Input Impedance of a Coaxial-Line Fed Probe in a Thick Coaxial-Line Waveguide

Edward Kai-Ning Yung, *Senior Member, IEEE*, Ze Ming Xie, and Ru Shan Chen, *Member, IEEE*

***Abstract*—The Green's functions for determining the electromagnetic fields inside a semiinfinite coaxial line due to a radially directed, infinitesimally thin, and short-current element have been derived. In addition to the TEM mode, TE and TM modes are also considered. Based on the Green's functions, a closed-form formula for determining the input impedance of a probe in a coaxial line terminated at an arbitrary load has been derived. Good agreement is observed between the theoretical results and experimental measurements over a wide frequency band for several configurations of interest. At low frequencies where the TEM mode is dominating, there is practically no difference between the results obtained by the rigorous analysis and those by a simple formula derived from the transmission-line theory. However, at frequencies where TE and TM modes are no longer insignificant, there is a noticeable discrepancy between the results obtained by the rigorous and not-so-rigorous methods.**

***Index Terms*—Coaxial line, Green's functions, input impedance, junctions, probe.**

I. Introduction

ANALYSIS OF a probe inside a waveguide is a classical problem. The basic techniques for finding its input impedance are given in most textbooks of microwave engineering [1], [2]. Having said that, rigorous evaluations of the input impedance of a probe in a rectangular waveguide [3], [4], cylindrical waveguide [5], and cavity [6] can only be found in recent literature. Based on the dyadic Green's functions, the input impedance of a probe in a rectangular waveguide loaded by a multilayered medium has been determined [7], [8]. On the other hand, experimental studies of a probe in a rectangular waveguide have been conducted in earlier years [9], [10]. Ironically, neither theoretical, nor experimental study of the excitation of the most commonly used waveguide (namely, the coaxial-line waveguide) has been reported in recent years.

The configuration of concern is a semiinfinite coaxial-line waveguide. Compared with a hollow one, a coaxial-line waveguide provides a better platform for studying the characteristics of a transverse electromagnetic wave propagating in a dielectric medium, including a complex or a compound one, at radio frequencies. By cutting a circumferential slot on its surface, the waveguide can be transformed into a dipole antenna, a widely used antenna in a base station of a mobile communications system. By drilling holes on its outer surface, the coaxial line can be converted into a leaky-wave antenna for uses in an indoors wireless communications system. Hence, an in-depth study of a thick coaxial line is of paramount interest in the dawning of the information age.

In some applications, a coaxial line is fed by a much thinner coaxial line connected longitudinally at one of its ends. Preferably, the characteristic impedance of the thicker coaxial line is made identical to that of the feeding line readily available in the market, say, 50 Ω. However, when the radii of the thicker line are manifold bigger than their counterparts in the thinner one, a tapered coaxial line is required to effect a better impedance matching over a wide frequency band. At very high frequency (VHF) and UHF frequencies, the length of this transition could be very long and expensive. To address this problem, it is proposed to feed the waveguide by a probe through a hole on its outer surface. At frequencies when the TEM mode is dominating, the input impedance of this three-port network can be determined by a simple formula derived from the basic transmission-line theory. Unfortunately, the effects of the higher order modes are more and more significant as the frequency increases. In this connection, a better excitation model has been contemplated and an efficient technique developed for a rigorous analysis.

II. Formulation

The coaxial-line waveguide of concern is perfectly shorted at one end and terminated at a matched load at the other, as depicted in Fig. 1. Various dimensions of the waveguide and coordinate systems are also defined in Fig. 1. The waveguide is filled by a lossless nonmagnetic dielectric with a relative permittivity of ε_r. It is excited by a radially directed probe through a hole on its outer surface. The probe itself is the protruded inner conductor of a much thinner coaxial line. For better efficiency, the probe is extended all the way to the inner conductor of the thicker coaxial line and is soldered onto it. The probe is placed approximately a quarter-wavelength from the shorted end.

