} & s_{44} & 1 + s_{44} \\ s_{31} & -s_{31} & 1 + s_{44} & s_{44} \end{pmatrix}. \tag{19}$$

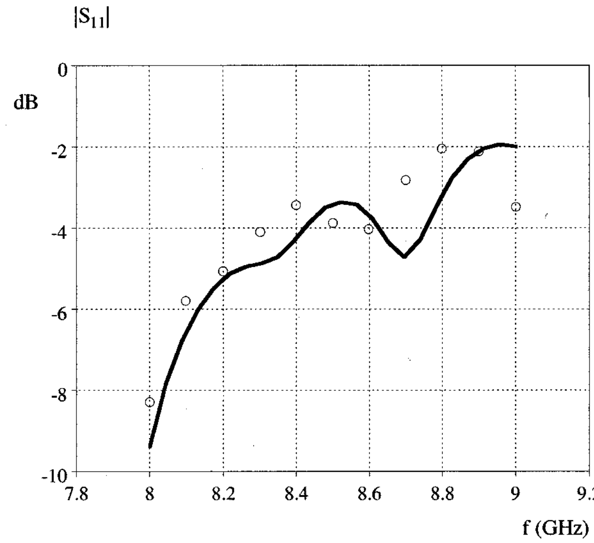
It is interesting to note that the S matrix is not symmetric since the input lines are not normalized to the same characteristic impedance [14]; in fact, for the leaky-mode lines corresponding to the two arms of the upper microstrip, the characteristic impedance is not defined at all. We would stress again the point that, for such ports, the scattering parameters are not related directly to a voltage or a current flow, but they are just the ratios between amplitude coefficients of a particular basis function used to represent the most significant part of the current on the strip, which are associated to the incident and reflected waves.

Once the scattering matrix describing the junction has been evaluated, it is possible to easily determine the input impedance seen by the feeding ports when all the other ports are suitably terminated. In particular, ports 2–4 (Fig. 3) can be closed with different lengths of open transmission lines (see, e.g., the schemes and the results of Figs. 4–7).

Moreover, the radiated field can be derived by considering the contribution of the currents of the EH_1 leaky mode having the incident and reflected waves calculated through the described circuit model (the relevant results will be presented in Figs. 8–10).



(a)



(b)

Fig. 4. (a) Circuit model of the slot-coupled leaky-wave antenna with a single feeding line. (b) Results for the scattering parameters as a function of the frequency for a basic junction as in Fig. 1: magnitude of S_{11} calculated with our model (circles) and with a commercial software (solid line). Parameters (see also Fig. 1): for both the upper and lower substrates, relative permittivity 3.05, and height $d_{l/u} = 0.508$ mm; lower strip: characteristic impedance 50 Ω, stub length $L_2 = \lambda/4$ (λ is the relevant wavelength of the EH_0 dominant mode); upper strip: $W_u = 1$ cm; $L_3 = L_4 = 8$ cm, slot: width 1 mm and length 5 mm.

III. Numerical Results

A. Implementation and Computational Features of the Code

The above-described analytical formulation has been implemented in a Fortran code whose main features are summarized here. The calculation of the quantities ΔV, Y_J, Y_M, and I, by which the scattering coefficients on the junction can be derived, requires the evaluation of integrals mainly in the spectral domain. The integration ranges can be reduced by exploiting the parity properties of the integrands; in the spectral domain, it is also necessary to properly deform the resulting integration path from the origin of the complex k_x- or k_y-plane to infinity in order to avoid the singularities of the integrands. The evaluation of the single integral for the ΔV parameter does not present any particular difficulty. For the calculation of the Y_J, Y_M, and I parameters, we have followed an approach such as in [15], performing a double spectral integration in rectangular coordinates

