Determination of Simple Equivalent Circuits of Interacting Discontinuities in Waveguides or Transmission Lines

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***Abstract*—It is well known that the generalized scattering matrix (GSM) of a microwave network, which includes one or more ports supporting evanescent modes, is nonunitary. This has hindered the formation of equivalent circuits since it has not been evident how to form the impedance or admittance matrix. This paper describes how the problem has been overcome, resulting in a method for the formation of simple equivalent circuits of interacting closely spaced discontinuities in a waveguide. The n-port immittance matrix corresponding to the nonunitary GSM is formed by normalizing the immittance matrix to real or imaginary portal impedances. As an example, an equivalent circuit for the even mode of a waveguide short-slot coupler is presented, and the effect of the evanescent TE_{30} mode in the coupling region is clearly expressed by an evanescent-mode waveguide in parallel with one supporting the dominant propagating mode. The method should find wide applications to problems involving interactions in waveguides.**

***Index Terms*—Equivalent circuits, hybrids, scattering matrices, transmission-line discontinuities, waveguide analysis, waveguide discontinuities.**

I. Introduction

A RECENT paper [1] describes methods for forming equivalent circuits of waveguide obstacles and multiport junctions. These are similar to those given in [2], but are based on numerical information derived from field theory analysis programs. Using these techniques, older results may be checked and improved if necessary, and equivalent circuits may be derived for previously unconsidered problems. The methods described in [1] apply to noninteracting obstacles or discontinuities, and the purpose here is to extend these to situations where interactions are important. The technique is applicable to scattering matrices derived by **any** field theory method, not simply numerical.

Considerable work on interacting discontinuities has been carried out, e.g., [3]–[8]. The older techniques [3], [4, Fig. 9] using mode matching presented results for the interaction effects as "proximity factors." The results have proven to be very useful in some instances, but are incomplete in many respects, e.g., are uncertain when applied to pairs of closely spaced discontinuities having dissimilar dimensions. Later, authors [5]–[8] form the generalized scattering matrix (GSM) with results expressed typically as truncated infinite series of transcendental functions. Although accurate, these previous theories are much more complicated when compared to the simple equivalent circuit formulation presented here.

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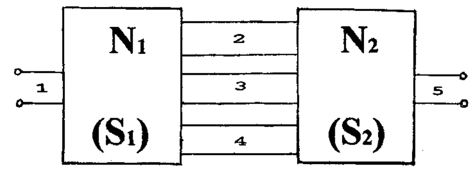

Fig. 1. Cascaded discontinuities defined by GSMs.

II. GSM

Modern numerical field analysis programs such as HFSS (available from Ansoft, Pittsburgh, PA, or Hewlett-Packard, Santa Rosa, CA), mode matching, or those based on the finite-difference time-domain (FDTD) method, are capable of deriving the GSM for microwave discontinuities. However, in addition to the propagating modes, it is necessary to specify the **accessible** modes, i.e., those which, although evanescent (i.e., below cutoff), propagate sufficiently to result in significant interaction effects. Other higher order modes are termed as **localized** and need not be specified as separate ports [6]. This does not mean that they are ignored since the field analysis program carries out an accurate calculation of the discontinuity. A separate port for an evanescent mode is specified only if it is required to characterize its interaction effect on a nearby discontinuity.

The situation is illustrated in Fig. 1, which shows discontinuities represented by networks N_1 and N_2 having GSMs $\mathbf{S}_1$ and $\mathbf{S}_2$. Three waveguides connecting N_1 and N_2 are shown in this example. Typically, there could be one propagating mode and two evanescent modes for cascaded two-port networks. Frequently, only one evanescent mode need be considered, and then there will be just two waveguides connecting N_1 and N_2, one propagating and one evanescent, as in the example given in Section III. It is also possible to have a condition where all of the connecting waveguides are evanescent, e.g., when analyzing an evanescent-mode filter.

One of the problems associated with the conventional GSM is that it is a nonunitary matrix [9], [10] i.e.,

$$\mathbf{S} \cdot \mathbf{S}^{\star} \neq \mathbf{I} \tag{1}$$

where $\mathbf{I}$ is the unit matrix.