As mentioned previously, the basic method to analyze a probe in a waveguide can be found in [1]. An equivalent circuit for analyzing a T-junction coaxial line has been developed in [11]. However, the latter method is not applicable because the parameters there were obtained by an experiment conducted at 3 GHz. Here, we are looking for a more rigorous analytical method. In essence, the technique used to derive a closed-form formula is an extension of that employed in determining the input impedance of a coaxial-line fed probe in a hollow cylindrical waveguide given in [5].

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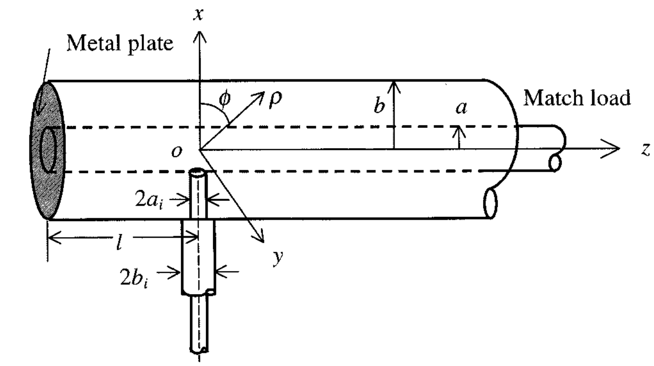

Fig. 1. Thick coaxial-line waveguide fed by a radially directed probe.

The radial component of the electric-type dyadic Green's functions at (ρ, ϕ, z) due to a ρ-directed impulse current element located at (ρ', ϕ', z') can be expressed as

$$\begin{aligned} G_{\rho\rho} = &\sum_{n=0}^{\infty}\sum_{m=1}^{\infty} \frac{j\alpha_{nm} S_n'(\gamma_{nm}\rho')\sinh[\alpha_{nm}(z'+l)]}{2\pi\sigma_{0n} w_{nm}\omega\varepsilon} \\ &\cdot S_n'(\gamma_{nm}\rho)\cos n(\phi-\phi')e^{-\alpha_{nm}(z+l)} \\ &- \frac{j\omega\mu}{\rho}\sum_{n=0}^{\infty}\sum_{m=1}^{\infty} \frac{n^2\alpha_{nm}' T_n(\gamma_{nm}'\rho')\sinh[\alpha_{nm}'(z'+l)]}{w_{nm}'\rho'} \\ &\cdot T_n(\gamma_{nm}'\rho)\cos n(\phi-\phi')e^{-\alpha_{nm}'(z+l)} \\ &+ \frac{\omega\varepsilon k\sinh[jk(z'+l)]}{w_{\mathrm{TEM}}\rho'}\frac{1}{\rho}e^{-jk(z+l)}, \qquad z' \le z \end{aligned} \tag{1}$$

$$\begin{aligned} G_{\rho\rho} = &\sum_{n=0}^{\infty}\sum_{m=1}^{\infty} \frac{j\alpha_{nm} S_n'(\gamma_{nm}\rho')e^{-\alpha_{nm}(z'+l)}}{2\pi\sigma_{0n} w_{nm}\omega\varepsilon} \\ &\cdot S_n'(\gamma_{nm}\rho)\cos n(\phi-\phi')\sinh[\alpha_{nm}(z+l)] \\ &- \frac{j\omega\mu}{\rho}\sum_{n=0}^{\infty}\sum_{m=1}^{\infty} \frac{n^2\alpha_{nm}' T_n(\gamma_{nm}'\rho')e^{-\alpha_{nm}'(z'+l)}}{w_{nm}'\rho'} \\ &\cdot T_n(\gamma_{nm}'\rho)\cos n(\phi-\phi')\sinh[\alpha_{nm}'(z+l)] \\ &+ \frac{\omega\varepsilon k e^{-jk(z'+l)}}{w_{\mathrm{TEM}}\rho'}\frac{1}{\rho}\sinh[jk(z+l)], \qquad -l < z < z' \end{aligned} \tag{2}$$

