Unified Analytical Expressions for Transimpedance and Equivalent Input Noise Current of Optical Receivers

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***Abstract*—Unified analytical expressions are derived for calculating the equivalent input noise current and transimpedance of optical receiver front ends with arbitrary input matching network topologies. To be independent of any transistor or amplifier type, noise parameters are used to describe the noise behavior of the active device. A new characteristic frequency-dependent value, called photodiode intrinsic conductance, is introduced. This figure-of-merit allows to compare the achievable equivalent input noise current and transimpedance of different types of photodiodes independently of amplifier type and geometry.**

***Index Terms*—Integrated optoelectronics, noise, optical receivers, photodetectors.**

I. Introduction

NOISE IN optical receivers can be divided into the following three categories:

- noise contributed by the optical source (such as laser relative intensity noise) or by the optical transmission path (optical excess noise);
- noise that arises from carrier generation or from carrier transport over the p-n junction;
- Johnson noise from the photodiode parasitics and noise from the front-end amplifier.

While the noise contributed by the first two items is dependent on the optical link architecture and on the dc current flow through the photodiode, respectively, the third item, i.e., electronics noise, can be optimized by a proper circuit design. However, noise optimization of circuit design can normally only be done at the expense of receiver's transimpedance. As it is not useful to lower the electronics noise much below the level caused by other noise sources mentioned above [1], the receiver designer can find an optimum compromise for the given application.

It has been shown that a noise matching network between a photodiode and amplifier can greatly enhance an optical receiver's noise performance [2], [3]. The noise representations classically used for receiver noise modeling are the Ogawa noise factor Γ [4], correlated drain and gate noise sources for p-i-n–FET front ends [5], or noise temperatures [6]. To be independent of a specific active device model and to achieve more accurate noise modeling at higher frequencies, it is advisable to use the four noise parameters: minimum noise figure $F_{\min}$, real and imaginary parts of optimum source admittance $Y_{\rm opt}$, and noise resistance R_n [7]. Furthermore, microwave design tools allow to directly simulate these noise parameters, even for different bias conditions [8].

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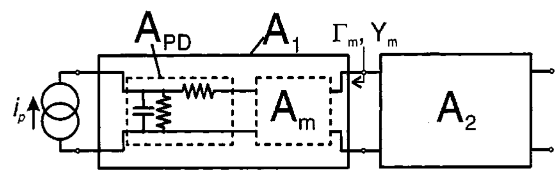

Fig. 1. Model of an optical receiver. $A_{\rm PD}$: photodiode equivalent circuit. A_m: matching network. A_2: active two-port.

In this paper, general expressions are derived for the equivalent input noise current, caused by the photodiode parasitics and the front-end amplifier, and the receiver's transimpedance. These equations can be used for arbitrary amplifier designs and photodiodes. The derived analytical expressions can be implemented in microwave design tools such that all techniques and software available for microwave amplifier design and optimization can be also used for optical receiver front-end design.

At higher frequencies, a photodiode equivalent circuit consisting only of junction capacitance and series resistance could be inaccurate. A photodiode intrinsic conductance is introduced. It can be calculated from an arbitrary photodiode equivalent circuit or can be directly extracted from measurements. This enables an easy calculation of the receiver's equivalent input noise current and transimpedance.

II. Theory

In this section, some expressions that allow an easy calculation of equivalent input noise current of optical receivers are derived. The model used to investigate the influence of an input matching network is depicted in Fig. 1. Basically, an optical receiver consists of a photodiode, some matching elements, and an amplifying part. The photodiode is commonly described by a current source driving a parallel junction capacitance and some additional parasitic elements. The equivalent-circuit elements of the photodiode model can be added to the two-port A_1 that describes the matching network. Thus, the calculation of the input equivalent noise current can be reduced to the case where an ideal current source drives a noisy two-port. The two-port A_2 stands for the active part of the receiver (either a single transistor or a complete amplifier).

In Section II-A, the equivalent input noise current of an arbitrary two-port driven by an ideal current source is calculated. The two-port is represented by its noise parameters. Section II-B is a short review of the noise parameter calculation of cascaded two-ports. Furthermore, the equivalent input noise current of cascaded two-ports is derived. In Section II-C, the results are applied to a simple example.