This makes the formation of equivalent circuits unclear since it is far from obvious that the conventional equations relating the normal scattering matrix (having ports with propagating modes only) to the impedance or admittance matrices, e.g., [1, eq. (1)], are applicable. However, it is well established that the GSM may be used to analyze complex circuits consisting of cascaded discontinuities by combining the GSMs of each discontinuity as described in numerous papers, e.g., [7], [8]. This suggests the possibility of the existence of a **simple** equivalent-circuit representation. Other properties of the GSM have been described in [9], while the formation of an alternate GSM, which is unitary, has been given in [10]. In the latter case, changes to existing well-established and entrenched software packages would be required for implementation.

The problem with the conventional GSM arises in the definition of the $\mathbf{S}$ matrix for a length ℓ of evanescent waveguide as

$$\begin{bmatrix} 0 & e^{-\alpha\ell} \\ e^{-\alpha\ell} & 0 \end{bmatrix} \tag{2}$$

which is nonunitary [10].

In the case of discontinuities, which are of zero thickness or length in the direction of propagation, the GSM will not contain any submatrices such as (2). The following theory has been tested only for such cases, and is probably inapplicable to more general thick discontinuities including evanescent lengths of waveguide, which will require further investigation. Here, the modified GSM suggested by Morini and Rozzi [10] may be required to give a solution. However the complex normalization process described below would still be required.

The GSM may be converted into an impedance matrix if it exists (i.e., if it is nonsingular) using the formula [1, eq. (1)], i.e.,

$$\mathbf{Z} = \mathbf{Z_o}^{1/2} \cdot (\mathbf{I} - \mathbf{S})^{-1} \cdot (\mathbf{I} + \mathbf{S}) \cdot \mathbf{Z_o}^{1/2} \tag{3}$$

where $\mathbf{Z_o}$ is the diagonal matrix of the n-port impedances, i.e.,

$$\mathbf{Z_o} = \begin{bmatrix} Z_{o1} & 0 & 0 & \cdot & \cdot & 0 \\ 0 & Z_{o2} & 0 & \cdot & \cdot & 0 \\ 0 & 0 & \cdot & & & 0 \\ \cdot & \cdot & & \cdot & & \\ \cdot & \cdot & & & \cdot & 0 \\ 0 & 0 & \cdot & & 0 & Z_{on} \end{bmatrix} \tag{4}$$

Now, of course, the problem here is that, in the case of evanescent modes, Z_{oi} is imaginary [2, pp. 27–28], but it has now been established that (3) is still valid in such cases. We merely set

$$Z_{oi} = jX_{oi} = X_{oi}e^{j\pi/2} \tag{5}$$

and

$$Z_{oi}^{1/2} = \pm X_{oi}^{1/2} e^{j\pi/4} \tag{6}$$

where X_{oi} is real for the ith nonpropagating port. The validity of this result is entirely reasonable since assuming that an equivalent circuit with network functions expressible in terms of the complex frequency variable exists, the impedance matrix is a real function of such a variable. Therefore, it is subject to the appropriate theorems relating to the complex plane, including that of analytic continuation. Hence, the formulas involving a waveguide below cutoff are essentially the same as those above cutoff when expressed in terms of the complex frequency variable. The result is also reinforced by examples wherein all impedance matrices derived from the above theory have been found to be purely imaginary, a necessary condition that (3) represents the impedance matrix of the multiport circuit.

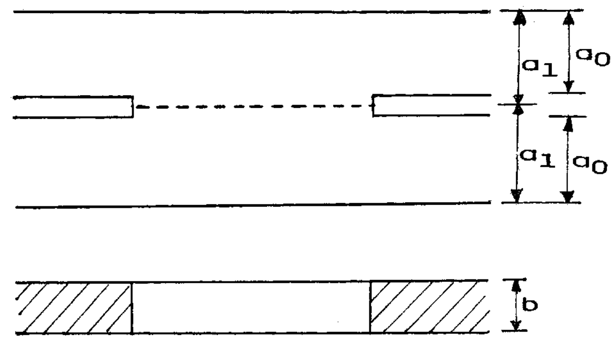

Fig. 2. Waveguide narrow-wall short-slot coupler.

Having formed this matrix (3), it may be used to derive an equivalent circuit using techniques described in [1]. In rare instances, the impedance matrix may be singular, in which case the admittance matrix may exist. Note that the impedance matrix is appropriate for representation of a parallel connection of ports, and the admittance matrix for a series connection [2]. In the unlikely event that both $\mathbf{Z}$ and $\mathbf{Y}$ matrices are singular, then the network probably consists mainly of ideal transformers, and a multiport transfer matrix, which always exists, would be derived.

An example for the case of a short-slot coupler is given here, with further details presented in [11].