where

$$S_n(\gamma_{nm}\rho) = [N_n(\gamma_{nm}a)J_n(\gamma_{nm}\rho) - J_n(\gamma_{nm}a)N_n(\gamma_{nm}\rho)] \tag{3}$$

$$T_n(\gamma_{nm}'\rho) = [N_n'(\gamma_{nm}'a)J_n(\gamma_{nm}'\rho) - J_n'(\gamma_{nm}'a)N_n(\gamma_{nm}'\rho)] \tag{4}$$

$$w_{\mathrm{TEM}} = -2\pi\omega^2\varepsilon^2\ln\frac{b}{a} \tag{5}$$

$$w_{nm} = \int_a^b \rho S_n^2(\gamma_{nm}\rho)\,d\rho \tag{6}$$

$$\begin{aligned} w_{nm}' = &\int_a^b 2\pi\gamma_{nm}'^2\alpha_{nm}'^2\sigma_{0n}T_n'^2(\gamma_{nm}'\rho)\rho\,d\rho \\ &+ \int_a^b 2\pi n^2\alpha_{nm}'^2\sigma_{0n}T_n^2(\gamma_{nm}'\rho)\frac{1}{\rho}\,d\rho. \end{aligned} \tag{7}$$

For the above equations, γ_{nm} and γ_{nm}' are the mth roots of $S_n(\gamma_{nm}b) = 0$ and $T_n(\gamma_{nm}'b) = 0$, respectively. The propagation constants are $\alpha_{nm} = \sqrt{\gamma_{nm}^2 - k^2}$, $\alpha_{nm}' = \sqrt{\gamma_{nm}'^2 - k^2}$, and $k = 2\pi/\lambda$, while σ_{0n} is the short-handed notation for

$$\sigma_{0n} = \begin{cases} 1, & n = 0 \\ 1/2, & n > 0. \end{cases} \tag{8}$$

In this paper, the probe is very thin; therefore, the probe current can be treated as a line current. Hence, the input impedance can be simplified into the following form:

$$\begin{aligned} Z_{\mathrm{in}} = \frac{-1}{(2\pi I_{\mathrm{in}})^2}\int_0^{2\pi}\int_0^{2\pi}\int_a^b\int_a^b I(\rho)G_{\rho\rho}(\rho,\phi,z;\rho',\phi',z') \\ \cdot I(\rho')\,d\rho'\,d\rho\,d\theta'\,d\theta \end{aligned} \tag{9}$$

where I_{in} is the current on the probe at the feeding point $\rho = b$.

Provided that the current distribution on the probe is known, Z_{in} can be determined by evaluating the integral given in (9). For a dangling probe, a sinusoidal distribution is assumed, but for a fully extended probe, a fairly uniform distribution is expected. This approximation is valid as long as the difference between the outer and inner radii is much smaller than a wavelength, i.e., $b - a \ll \lambda$. It then follows that Z_{in} can be expressed in terms of its modal components

$$Z_{\mathrm{in}} = Z_{\mathrm{TEM}} + \sum_{n=0}^{\infty}\sum_{m=1}^{\infty} Z_{\mathrm{TM}}^{nm} + \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} Z_{\mathrm{TM}}^{nm} \tag{10}$$

where

$$Z_{\mathrm{TEM}} = \left[\sin^2(kl) + j\sin(kl)\cos(kl)\right]\frac{1}{2\pi}\sqrt{\frac{\mu}{\varepsilon}}\ln\frac{b}{a} \tag{11}$$

$$\begin{aligned} Z_{\mathrm{TM}}^{nm} = &\,j\frac{(-1)\alpha_{nm}(1 - e^{-2\alpha_{nm}l})}{4\pi\sigma_{0n}w_{nm}\omega\varepsilon} \\ &\cdot \int_a^b S_n'(\gamma_{nm}\rho)\,d\rho\int_a^b S_n'(\gamma_{nm}\rho')F(\alpha_{nm},\rho')\,d\rho' \end{aligned} \tag{12}$$

$$\begin{aligned} Z_{\mathrm{TM}}^{nm} = &\,j\frac{\omega\mu n^2(1 - e^{-2\alpha_{nm}'l})\alpha_{nm}'}{2w_{nm}'} \\ &\cdot \int_a^b \frac{T_n(\gamma_{nm}'\rho)}{\rho}\,d\rho\int_a^b \frac{T_n(\gamma_{nm}'\rho')}{\rho'}F(\alpha_{nm}',\rho')\,d\rho'. \end{aligned} \tag{13}$$