A. Calculation of the Equivalent Input Noise Current

First, an expression for the equivalent input noise current of a generic two-port is derived. In microwave circuit design, the noise behavior of two-ports is commonly described by the four noise parameters: minimum noise figure $F_{\min}$, real and imaginary part of the optimum source admittance G_{opt} and B_{opt}, and noise resistance $R_n = r_n Z_0$ with Z_0 being the characteristic impedance chosen to be real. As described in [9], the noisy two-port can be replaced by a noise-free, but otherwise unchanged, two-port in conjunction with a voltage and a current noise source preceding the two-port. A correlation admittance $Y_{\text{cor}} = G_{\text{cor}} + \mathrm{j}B_{\text{cor}}$ can be added so that the introduced current and voltage noise sources are not correlated. The noise voltage v_n and current i_n can be described by an equivalent noise resistance R_n and an equivalent noise conductance G_n

$$\overline{|v_n|^2} = 4kT_0\, df\, R_n \quad \overline{|i_n|^2} = 4kT_0\, df\, G_n \tag{1}$$

where T_0 is the reference temperature. The noise conductance G_n and the correlation admittance Y_{cor} can be derived from the noise parameters [9, eq. (21)–(23)]

$$\begin{aligned} B_{\text{cor}} &= -B_{\text{opt}} \\ G_{\text{cor}} &= \frac{F_{\min} - 1}{2R_n} - G_{\text{opt}} \\ G_n &= \frac{-(F_{\min} - 1)^2}{4R_n} + (F_{\min} - 1)G_{\text{opt}}. \end{aligned} \tag{2}$$

The resulting equivalent circuit is depicted in Fig. 2(a). If the two-port is preceded by a current source with source admittance Y_S, one can replace the noise sources and correlation admittance by a single current noise source in parallel to the input current source [see Fig. 2(b)]. From principle of superposition, it follows that the equivalent input noise current i_{eq} is given by

$$\begin{aligned} \overline{|i_{\text{eq}}|^2} &= \overline{|i_n + v_n(Y_S + Y_{\text{cor}})|^2} \\ &= 4kT_0\, df \left[(F_{\min} - 1)G_S + R_n|Y_S - Y_{\text{opt}}|^2\right] \end{aligned} \tag{3}$$

in which G_S is the real part of the source admittance. Replacing the admittances by reflection coefficients Γ_S, Γ_{opt} results in

$$\overline{|i_{\text{eq}}|^2} = \frac{4kT_0\, df}{|1 + \Gamma_S|^2 Z_0} \left[(F_{\min} - 1)\,(1 - |\Gamma_S|^2) + 4r_n \left|\frac{\Gamma_{\text{opt}} - \Gamma_S}{\Gamma_{\text{opt}} + 1}\right|^2\right]. \tag{4}$$

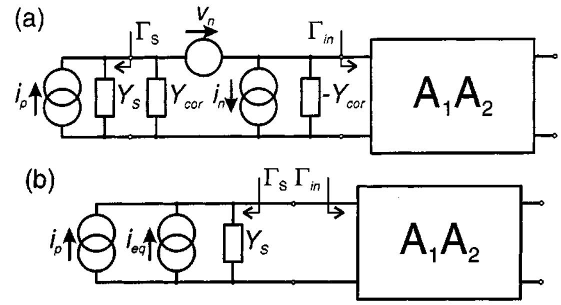

Fig. 2. (a) Equivalent circuit for calculating the equivalent input noise current. (b) Resulting circuit.

As mentioned above, the equivalent circuit elements of the photodiode can be added to the two-port A_1. Therefore, it can be assumed that the current source at the two-port input is an ideal one. That results in a source admittance $Y_S = 0$ and a source reflection coefficient $\Gamma_S = 1$. Equation (4) then becomes

$$\overline{|i_{\text{eq}}|^2} = 4kT_0\, df\, R_n|Y_{\text{opt}}|^2 = 4kT_0\, df \frac{r_n}{Z_0} \left|\frac{1 - \Gamma_{\text{opt}}}{1 + \Gamma_{\text{opt}}}\right|^2. \tag{5}$$

From (5), one can notice that a low product of the noise resistance R_n and the squared magnitude of optimum source admittance Y_{opt} is needed to achieve a low equivalent input noise current.