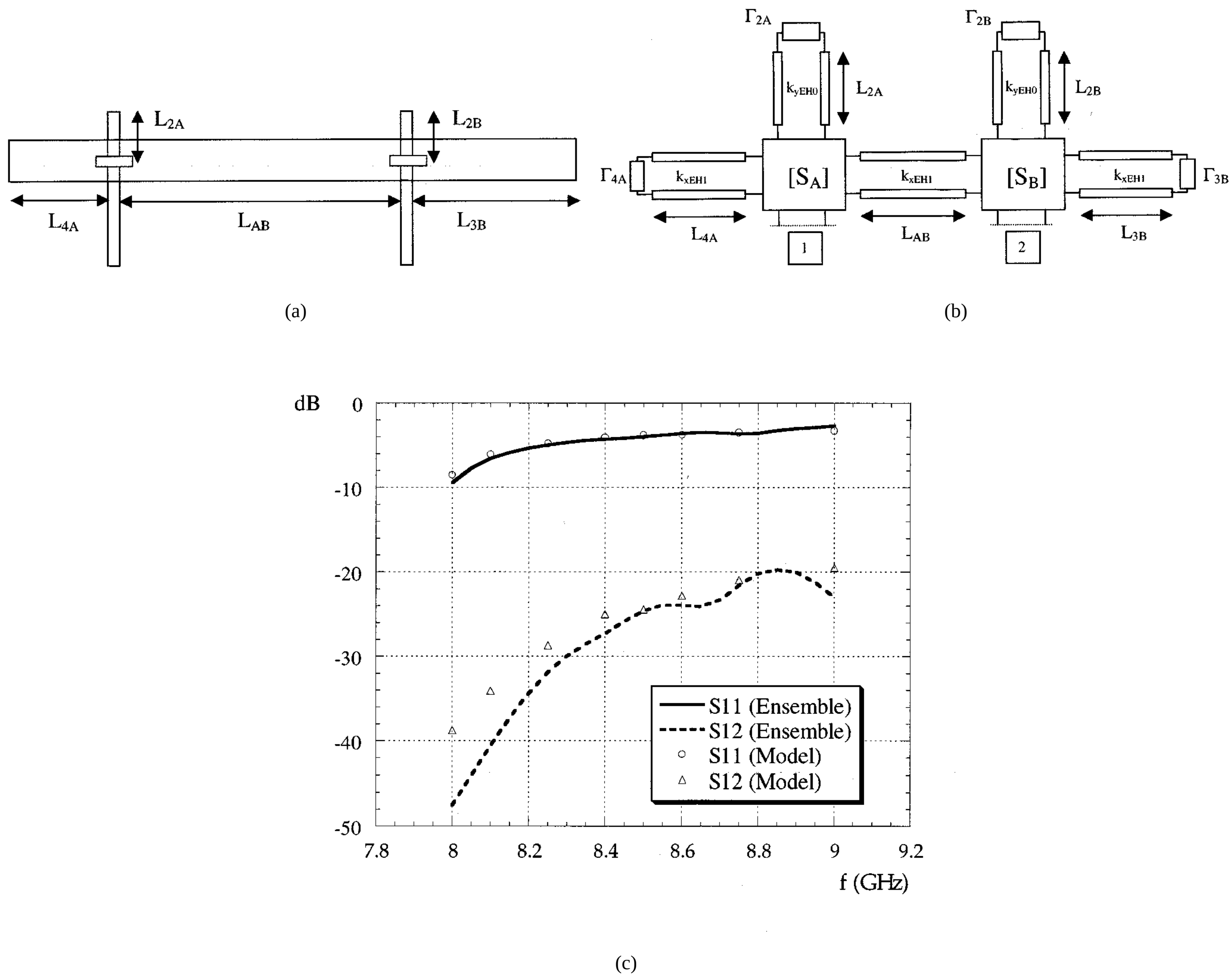


Fig. 5. (a) Geometry of a double-fed antenna. (b) Relevant circuit model. (c) Results for the scattering parameters as a function of the frequency: reflection coefficient $S_{11} = S_{22}$ and transmission coefficient $S_{12} = S_{21}$. Parameters: $L_{4A} = L_{AB} = L_{3B} = 7.5$ cm, $L_{2A} = L_{2B} = \lambda/4$, other parameters as in Fig. 4.

by means of an adaptive integration routine (Romberg method) [16].