For the above equation, $F(\alpha, \rho)$ is defined by

$$F(\alpha,\rho) = \frac{1}{2\pi}\int_0^{2\pi} e^{-\alpha a_i|\sin(\theta)|}\cos\left[n\tan^{-1}\left(\frac{a_i\cos\theta}{\rho}\right)\right]\,d\theta. \tag{14}$$

Note that Z_{TEM} is identical to the input impedance of a probe in a semiinfinite coaxial line, as derived from the basic transmission-line theory. Moreover, both Z_{TM}^{nm} and Z_{TE}^{nm} are pure imaginary because they account for the contributions of the evanescent modes that support no energy propagation.

III. Analytical Results and Experimental Measurements

The input impedance of the coaxial line excited by a coaxial-line fed probe is obtained by evaluating (10) and the associated integrals given in (12)–(14). The variation of Z_{in}

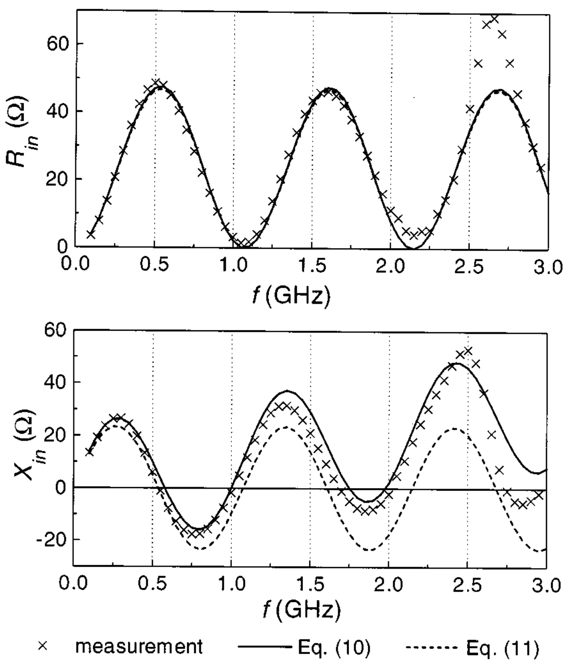

Fig. 2. Input impedance of the probe in a waveguide terminated at a matched load as a function of frequency for $l = 139.8$ mm, $a = 3.13$ mm, $b = 6.93$ mm, $a_i = 0.50$ mm, and $b_i = 2$ mm.

with respect to frequency is sketched in Fig. 2 in the range of 100 MHz to 3 GHz, for $a = 3.13$ mm, $b = 6.93$ mm, $l = 139.8$ mm, and $\varepsilon_r = 1$. In this frequency range, the difference of the radii varies from 0.00126λ to 0.0378λ; therefore, the short probe assumption is always valid. The input resistance reaches its first peak at 540 MHz, at which the probe is placed one quarter-wavelength from the shorted end. The input resistance reaches its second and third peaks when l is approximately equal to three and five quarter-wavelengths. Since $Z_{\rm TM}^{nm}$ and $Z_{\rm TE}^{nm}$ are no longer insignificant at these frequencies, there are noticeable discrepancies between the input reactance obtained by (10) and (11) only.