III. Example: Short-Slot Waveguide Coupler (Even-Mode Circuit)

Here, a simple form of the short-slot coupler will be treated, one having no end transitions to standard waveguide dimensions, and having no central matching element, as indicated in Fig. 2. It turns out that the matching element is not required for couplings in the 4.5–7-dB range. The common wall is removed entirely in the coupling slot region, and the results are independent of the waveguide narrow dimension.

The coupler may be analyzed by forming the even- and odd-mode circuits. In the example shown in Fig. 2, the odd-mode circuit is simply a length of waveguide with small (almost insignificant) steps at the ends of the coupling region, which is easily analyzed. Hence, the coupler problem reduces mainly to the formation of the equivalent circuit of the even mode. This is assumed to support appropriate TE_{10} modes in the terminating waveguides and in the coupling region, but also the TE_{30} mode as an accessible nonpropagating mode in the coupling region. The TE_{30} mode is very significant since the wide waveguide formed by the coupling region has a width selected to locate the TE_{30} mode cutoff frequency slightly above the operating frequency band. Other higher order modes, such as TE_{50}, appear to have negligible effect, and are localized modes.

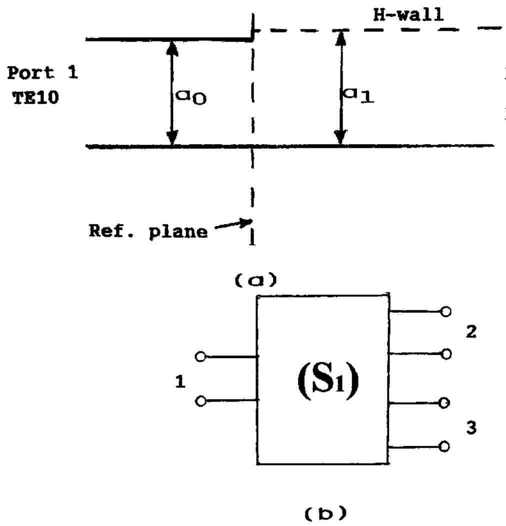

Fig. 3. (a) Even-mode junction discontinuity. (b) GSM representation of the junction discontinuity.

It should be noted that the TE_{20} mode corresponds to the odd-mode network and the magnetic wall of the even-mode circuit ensures that it is not supported in that mode.

The particular equivalent circuit to be formed is for a three-port circuit formed by the junction of an input waveguide and the even mode of the short-slot coupling region (supporting the TE_{10} and TE_{30} evanescent modes each considered as distinct waveguides), as shown in Fig. 3. The coupler considered here is intended to operate at a center frequency of 19.95 GHz and was analyzed over the 18.98–20.95-GHz band. The GSM was formed at a number of frequencies using both Ansoft and Hewlett-Packard versions of HFSS. The Hewlett-Packard version gave a GSM having a normal reciprocal GSM with $S_{ik} = S_{ki}$. The Ansoft version differed from Hewlett-Packard by phase factors of $\pm\pi/4$, for those S_{ik} terms for which i or k denote an evanescent mode (except for $i = k$). This does not represent an error in one sense since normal operations using the GSM matrices, such as combining cascaded circuits to form an overall matrix, are unaffected, the phase differences canceling out to give identical results in both Ansoft and Hewlett-Packard versions of HFSS. However, it is important to have a reciprocal matrix when forming the impedance matrix, and it was simple to make the necessary corrections to the GSM formed by Ansoft. For example, considering a three-port circuit with ports 1 and 2 propagating and port 3 evanescent, it was necessary to add $\pi/4$ to the phase of S_{31} and S_{32} and to subtract $\pi/4$ from the phases of S_{13} and S_{23}. This gave exact agreement with Hewlett-Packard.

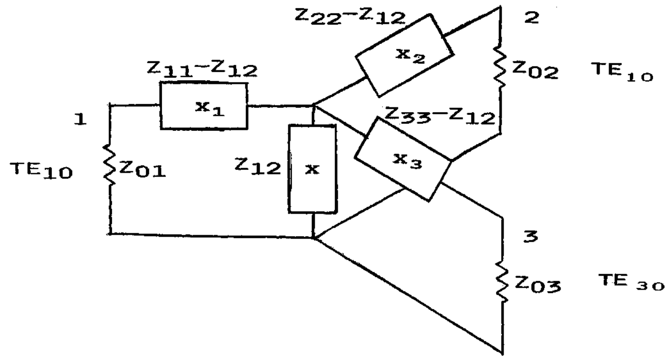

Fig. 4. Simplified three-port equivalent circuit.