The evaluation of the I parameter deserves some additional comments since it is directly related to a suitable contribution on the junction due to the incident EH_1 leaky mode excited at a finite distance D from the center of the slot. The currents of the incident mode have been written as

$$J_{ux}^{\mathrm{inc}}(x,y) = \sum_{m=1}^{M} A_m^{\mathrm{LW}} T_m^x(y) \Big[e^{-jk_{x\,\mathrm{LW}}(x+D)} u_{-1}(x+D) - e^{jk_{x\,\mathrm{LW}}(x+D)} u_{-1}(-x-D) \Big]$$
$$J_{uy}^{\mathrm{inc}}(x,y) = \sum_{n=1}^{N} B_n^{\mathrm{LW}} T_n^y(y) \Big[e^{-jk_{x\,\mathrm{LW}}(x+D)} u_{-1}(x+D) + e^{jk_{x\,\mathrm{LW}}(x+D)} u_{-1}(-x-D) \Big] \tag{20}$$

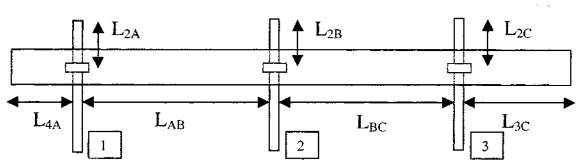
where the propagation constant $k_{x\,\mathrm{LW}}$ and the A_m^{LW} and B_n^{LW} coefficients relevant to the EH_1 mode have previously been determined with a spectral-domain analysis of the modal dispersion characteristics. As already observed, since this mode cannot be considered as incident from an infinite distance, the quantity I has been calculated for increasing finite values of D. The results thus obtained show that I is mostly independent of D as soon as D is greater than about half a guided wavelength. For distances D greater than some wavelengths, however, the calculation shows numerical instability due to the exponential amplification of the incident mode. A typical value of one wavelength for D has then been used in our simulations.

As far as the computational aspects of the code are concerned, typical calculation times for the determination of the scattering parameters of a slot junction at a fixed frequency are of the order of a few minutes on a standard personal computer.

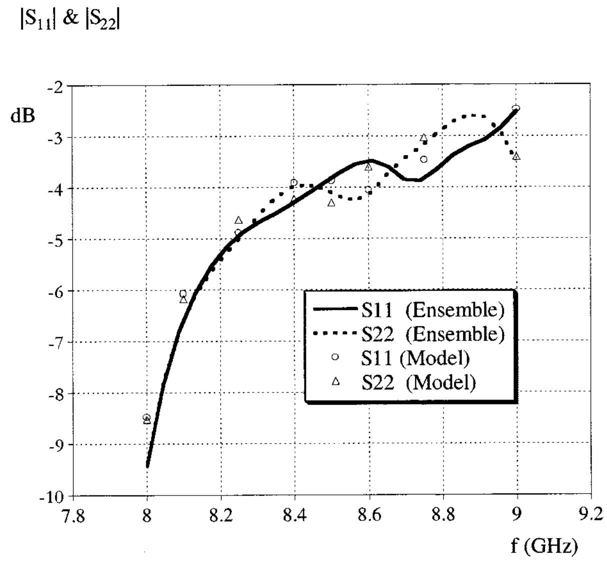
B. Scattering and Impedance Parameters in Various Configurations

In order to check the validity of the model, we have considered various simulated structures, also analyzed with a software (i.e., Ensemble) that uses a full-wave spatial-domain MoM.

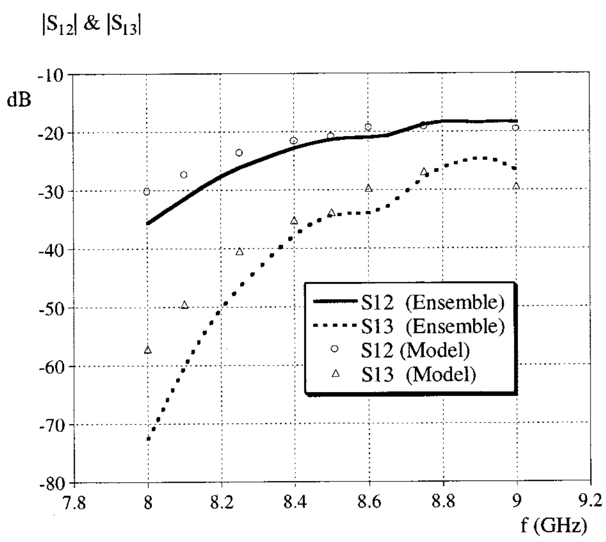
The first numerical tests have regarded the evaluation of the scattering parameters for various configurations of leaky-wave