B. Noisy Cascaded Two-Ports

As depicted in Fig. 1, an optical receiver can be divided in a passive and an active two-port. The passive two-port consists of the photodiode equivalent circuit and a matching circuit. The active two-port describes the front-end amplifier. Both two-ports are noisy. For calculating an overall equivalent input noise current, one has to calculate current and noise sources and correlation admittances for both two-ports and to transform them to the input of the first two-port. In [10], a method is presented that simplifies this calculation. Following this method, the noise behavior of two cascaded two-ports can be described by the following equation:

$$\mathbf{C_A} = \mathbf{C_{A1}} + \mathbf{A_1 C_{A2} A_1^\dagger} \tag{6}$$

where $\mathbf{C_{A1}}$, $\mathbf{C_{A2}}$, and $\mathbf{C_A}$ denote the correlation matrix in a chain representation of the first, second, and resulting two-ports, respectively. $\mathbf{A_1}$ and $\mathbf{A_1^\dagger}$ denotes the chain matrix representation of the first two-port A_1 and its Hermitian conjugate, respectively.

The correlation matrix of a two-port can be calculated using the four noise parameters [10]

$$\mathbf{C_A} = 4kT_0\, df \begin{pmatrix} R_n & \frac{F_{\min} - 1}{2} - R_n Y_{\text{opt}}^* \\ \frac{F_{\min} - 1}{2} - R_n Y_{\text{opt}} & R_n |Y_{\text{opt}}|^2 \end{pmatrix}. \tag{7}$$

Introducing (7) into (6) results in an expression for the product $R_n|Y_{\rm opt}|^2$ in (5)

$$R_n|Y_{\rm opt}|^2 = R_{n,1}|Y_{\rm opt,1}|^2 + {\rm Re}\{a_{21}^* a_{22}\}(F_{\rm min,2}-1) + R_{n,2}(|a_{21}|^2 + |a_{22}Y_{\rm opt,2}|^2 - 2{\rm Re}\{a_{21}^* a_{22} Y_{\rm opt,2}\}) \quad (8)$$

where a_{ij} are the coefficients of the chain matrix $\mathbf{A_1}$. $R_{n,1}$ and $Y_{\rm opt,1}$ denote noise resistance and optimum source admittance of the passive two-port A_1, respectively, whereas $R_{n,2}$, $F_{\rm min,2}$, and $Y_{\rm opt,2}$ denote the noise parameters of the active two-port A_2. For linear two-ports, the correlation matrix in a chain representation can be derived from the correlation matrix in an impedance or admittance representation [10]

$$\begin{aligned}\mathbf{C_A} &= \begin{pmatrix} 1 & -a_{11} \\ 0 & -a_{21} \end{pmatrix} \mathbf{C_Z} \begin{pmatrix} 1 & 0 \\ -a_{11}^* & -a_{21}^* \end{pmatrix} \\ &= \begin{pmatrix} 0 & a_{12} \\ 1 & a_{22} \end{pmatrix} \mathbf{C_Y} \begin{pmatrix} 0 & 1 \\ a_{12}^* & a_{22}^* \end{pmatrix}. \end{aligned} \quad (9)$$

If the two-port is passive, the correlation matrix in an impedance or admittance representation is given by [10]

$$\begin{aligned}\mathbf{C_Z} &= 4kT_0\, df {\rm Re}\{\mathbf{Z}\} \\ \mathbf{C_Y} &= 4kT_0\, df {\rm Re}\{\mathbf{Y}\}\end{aligned} \quad (10)$$

where $\mathbf{Z}$ and $\mathbf{Y}$ denote the impedance and admittance matrix of the matching two-port, respectively. Substituting (10) into (9) and comparing the result with (7) gives an expression for the first term in (8)

$$R_{n,1}|Y_{\rm opt,1}|^2 = |a_{21}|^2{\rm Re}\{z_{22}\} = |a_{21}|^2{\rm Re}\left\{\frac{a_{22}}{a_{21}}\right\}. \quad (11)$$

Substituting (11) and (8) into (5) yields the following expression for the equivalent input noise current:

$$\overline{|i_{\rm eq}|^2} = 4kT_0\, df \Big[{\rm Re}\{a_{21}^* a_{22}\}F_{\rm min,2} + R_{n,2}\big(|a_{21}|^2 + |a_{22}Y_{\rm opt,2}|^2 - 2{\rm Re}\{a_{21}^* a_{22} Y_{\rm opt,2}\}\big)\Big]. \quad (12)$$