In the case of the coupler of Fig. 2 having $a_o = 0.395$ in and $a_1 = 0.400$ in, i.e., a common wall thickness of 0.010 in, the following GSM was found at 19.95 GHz:

$$\begin{bmatrix} .16710e^{j103.1458} & .98594e^{j4.5733} & .43943e^{j159.1279} \\ .98594e^{j4.5733} & .16710e^{j86.0008} & .40758e^{j124.0504} \\ .43943e^{j159.1279} & .40758e^{j124.0504} & .60564e^{-j162.750} \end{bmatrix} \tag{7}$$

where the arguments of the exponents are in degrees. We note that while the entire matrix is nonunitary, the 2×2 matrix of the propagating modes (i.e., the above matrix with row and column 3 deleted) is unitary. Actually, the reduced matrix represents the discontinuity ignoring the effect of the TE_{30} mode.

It is instructive to retrace the operations that led to the formulation of the theory. Initially, it was decided to form the normalized impedance matrix given by the formula

$$\mathbf{Z_1} = (\mathbf{I} - \mathbf{S})^{-1} \cdot (\mathbf{I} + \mathbf{S}). \tag{8}$$

Using $\mathbf{S}$, given by (7), the normalized impedance matrix becomes (9), shown at the bottom of this page. Now the impedance matrix of a lossless network must be purely imaginary, which clearly (9) is not. An immediate simplification is to change the arguments, which are almost 0°, 90°, and 225° to those exact values. The deviations represent roundoff errors in the data since only four or five significant digits were used.

It was immediately noted that the Z_{13} and Z_{23} terms could be made imaginary if modified by a phase of 45° and that Z_{33} should be modified by a phase factor of 90°. This led to the conclusion that the matrix had to be normalized with respect to the complex characteristic impedance of the nonpropagating port 3, and that one should use (3) where here the square root of the diagonal matrix (4) is

$$\mathbf{Z_o}^{1/2} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{-j45} \end{bmatrix} \tag{10}$$

$$\begin{bmatrix} 26.0371e^{j89.9984} & 24.9482e^{j89.9962} & 13.0541e^{j224.9973} \\ 24.9482e^{j89.9962} & 24.0075e^{j89.9942} & 12.3760e^{j224.9948} \\ 13.0541e^{j224.9973} & 12.3760e^{j224.9948} & 6.7214e^{-j0.0039} \end{bmatrix} \tag{9}$$

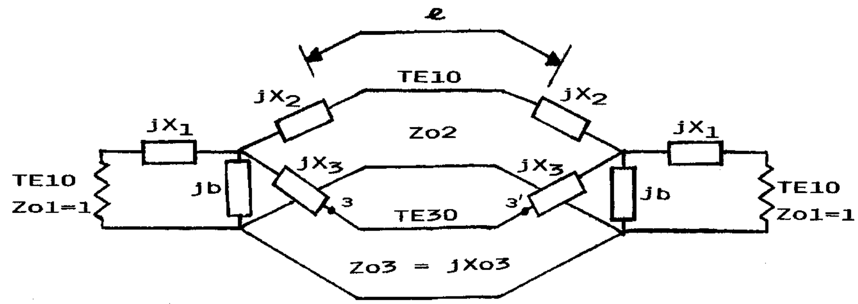

Fig. 5. Equivalent circuit of the even-mode circuit for the coupler, consisting of the two junctions spaced by waveguides of length ℓ.

the resulting normalized impedance matrix becomes

$$\mathbf{Z_n} = j \cdot \begin{bmatrix} 26.0371 & 24.9482 & 13.0541 \\ 24.9482 & 24.0075 & 12.3760 \\ 13.0541 & 12.3760 & 6.7214 \end{bmatrix} \qquad (11)$$

which is purely imaginary, as required.

The validity of this impedance matrix was shown by applying it to the analysis of the even-mode circuit and comparing the results to those computed by well-known procedures using the GSM, e.g., [7], [8], and also by direct HFSS analysis, giving identical results, as described below in more detail.

Having derived the impedance matrix of the three-port, it is desirable to find an equivalent-circuit representation. This has been described in [1], and the relevant parts of the procedure will be reproduced here for completeness.