(a)



(b)



(c)

Fig. 6. (a) Geometry of a triple-slot antenna. (b) Results for the scattering parameters as a function of the frequency: reflection coefficients $S_{11} = S_{33}$ and S_{22}. (c) Transmission coefficients $S_{12} = S_{21} = S_{23} = S_{32}$ and $S_{13} = S_{31}$. Parameters: $L_{4A} = L_{AB} = L_{BC} = L_{3C} = 7.5$ cm, $L_{2A} = L_{2B} = L_{2C} = \lambda/4$, other parameters as in Fig. 4.

microstrip structures. For a configuration of the type shown in Fig. 1, we have the circuit scheme of Fig. 4(a), in which the upper and lower transmission lines are closed on loads having reflection coefficients Γ_2, Γ_3, and Γ_4, which model the open-end termination of the microstrips and whose value has been assumed equal to unity as a first approximation. We present in Fig. 4(b) the results of the magnitude of the s_{11}-parameter for a frequency range and a choice of the physical quantities where the radiating microstrip can have a leaky behavior. (In all these simulations, the physical parameters have been chosen for microwave applications.) The agreement between our results and those of the compared method is satisfactory, but it is seen that the computing times of the commercial software are more than one order of magnitude greater. Our approach, therefore, strongly reduces the efforts to reach an efficient design of such structures.

The proposed model also allows for analyses of more complicated structures, where, for instance, several radiating strips can be excited to form an array, or multiple feeding lines can also excite a length of a microstrip leaky radiator. In these cases, the circuit model is easily achievable by connecting the four-port networks associated to each slot with suitable lengths of transmission lines equivalent to the leaky mode. In Fig. 5(a), we show the configuration of a double-slot leaky-wave radiator having the circuit scheme of Fig. 5(b). In Fig. 5(c), both the reflection and transmission coefficients (input at the first and output at the second feeding strip) are calculated and compared with the results of the commercial software (due to the symmetries, these are actually the only two characterizing parameters of the structure). The agreement still appears to be very good.

Finally, we illustrate the results for a microstrip fed by three different lines, according to the configuration of Fig. 6(a). Fig. 6(b) presents compared results for the reflection coefficients for the first and second feeding strips (the third one has the same properties of the first due to the symmetries). Fig. 6(c) presents compared results for the transmission coefficients, with input at the first strip and outputs at the second and third strips. In these cases, while good accuracy is maintained, the economy in memory storage and computing times of our approach becomes more and more conspicuous.

A different result for the basic network parameters is given in Fig. 7. In the case of a structure as the one shown in Figs. 1 and 4, a Smith chart is presented where we have calculated and again compared with Ensemble the values of the series impedance that, in the equivalent circuit, resumes the contribution of the discontinuity on the feeding strip due to the coupling slot. Both for the real part (which describes the radiative contribution) and the imaginary part (which takes into account the reactive effect), it is seen that the accord is fully adequate.

C. Currents and Radiation Patterns

Quite encouraging results have further been obtained as concerns the explicit description of the radiation features of the structures. In fact, as said, once the junction is represented by the scattering matrix, it is possible to derive the proper amount of the currents on the upper microstrip, and then to calculate the radiation pattern of the antenna.

A typical distribution of the currents on the radiating microstrip is presented in Fig. 8 for a structure as in Fig. 1. To optimize the behavior as a leaky-wave antenna in these cases,

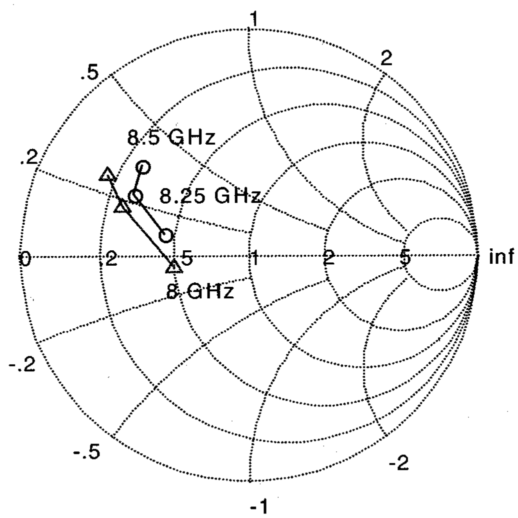


Fig. 7. Smith-chart representation of the series impedance that models the slot discontinuity on the lower feeding microstrip line for a single slot leaky-wave antenna: comparison between the values calculated with our model (circles) and those obtained with the commercial software (triangles) in a frequency range from 8 to 8.5 GHz. Parameters: as in Fig. 4.