The second part of (12) vanishes if

$$Y_{\rm opt,2} = G_{\rm opt,2} + {\rm j}B_{\rm opt,2} = a_{21}/a_{22}. \quad (13)$$

In this case, the output conductance a_{21}/a_{22} of the matching network and photodiode becomes $Y_{\rm opt,2}$. Therefore, the minimum achievable equivalent input noise current is given by

$$\overline{|i_{\rm eq,min}|^2} = 4kT_0\, df |a_{21}|^2 F_{\rm min,2}{\rm Re}\left\{\frac{1}{Y_{\rm opt,2}}\right\}. \quad (14)$$

C. Derivation of Equivalent Input Noise Current for Two-Element Photodiode Model

Equation (14) suggests that, by reducing $|a_{21}|$, the minimum equivalent input noise current can be decreased. Unfortunately, this is not possible. This is illustrated in the following example by using a simple two-element photodiode equivalent circuit, consisting of a junction capacitance C_i and series resistance R_s. It is assumed that the matching network is reciprocal and lossless. If a chain representation $\mathbf{A_m}$ is chosen for the matching two-port A_m with its coefficients $a_{ij,m}$, reciprocity can be written as [11]

$$\Delta\mathbf{A_m} = a_{11,m}a_{22,m} - a_{12,m}a_{21,m} = 1 \quad (15)$$

and losslessness as

$$\mathbf{A_m} = \begin{pmatrix} {\rm Re}\{a_{11,m}\} & j{\rm Im}\{a_{12,m}\} \\ j{\rm Im}\{a_{21,m}\} & {\rm Re}\{a_{22,m}\} \end{pmatrix} = \begin{pmatrix} m_{11} & jm_{12} \\ jm_{21} & m_{22} \end{pmatrix} \quad (16)$$

where m_{ij} are real coefficients. Multiplying the chain matrix

$$\mathbf{A_{PD}} = \begin{pmatrix} 1 & R_s \\ j\omega C_i & 1 + j\omega C_i R_s \end{pmatrix} \quad (17)$$

representing the photodiode equivalent circuit and $\mathbf{A_m}$ given in (16) yields

$$\begin{aligned}\mathbf{A_1} &= \mathbf{A_{PD}}\mathbf{A_m} \\ &= \begin{pmatrix} m_{11} + jm_{21}R_s & m_{22}R_s + jm_{12} \\ -\omega C_i R_s m_{21} + j(\omega C_i m_{11} + m_{21}) & m_{22} - \omega C_i m_{12} + j\omega C_i R_s m_{22} \end{pmatrix}. \end{aligned} \quad (18)$$

Equations (13) and (15) give three equations for the four unknown m_{ij}. The two solutions are

$$\begin{aligned} m_{11} &= G_{\rm opt,2}m_{22}R_s - \frac{m_{22}B_{\rm opt,2}}{\omega C_i} + \frac{(G_{\rm opt,2} + \omega C_i R_s B_{\rm opt,2})\sqrt{G_{\rm opt,2}R_s(1 - G_{\rm opt,2}m_{22}^2 R_s)}}{\omega C_i R_s G_{\rm opt,2}} \\ m_{12} &= \frac{m_{22}}{\omega C_i} - \frac{\sqrt{G_{\rm opt,2}R_s(1 - G_{\rm opt,2}m_{22}^2 R_s)}}{G_{\rm opt,2}} \\ m_{21} &= m_{22}B_{\rm opt,2} - \frac{\sqrt{G_{\rm opt,2}R_s(1 - G_{\rm opt,2}m_{22}^2 R_s)}}{R_s} \end{aligned} \quad (19)$$

and

$$\begin{aligned} m_{11} &= G_{\rm opt,2}m_{22}R_s - \frac{m_{22}B_{\rm opt,2}}{\omega C_i} - \frac{(G_{\rm opt,2} + \omega C_i R_s B_{\rm opt,2})\sqrt{G_{\rm opt,2}R_s(1 - G_{\rm opt,2}m_{22}^2 R_s)}}{\omega C_i R_s G_{\rm opt,2}} \\ m_{12} &= \frac{m_{22}}{\omega C_i} + \frac{\sqrt{G_{\rm opt,2}R_s(1 - G_{\rm opt,2}m_{22}^2 R_s)}}{G_{\rm opt,2}} \\ m_{21} &= m_{22}B_{\rm opt,2} + \frac{\sqrt{G_{\rm opt,2}R_s(1 - G_{\rm opt,2}m_{22}^2 R_s)}}{R_s} \end{aligned} \quad (20)$$

if $|m_{22}| < 1/\sqrt{R_s G_{\rm opt,2}}$.

