

# Short Papers

## An Error in the *Waveguide Handbook*

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**Abstract**—This paper corrects a long-standing error that appears in a formula given in the literature for the admittance of a parallel-plate line radiating into a half-space.

**Index Terms**—Equivalent circuits, parallel-plate transmission line.

### I. INTRODUCTION

In this paper, a correction is provided to a long-standing error in the susceptible component of a published formula [1] for the normalized admittance presented to a parallel-plate line when it radiates into a half-space. Numerical values of both parts of the admittance are provided for the range of frequencies extending up to the onset of the first higher order mode. The paper discusses a means for circumvention of difficulties that arise from a logarithmic singularity, which lies in the range of an integral in terms of which the formula is presented.

### II. THE CORRECTION

We have recently discovered an error in one of the formulas that appears in [1, p. 184, eq. (2a)]. It relates to the susceptible component of the admittance of a parallel-plate waveguide radiating into a half-space ( $E$ -plane), where [1, eq. (2a)] should read

$$\frac{B}{Y_o} = - \int_0^{kb} N_o(x) dx + N_1(kb) + \frac{2}{\pi} \frac{1}{kb}. \quad (1)$$

The error is in the omission of the minus sign before the integral on the right-hand side. However, the result for the conductive component, given in [1, p. 184, eq. (1a)] is correct as printed. Interestingly, so is the graph on [1, p. 185], which correctly displays the functional forms of both  $G$  and  $B$ .

The integral in the susceptible component is not one easily reduced in terms of a few simple known functions, and since the integrand has a logarithmic singularity at the lower limit of integration, is not absolutely trivial to evaluate, even numerically. Due to the limited range of arguments  $kb$  that are of interest, we chose to integrate the polynomial approximation for a zeroth-order Neumann function given in [2, eq. (9.4.2)]. This explicitly exposes the logarithmic term and allows it to be handled analytically and separately from the rest of the approximating polynomial.

On this basis, we present Table I, which lists normalized conductances and susceptances for  $b/\lambda < 0.48$  and should spare any user requiring numerical values the need to follow a similar course. For the application that we had in mind, this is sufficient. Cutoff of the  $TM_{01}$  mode corresponds to a half-wavelength plate separation, meaning that

TABLE I  
NORMALIZED ADMITTANCE FOR PARALLEL-PLATE WAVEGUIDE  
RADIATING IN A HALF-PLANE

$b/\lambda$	$G/Y_o$	$B/Y_o$	$b/\lambda$	$G/Y_o$	$B/Y_o$
0.01	0.0314	0.0870	0.25	0.7095	0.5221
0.02	0.0628	0.1468	0.26	0.7318	0.5201
0.03	0.0941	0.1961	0.27	0.7535	0.5173
0.04	0.1253	0.2385	0.28	0.7746	0.5136
0.05	0.1564	0.2756	0.29	0.7950	0.5092
0.06	0.1874	0.3084	0.30	0.8147	0.5040
0.07	0.2181	0.3377	0.31	0.8338	0.4982
0.08	0.2487	0.3640	0.32	0.8521	0.4918
0.09	0.2790	0.3874	0.33	0.8698	0.4848
0.10	0.3090	0.4084	0.34	0.8867	0.4772
0.11	0.3388	0.4272	0.35	0.9030	0.4692
0.12	0.3682	0.4439	0.36	0.9185	0.4607
0.13	0.3972	0.4586	0.37	0.9333	0.4517
0.14	0.4259	0.4716	0.38	0.9474	0.4424
0.15	0.4542	0.4829	0.39	0.9608	0.4327
0.16	0.4820	0.4926	0.40	0.9735	0.4228
0.17	0.5094	0.5008	0.41	0.9855	0.4125
0.18	0.5363	0.5077	0.42	0.9968	0.4020
0.19	0.5627	0.5132	0.43	1.0074	0.3913
0.20	0.5886	0.5174	0.44	1.0173	0.3805
0.21	0.6139	0.5205	0.45	1.0266	0.3695
0.22	0.6387	0.5225	0.46	1.0352	0.3584
0.23	0.6629	0.5233	0.47	1.0430	0.3472
0.24	0.6865	0.5232	0.48	1.0503	0.3360

Table I covers more or less all of the range for which the line is guaranteed to convey only a TEM mode.

The approximating polynomial begins to lose accuracy for plate separations exceeding  $0.48\lambda$ . However, should one wish to extend Table I to greater separations, [2, eq. (9.4.3)] provides a large argument formula for the Neumann function, giving seven-figure accuracy for any argument greater than three. Since it is not in the form of a simple polynomial and, thus, is not so simply integrated analytically as is the small argument formula, for this case, numerical integration would be simpler with the range of integration broken at  $b/\lambda = 0.48$  and with Table I used to provide the first segment.

### III. CONCLUSION

This paper has corrected a long-standing error that appears in [1, p. 184, eq. (2a)] for the susceptible component of the admittance of a parallel-plate line radiating into a half-space, and provides a table (see Table I) of numerical values for both the real and imaginary parts of the admittance, which covers the full range of purely TEM-mode propagation in the line.

### REFERENCES

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