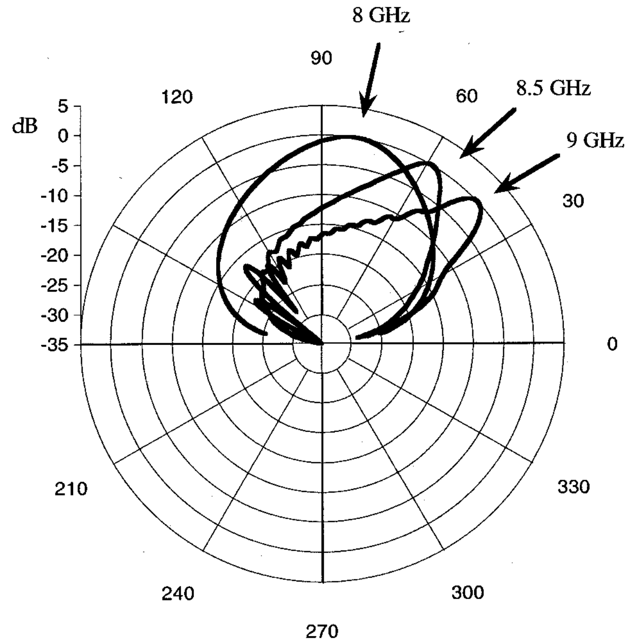
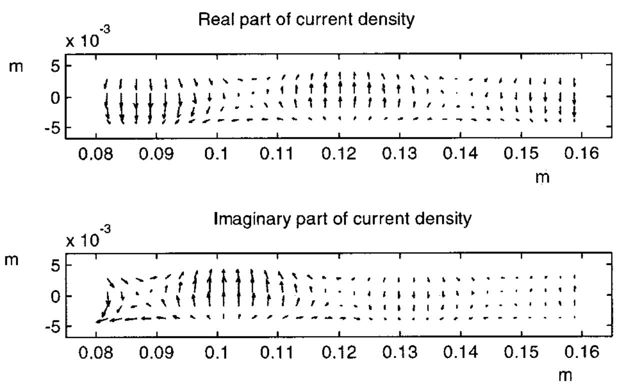
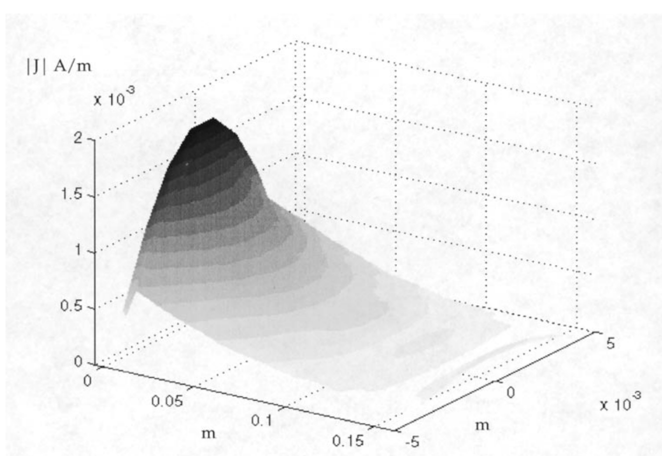


Fig. 9. Radiation patterns of the single-slot leaky-wave antenna in the elevation plane, for the frequency range of 8–9 GHz. Parameters (see Fig. 1): $L_3 = 2.5$ mm, $L_4 = 16$ cm, other parameters as in Fig. 4.