Inserting one of these solutions in (18) results in

$$|a_{21}| = \omega C_i |Y_{\rm opt,2}|\sqrt{\frac{R_s}{G_{\rm opt,2}}} \quad (21)$$

which is no longer dependent on m_{22}. Substituting (21) into (14) results in the expression for the minimum equivalent input noise current

$$\overline{|i_{\text{eq,min}}|^2} = 4kT_0\ df(\omega C_i)^2 R_s F_{\text{min},2}. \tag{22}$$

This simplified result for the minimum equivalent input noise current of a photodiode represented by a junction capacitance C_i and a series resistance R_s can also be found in [12].

III. Unified Expressions for Equivalent Input Noise Current and Transimpedance

A. Equivalent Input Noise Current

According to (12), the equivalent input noise current of an arbitrary active two-port A_2 preceded by a passive matching network and photodiode equivalent circuit can be expressed by the simple equation

$$\overline{|i_{\text{eq}}|^2} = \overline{|i_{\text{eq,min}}|^2} + \overline{|i_{\text{eq},i}|^2}\frac{R_{n,2}}{G_m}\ |Y_m - Y_{\text{opt},2}|^2 \tag{23}$$

with

$$Y_m = a_{21}/a_{22} \tag{24}$$

and G_m input admittance of matching network and photodiode equivalent circuit (see Fig. 1) and its real part, respectively. For an arbitrary network A_1, the squared noise currents $\overline{|i_{\text{eq,min}}|^2}$ and $\overline{|i_{\text{eq},i}|^2}$ are given by

$$\begin{aligned}\overline{|i_{\text{eq,min}}|^2} &= 4kT_0\ df\text{Re}\{a_{21}^* a_{22}\}F_{\text{min},2}\\ \overline{|i_{\text{eq},i}|^2} &= 4kT_0\ df|a_{22}|^2 G_m.\end{aligned} \tag{25}$$

It can be shown that if a more general equivalent circuit than the one in the previous section is chosen by a chain matrix $\mathbf{A_{PD}}$ with coefficients $a_{ij,PD}$ and a lossless and reciprocal matching network A_m is used, we get an expression for the minimum achievable equivalent input noise current that is independent of any matching network

$$\overline{|i_{\text{eq,min}}|^2} = 4kT_0\ df\text{Re}\{a_{21,PD}^* a_{22,PD}\}F_{\text{min},2}. \tag{26}$$

The parameter $\overline{|i_{\text{eq},i}|^2}$ is then given by

$$\overline{|i_{\text{eq},i}|^2} = 4kT_0\ df\text{Re}\{a_{21,PD}^* a_{22,PD}\}. \tag{27}$$

Comparing (26) and (27) with (23) and remembering that the noise figure of a two-port is given by

$$F = F_{\text{min}} + \frac{R_n}{G_m}\ |Y_m - Y_{\text{opt}}|^2 \tag{28}$$

we can, under the above-mentioned restrictions, calculate the squared equivalent input noise current by multiplying the noise figure of the amplifying two-port F_2 with the squared photodiode intrinsic noise current $\overline{|i_{\text{eq},i}|^2}$

$$\overline{|i_{\text{eq}}|^2} = F_2\overline{|i_{\text{eq},i}|^2}. \tag{29}$$

Thus, the already well-known techniques for designing low-noise amplifiers can also be used for designing low-noise photoreceivers. For example, circles of constant equivalent input noise current can be plotted in a Smith chart.