In order to verify the validity of the analytical technique, an experimental study has been conducted. A long coaxial line with the aforementioned radii is constructed. It is fed by the extended inner conductor of a thin 50-Ω coaxial cable, via an SMA connector mounted on the outer surface of the thicker line. The matched load is facilitated by connecting a 50-Ω terminator longitudinally to the other end of the coaxial waveguide through an N-type socket. Using an HP8753C network analyzer, the input impedance of the probe is measured. For easy comparison, the experimental data are also plotted in Fig. 2. At VHF frequencies, the experimental figures, the analytical results obtained by (10) and (11) are close, within the acceptable error of measurement. On the other hand, reasonably good agreement between the analytical results obtained by the rigorous method and measurements are observed until the frequency reaches 2.5 GHz. Beyond that, the difference between the theoretical and experimental results is not reconcilable. Since the theoretical derivation contains no assumption that may cause an error of this magnitude, the discrepancy must be related to the experimental setup.

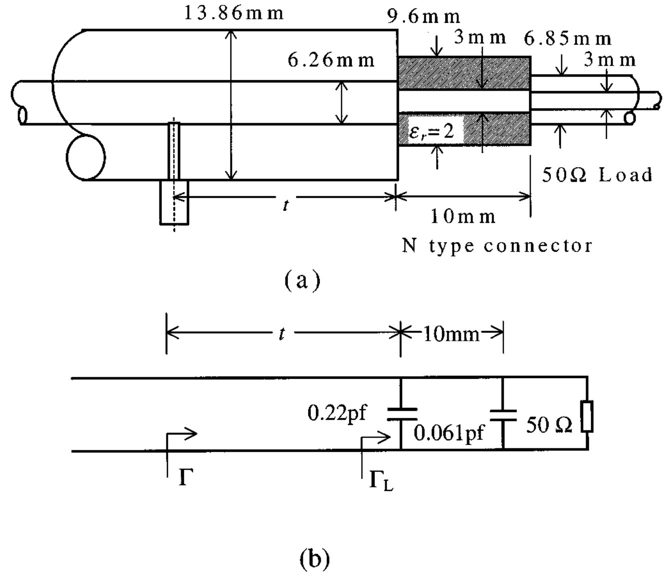

Fig. 3. Coaxial line terminated at a 50-Ω load via an N-type connector.

After an in-depth study of the experimental setup, it is realized that the error is probably due to the "matched load." As depicted in Fig. 3(a), the thick coaxial line is twice as big as the N-type connector, and the latter is also twice as big as the 50-Ω load. The abrupt change of this magnitude in both radii inevitably causes wave reflections. It is equivalent to a shunting capacitance load. Based on the techniques given in [12], the capacitance can be determined, and the equivalent circuit of the "matched load" is shown in Fig. 3(b). Due to an unmatched load, the wave is reflected and the input impedance of the probe must be modified to

$$Z_{\rm in}^{L} = Z_{\rm TEM}^{L} + \sum_{n=0}^{\infty}\sum_{m=1}^{\infty} Z_{\rm TM}^{nm} + \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} Z_{\rm TM}^{nm} \tag{15}$$

$$Z_{\rm TEM}^{nm} = \frac{1}{2\pi}\sqrt{\frac{\mu}{\varepsilon}}\ln\frac{b}{a} \cdot \frac{(1-\Gamma^2)\sin^2(\beta l) + j(1+\Gamma)^2\sin(\beta l)\cos(\beta l)}{(1+\Gamma)^2\cos^2(\beta l) + (1-\Gamma)^2\sin^2(\beta l)} \tag{16}$$

in which Γ is the reflection coefficient, as observed by the probe, given by $\Gamma = \Gamma_L e^{-j2\beta t}$. The input impedance of a probe in a coaxial-line waveguide terminated at an arbitrary load, as given by (15), is plotted in Fig. 4 as a function of frequency. Very good agreement between the theoretical and experimental results is restored even at high frequencies.