(a)



(b)

Fig. 8. (a) Vector plots of real and imaginary part of the current density excited on the final portion of the radiating strip. (b) Surface plot of the magnitude of the current density on the whole radiating strip. Parameters (see Fig. 1): $L_3 = 2.5$ mm, $L_4 = 16$ cm, other parameters as in Fig. 4.

the slot is placed near one end of the upper strip. It may be seen that the assumption for which the current distribution can be resumed by the only contribution of the EH_1 leaky mode appears to be quite valid. The calculated current–density contributions on the radiating upper strip are presented in Fig. 8(a) in vector form as arrow plots (real and imaginary parts); the results of our approach show a significant similarity with the configuration of the current derivable by the compared software. It is seen that the dominant current component is transversely directed with the proper symmetries. Both the phase and amplitude current behaviors are globally well predicted. (Just around the coupling slot element, the full-wave approach can well represent discontinuity effects, which are rather localized and do not affect the current behavior after a short distance.) In particular, it can be seen in Fig. 8(b) that the amplitude of the current on the upper strip tends to decrease along the line with an appropriate exponential amount, according to a pattern dominated by the complex propagation wavenumber $k_{x\,\mathrm{LW}}$ of the EH_1 leaky mode.

Finally, from the determination of the radiating currents, we have calculated the radiation patterns as well. In Fig. 9, for a structure as in Fig. 1, we show, in a polar form, the typical behavior of the radiation pattern that is scanned by varying the frequency in the elevation plane (H-plane of the antenna), as is usual in leaky-wave antennas. The presence of just one main lobe is related to the fact that the radiating microstrip is now fed near one of its ends.

In Fig. 10, for fixed frequencies in the leaky range of the upper microstrip, we show the entire comparisons between the radiation patterns calculated through Ensemble and our approach. Fig. 10(a)–(c) gives the radiation patterns as a function of the