B. Transimpedance

The transimpedance is defined as the magnitude of the ratio of output voltage v_L at a load impedance Z_L and photocurrent i_p through the photodiode. From a scattering-parameter definition, the transimpedance can be calculated to be

$$Z_T = \left|\frac{v_L}{i_p}\right| = \left|Z_0\frac{S_{21}(1+\mathit{\Gamma}_L)}{(1-S_{22}\mathit{\Gamma}_L)(1-S_{11}) - S_{12}S_{21}\mathit{\Gamma}_L}\right| \tag{30}$$

where $\mathit{\Gamma}_L$ and S_{ij} denote the load reflection coefficient and scattering parameters of an arbitrary network, respectively. If the load reflection coefficient is chosen to be zero, (30) results in

$$Z_T = \left|Z_0\frac{S_{21}}{1-S_{11}}\right|. \tag{31}$$

For the two cascaded two-ports A_1 and A_2, the transimpedance for $\mathit{\Gamma}_L = 0$ is given by

$$Z_T = Z_0\left|\frac{S_{21,A2}}{a_{21}(1+S_{11,A2})Z_0 + a_{22}(1-S_{11,A2})}\right| \tag{32}$$

in which $S_{ij,A2}$ are the scattering parameters of two-port A_2. As before, the first two-port A_1 consists of the photdiode equivalent circuit A_{PD} and a matching network A_m, which is assumed to be lossless and reciprocal. One can then write using (24)

$$\begin{aligned}Z_T &= Z_0\sqrt{\frac{G_m}{G_{i,PD}}}\ \left|\frac{S_{21,A2}}{(1+S_{11,A2})Y_m Z_0 + (1-S_{11,A2})}\right|\\ &= \sqrt{\frac{Z_0(1-|\mathit{\Gamma}_m|^2)}{G_{i,PD}}}\ \left|\frac{S_{21,A2}}{2(1-S_{11,A2}\mathit{\Gamma}_m)}\right|\end{aligned} \tag{33}$$

where

$$G_{i,PD} = \text{Re}\{a_{21,PD}^* a_{22,PD}\}. \tag{34}$$

Remembering that the transducer power gain for $\mathit{\Gamma}_L = 0$ is given by [13]

$$G_T = \frac{1-|\mathit{\Gamma}_m|^2}{|1-S_{11}\mathit{\Gamma}_m|^2}\ |S_{21}|^2 \tag{35}$$

one can calculate the transimpedance by

$$Z_T = \frac{1}{2}\sqrt{G_T\frac{Z_0}{G_{i,PD}}}. \tag{36}$$

With (33), it is possible to plot circles of constant transimpedance in a Smith chart. In Fig. 3, circles of constant equivalent input noise current and constant transimpedance at a frequency of 10 GHz are plotted for a photodiode with a junction capacitance of 110 fF and a series resistance of 10 Ω. The model parameters are given in Table I. The parameters used for the active two-port represent a GaAs-based 0.15-μm gate-length pseudomorphic HEMT with inductive series feedback. As known from constant gain and constant noise-figure circles, it is, in general, not possible to achieve a simultaneous matching for minimum equivalent input noise current and maximum transimpedance.

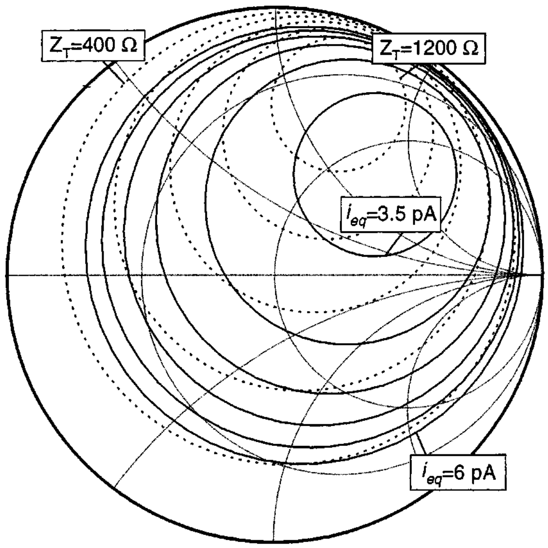

Fig. 3. Circles of constant equivalent input noise current in a square-root hertz bandwidth (solid) and circles of constant transimpedance (dotted) at 10 GHz. The utilized model parameters are given in Table I.