Other than frequency, the only variable in the present setup is l: the separation between the probe and the shorted end. Hence, the input impedance of the probe obtained by (15) is plotted in Fig. 5 as a function of l at $f = 1$ GHz. Excellent agreement between the theoretical results and experimental data is observed. Also plotted in Fig. 5 are the results obtained by the transmission-line theory, as given by (11). Here, the superiority of the rigorous method is clearly demonstrated. It is worthwhile to mention that at $l = 75$ mm (a quarter-wavelength), the input impedance obtained by (15) is $(42.8 + j11.72)$ Ω, and by (11)

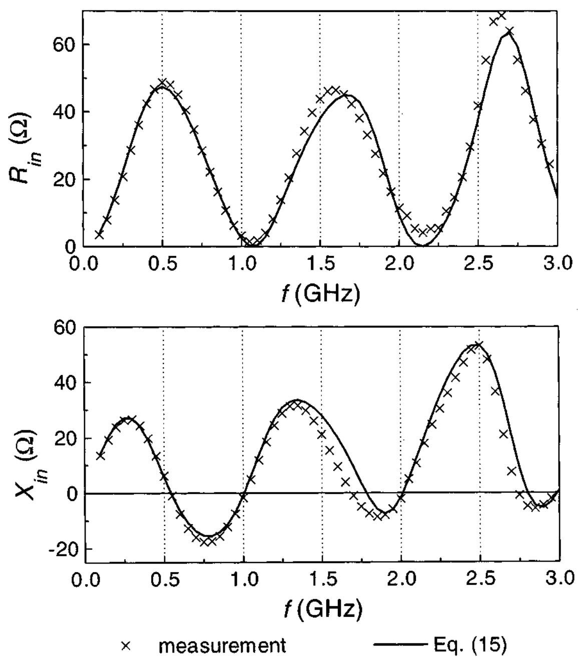

Fig. 4. Input impedance of the probe in a waveguide, as given by (15), as a function of frequency for $l = 139.8$ mm, $a = 3.13$ mm, $b = 6.93$ mm, $a_i = 0.50$ mm, and $b_i = 2$ mm.

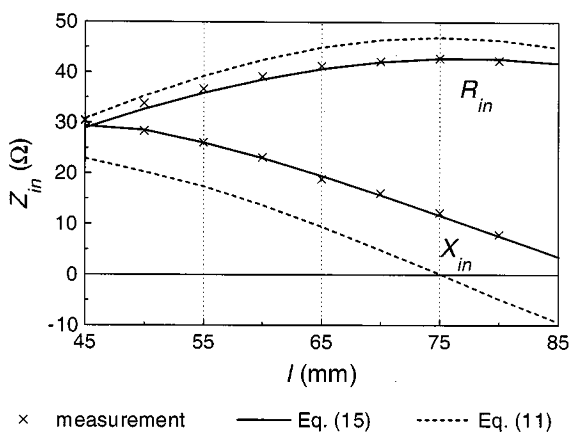

Fig. 5. Input impedance of the probe in a waveguide, as given by (15), as a function of probe location for $f = 1$ GHz, $a = 3.13$ mm, $b = 6.93$ mm, $a_i = 0.50$ mm, and $b_i = 2$ mm.

it is $(47 + j0)\ \Omega$. In other words, the contributions due to $Z_{\rm TM}^{nm}$ and $Z_{\rm TE}^{nm}$ cannot be ignored even at $f = 1$ GHz.

IV. Concluding Remarks

In this paper, a closed-form formula has been derived for determining the input impedance of a probe in a coaxial-line waveguide terminated at an arbitrary load. Excellent agreement between the theoretical results and experimental data has been observed. The formula yields much better results than those obtained by the transmission-line theory. It is also unveiled that the higher order modes cannot be ignored, even at low frequencies.

In deriving the formula given in (10), a constant current distribution on the probe is assumed. If this current is treated as an unknown, the Green's functions given in (1) and (2) can be used to determine the electric field on the surface of the antenna, i.e., the probe. The unknown current distribution can then be evaluated by enforcing that the electric field be vanishingly small on the surface of the antenna, and by solving the equation by the method of moments. The latter step is bypassed because the processes involved are very tedious and laborious. As is illustrated in Fig. 5, the constant current distribution is an excellent assumption that yields extremely good results. Thus, we conclude that the closed-form formula given in (15) is adequate for nearly all applications.

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