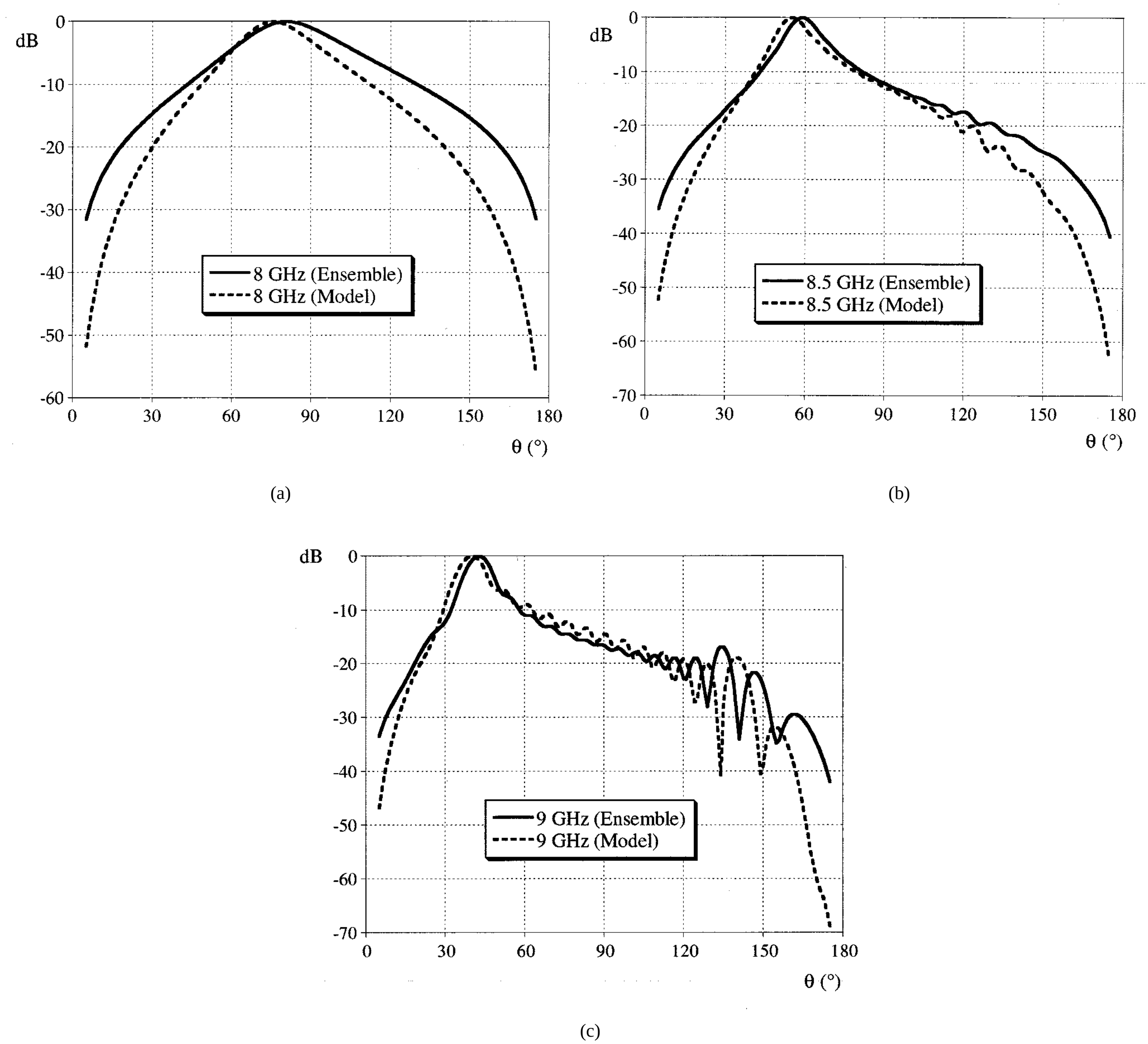


Fig. 10. Comparisons between the radiation patterns versus θ in the elevation plane, calculated with our model and with Ensemble, for the frequency: (a) 8 GHz, (b) 8.5 GHz, and (c) 9 GHz.

angle θ in the elevation plane, at the frequency of 8, 8.5, and 9 GHz, respectively. A valid prediction is globally achieved for the pointing angle, beamwidth, and behavior of the sidelobes.

The general good agreement of the numerical results confirms the validity of the hypotheses and of the formulation of our approach. It results that, under suitable conditions, the radiation performance of a finite-length microstrip line can be approximated in a valid way by the current contribution of the leaky wave, and the coupling phenomena due to the slot-coupled feeding strip are quantified in the appropriate fashion.

IV. Conclusion

We have investigated the possibility to develop a circuit model that can efficiently describe the excitation and radiation features in typical printed-circuit leaky-wave structures, in order to reach straightforward analysis and design procedures. The present formulation, which requires unusual associations of equivalent transmission lines, allows us to characterize a coupling junction between feeding and radiating lines through a particular scattering matrix. A quantification has been derived expressly for slot-coupled microstrip antennas, both with single and multiple feeders and/or radiators.

The significant and original aspect of this innovative theoretical approach is represented by the fact that a simple circuit model is achievable to describe types of problems for which transmission lines could not be associated in a conventional fashion. It is shown that equivalent scattering parameters can be used to evaluate the coupling effect between a feeding line acting on a usual propagation mode and a radiating line, which can support a leaky mode having an improper (nonspectral) nature.

The advantageous implications of such a circuit model are manifold. As in conventional microwave circuits, a convenient scattering matrix also gives immediate information for the behavior of printed-circuit radiating structures employing traveling waves. Moreover, the achievement of the scattering parameters in a numerical form is very economic and efficient; the computational efforts are limited since the typical required spectral-domain integral calculations need rather limited resources of memory storage and short computing times. The advantages in efficiency with respect to different full-wave approaches (such as those based on commercial software) are particularly evident as the structures become more and more complex (multiple-fed radiators, arrays, etc.) considering also that leaky-wave radiators are typically many wavelengths long. Since the involved physical space can become rather wide, the conventional numerical techniques may, in fact, present dramatic problems related to memory requirements, computation time, and also precision. On the contrary, the approach proposed here is immediately extended to complicated topologies without appreciable deterioration of efficiency and accuracy. An attractive feature is that the model quantifies not only the input impedance of the structure, but also its antenna properties, obtaining with quite satisfactory precision, as a function of the physical quantities involved, the radiation patterns from the characterization of the leaky-wave currents.

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Paolo Burghignoli (S'97) was born in Rome, Italy, on February 18, 1973. He received the Laurea degree (*cum laude*) in electronic engineering from "La Sapienza" University of Rome, Rome, Italy, in 1997, and is currently working toward the Ph.D. degree at "La Sapienza" University of Rome in applied electromagnetics.