TABLE I
PARAMETERS FOR EQUIVALENT INPUT NOISE CURRENT AND TRANSIMPEDANCE CIRCLES CALCULATION

Parameter	Symbol	Value
Photodiode parameters:		
Intrinsic conductance	$G_{i,PD}$	0.478 mS
Active two-port parameters:		
Minimum noise figure	$F_{min,2}$	1.38 dB
Noise resistance	$R_{n,2}$	30.26 Ω
Optimum input admittance	$Y_{opt,2}$	(5.5 - j 7.7) mS
Scattering parameters	$S_{11,A2}$	0.28 - j 0.74
	$S_{21,A2}$	-3.29 + j 4.32

IV. Intrinsic Conductance and Noise Current

In the previous section, two new parameters were introduced: a conductance $G_{i,PD}$, describing intrinsic properties of the photodiode, and an intrinsic noise current $i_{\text{eq},i}$. The relation between these parameters is given by

$$\overline{|i_{\text{eq},i}|^2} = 4kT_0\ df\ G_{i,PD}. \tag{37}$$

Rewriting (29) as follows:

$$\overline{|i_{\text{eq}}|^2} = 4kT_0\ df\ G_{i,PD}F_2 \tag{38}$$

and from (36), it can be seen that the equivalent input noise current is proportional and the transimpedance is inversely proportional to the square root of the photodiode intrinsic conductance.

For the above-used simple equivalent circuit, the intrinsic conductance

$$G_{i,PD} = (\omega C_i)^2 R_s \tag{39}$$

is obtained. The calculation of the intrinsic conductance from the photodiode equivalent circuit has one disadvantage: The result cannot be more accurate than the extraction of the equivalent-circuit elements. Especially at higher frequencies, it is very difficult to find valid equivalent circuits. Therefore, it is preferable to directly extract the intrinsic conductance from measurements, as shown below.

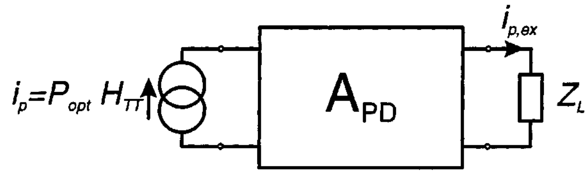

Fig. 4. Photodiode model for calculating the intrinsic conductance.

The frequency-dependent photodiode responsivity in amperes/watts (A/W) is given by

$$R = \frac{i_{p,\text{ex}}}{P_{\text{opt}}} = H_{TT}H_{EC} \tag{40}$$

where

$$H_{TT} = \frac{i_p}{P_{\text{opt}}} \tag{41}$$

is the transfer function due to the carrier transit time in the photodiode absorption region and

$$H_{EC} = \frac{i_{p,\text{ex}}}{i_p} = \frac{1}{a_{21,PD}Z_L + a_{22,PD}} \tag{42}$$

is the current transfer function due to the photodiode parasitics. P_{opt}, Z_L, and $i_{p,\text{ex}}$ denote the incident optical power, load impedance, and current flowing through the load (see Fig. 4), respectively. The magnitude of the responsivity can be measured up to the highest frequencies, e.g., using a heterodyne measurement setup [14]. If the carrier transit time behavior of the photodiode can be calculated (for p-i-n diodes, see e.g., [15]) or simulated by a semiconductor device simulation tool, $|H_{EC}|$ can be determined from (40). Furthermore, the photodiode reflection coefficient can be measured from which the photodiode output conductance $Y_{PD} = a_{21,PD}/a_{22,PD}$ can be calculated. The photodiode intrinsic conductance is then given by

$$G_{i,PD} = \left|\frac{Y_{PD}}{H_{EC}(Z_L Y_{PD} + 1)}\right|^2 \text{Re}\left\{\frac{1}{Y_{PD}}\right\}. \tag{43}$$

V. Conclusion

A new characteristic figure for photodiodes, the so-called photodiode intrinsic conductance, has been introduced in this paper. As the equivalent input noise current is proportional and the transimpedance is inversely proportional to the square root of the photodiode intrinsic conductance, the achievable equivalent input noise current and transimpedance can be compared for different photodiodes to be used in an optical receiver independently of the front-end amplifier. It also simplifies the design of optical receivers because all methods known from microwave amplifier synthesis and design can be applied.

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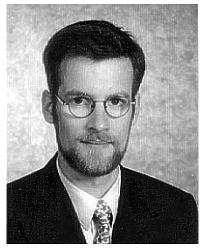

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