The direct representation of the three-port impedance matrix as given in [2, Fig. 3.3] is very complicated and not really of direct use. It was shown in [1] that a simple equivalent circuit results if the port impedances are scaled to give

$$Z_{12} = Z_{13} = Z_{23}. \qquad (12)$$

The scaling is carried out mathematically by an operation similar to that implied by (3), i.e., matrix (11) is pre- and post-multiplied by the square root of diagonal matrices

$$\begin{bmatrix} Z_{o1} & 0 & 0 \\ 0 & Z_{o2} & 0 \\ 0 & 0 & Z_{o3} \end{bmatrix}. \qquad (13)$$

The result is that the elements of the original matrix (11) denoted by Zn_{ij} become

$$Z_{ij} = Zn_{ij} \cdot (Z_{oi} \cdot Z_{oj})^{1/2}. \qquad (14)$$

Note that, at this stage, Z_{o3} may be a real number since the imaginary normalization has already been carried out by the operation given by matrix (10), leading to (11).

It is convenient to take $Z_{o1} = 1$ without loss of generality. Conditions (12) are then satisfied if we take

$$Z_{o2} = (Zn_{13}/Zn_{23})^2 \quad \text{and} \quad Z_{o3} = (Zn_{12}/Zn_{23})^2. \qquad (15)$$

The equivalent circuit now takes the simple form given in Fig. 4. In the case of matrix (11), we find

$$Z_{o2} = 1.11258 \quad Z_{o3} = jX_{o3} = 4.06366 \qquad (16)$$

and this impedance matrix becomes

$$j \begin{bmatrix} 26.0371 & 26.3149 & 26.3149 \\ 26.3149 & 26.7100 & 26.3149 \\ 26.3149 & 26.3149 & 27.3132 \end{bmatrix}. \qquad (17)$$

The element values of Fig. 4 for the case of matrix (17) are

$$\begin{aligned} X_1 &= -0.2778 \\ X_2 &= 0.3951 \\ X_3 &= 0.9983 \\ X &= Z_{12} = 26.3149 \end{aligned} \qquad (18)$$

and the susceptance to ground is jB, where $B = -1/X = -0.038$.

We can now form the equivalent circuit of the entire even mode of the slot coupler, shown in Fig. 5, and consisting of two equivalent circuits, as in Fig. 4, with ports 2 and 3 of each such equivalent circuit connected by appropriate transmission lines having the derived characteristic impedances (16). The connecting lines have characteristic impedances Z_{o2} (real) and jX_{o3}, and the propagation constants are those of the TE_{10} mode for line 2 and for the TE_{30} mode for line 3, the latter being given by the real attenuation coefficient α. Hence, the transfer matrix of line 3 of length ℓ is

$$\begin{bmatrix} \cosh \alpha\ell & jX_{o3} \sinh \alpha\ell \\ -j(\sinh \alpha\ell)/X_{o3} & \cosh \alpha\ell \end{bmatrix} \qquad (19)$$

and the analysis of the circuit is quite straightforward. As stated previously, this analysis gives results identical to those obtained from an HFSS analysis of the entire even-mode circuit.

The effect of the TE_{30} mode is now clearly delineated. For example, its effect is less important for long slots or widely separated discontinuities because of the increased attenuation of the line connecting ports 3 and 3′ with increasing slot length. In this example, it has been found that the elements of the equivalent circuit have smooth monotonic variations with frequency over the 18.95–20.95-GHz band. Further discussion of the short-slot coupler is given in [11].

One of the advantages of obtaining the equivalent circuit is a reduction by a factor of two of the number of variables compared with the scattering matrix, where there are six complex numbers rather than six single-valued numbers. Theoretically, the scattering matrix also has only six independent variables since the complex variables are constrained by certain conditions on the GSM [9], [10], but such conditions are not always expressible in simple terms. Also, the scattering matrix gives little information relating to an equivalent circuit—generation of an immittance matrix is required.

The theory has wide implications since it may be applied to any problem where interaction between neighboring obstacles is important. As previously stated, the theory is also applicable to problems having more than one interacting accessible evanescent mode.

IV. Conclusion

GSMs derived from numerical field analysis programs such as HFSS may be used to generate equivalent circuits when the GSM is correctly normalized to the real and reactive (or imaginary) port impedances. The equivalent circuits are expected to give much clearer physical insight into the form of interaction between neighboring discontinuities. Compared with the scattering matrix, the circuit effectively reduces by a factor of two the number of variables used to characterize the circuit.

The method was applied to characterize the even-mode circuit of a waveguide slot coupler, giving results identical to those derived by direct numerical analysis.

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