His scientific interests include analysis and design of planar leaky-wave antennas and general numerical methods for the analysis of passive guiding and radiating microwave structures.

Carlo Di Nallo (S'95–A'96) was born in Varese, Italy, in 1967. He received the Laurea degree (*cum laude*) and the Doctorate degree in applied electromagnetics from the "La Sapienza" University of Rome, Rome, Italy, in 1992 and 1996 respectively.

In 1992, he joined the Department of Electronic Engineering, "La Sapienza" University of Rome. From May 1995 to October 1995, he was at the University of Houston, Houston, TX, as a Visiting Scholar. His research interests concern numerical modeling of microwave passive components, planar antenna analysis and design, and theoretical and experimental studies on leaky waves.

Fabrizio Frezza (S'87–M'90–SM'95) received the Laurea degree (*cum laude*) in electronic engineering and the Doctorate degree in applied electromagnetics from "La Sapienza" University of Rome, Rome, Italy, in 1986 and 1991, respectively.

In 1986, he joined the Electronic Engineering Department, "La Sapienza" University of Rome, where he has become a Researcher (1990), a temporary Professor (1994), and an associate Professor of electromagnetic fields (1998). His main research activity concerns guiding structures, antennas and resonators for microwaves and millimeter waves, numerical methods, scattering, optical propagation, plasma heating, and anisotropic media.

Dr. Frezza is a member of Sigma Xi, the Electrical and Electronic Italian Association (AEI), the Italian Society of Optics and Photonics (SIOF), the Italian Society for Industrial and Applied Mathematics (SIMAI), and the Aeronautics and Astronautics Italian Association (AIDAA).

Alessandro Galli (S'91–M'96) received the Laurea degree in electronic engineering and the Ph.D. degree in applied electromagnetics from "La Sapienza" University of Rome, Rome, Italy, in 1990 and 1994, respectively.

In 1990, he joined the Electronic Engineering Department, "La Sapienza" University of Rome, for his research activity. His scientific interests mainly involve electromagnetic theory and applications, particularly regarding the analysis and design of passive devices (dielectric and anisotropic waveguides and resonators) and of leaky-wave antennas for microwaves and millimeter waves. He has also been active in bioelectromagnetics (interaction mechanisms).

Dr. Galli received the Quality Presentation Recognition Award presented by the IEEE Microwave Theory and Technique Society (IEEE MTT-S) in 1994 and 1995.

Paolo Lampariello (M'73–SM'82–F'96) was born in Rome, Italy, on May 17, 1944. He obtained the Laurea degree (*cum laude*) in electronic engineering from the University of Rome, Rome, Italy, in 1971.

In 1971, he joined the Institute of Electronics, University of Rome, where he was Assistant Professor of electromagnetic fields. Since 1976, he has been engaged in educational activities involving electromagnetic-field theory. He became a Professor of electromagnetic fields in 1986. From November 1988 to October 1994, he served as Head of the Department of Electronic Engineering, "La Sapienza" University of Rome, Rome, Italy. Since November 1993, he has been the President of the Council for Electronic Engineering Curriculum at the "La Sapienza" University of Rome. Since September 1995, he has been the President of the Center Interdepartmental for Scientific Computing, "La Sapienza" University of Rome. From September 1980 to August 1981, he was a NATO Post-Doctoral Research Fellow at the Polytechnic Institute of New York, Brooklyn. He has been engaged in research in a wide variety of topics in the microwave field, including electromagnetic and elastic wave propagation in anisotropic media, thermal effects of electromagnetic waves, network representations of microwave structures, guided-wave theory with stress on surface waves and leaky waves, traveling-wave antennas, phased arrays, and, more recently, guiding and radiating structures for the millimeter and near-millimeter wave ranges.

Prof. Lampariello is a member of the Associazione Elettrotecnica ed Elettronica Italiana (AEI). He is currently the President of the Specialist Group "Electromagnetism" of the AEI. He is a past chairman of the Central and South Italy Section of the IEEE and a past chairman of the IEEE Microwave Theory and Techniques Society (IEEE MTT-S)/IEEE Antennas Propagation Society (IEEE AP-S) Joint Chapter of the same section.